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Allocating raw materials to competing projects

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ABSTRACT

This paper considers the problem of material allocation to competing self-interested agents. A novel resource allocation model is presented and studied in a mechanism design setting without using money as incentive. The novelties and specialties of our contribution include that the materials are supplied at different dates, the jobs requiring them are related with precedence relations, and the utilities of the agents are based on the tardiness values of their jobs. We modify a classical scheduling algorithm for implementing the Serial Dictatorship Mechanism, which is then proven to be truthful and Pareto-optimal.

1. Introduction

Recently, there has been a growing interest in game theoretical analysis in the supply chain management and scheduling research communities. For large-scale manufacturing systems embedded in a strategic setting, agents often possess private information, and since they are self-interested, they intend to manipulate the outcome of the system for their benefit. Allocation of multiple goods or resources is a frequently studied optimization problem of this sort. When the protocol controlling the system behavior includes monetary transfers, like in case of supply chains, setting the payments appropriately can be used to make manipulations ineffective (see e.g., Egri & Váncza, 2012). If such transfers are not allowed, usually only dictatorial mechanisms can prevent manipulations (see e.g., Abdulkadroğlu & Sönmez, 1998). In this paper, we study this latter situation specialized for an industrial project scheduling application. The novelties and specialties of our contribution are: (i) the raw materials are supplied over the scheduling time horizon at different dates, (ii) the jobs requiring the materials are related with precedence relations, and (iii) the utilities of the agents are not arbitrary, but based on the tardiness values of their jobs.

More specifically, we consider a project scheduling problem with *non-renewable* (consumable) resource constraints, where each project is owned by a self-interested agent. The projects consist of *jobs* that compete for commonly used resources (materials). In this paper we consider only raw materials, but other non-renewable resources are also conceivable, such as energy and money (Gafarov, Lazarev, & Werner, 2011; Grigoriev, Holthuijsen, & van de Klundert, 2005). Even computational resources—such as CPU, memory and network bandwidth—are frequently modeled as consumable resources in cloud infrastructures (see e.g., Kash, Procaccia, & Shah, 2014). The materials are consumed by the jobs and they have an initial stock which is replenished over time at given dates and in known quantities. The jobs have to be executed while meeting *precedence* and *resource constraints*. That is, each job may have some predecessors, all of which have to be completed prior to starting the job, and it may require some materials which have to be on stock when starting the job. Once the job is started, the stock levels of the respective resources are decreased by the required quantities. The stock levels can never be negative, so if the initial stock of some resource is not enough to complete all the jobs, some of them have to be delayed in order to meet the resource constraints. Each project has a *due date*, and if it is completed afterwards, it will be tardy. A *schedule* specifies the start time of each job, and it is *feasible* if all the precedence and resource constraints are satisfied, see Fig. 1 for an illustration. The figure depicts a schedule (on top) and the corresponding resource consumption (on the bottom). The schedule of the jobs is represented by dark rectangles where their left and right ends correspond to the start and end times. For the sake of intelligibility, the jobs of the projects are separated vertically, and different light rectangles contain the jobs of different projects. The thin arrows indicate the precedence relations between the jobs (in a feasible schedule the arrows always point to the right), while the thin vertical lines show the times of supply. Thick vertical lines mark the due dates of the projects, and to the right of these lines, the background is diagonally hatched in order to indicate that any job in this area is tardy. Thick, two-ended arrows denote the tardiness of the projects (if exist). In addition, Fig. 1 shows the cumulative demand of a resource implied by the schedule as well as its supply.

Throughout the paper we assume that the total supply from each resource equals the total demand in order to guarantee the existence of feasible schedules. Since the materials are replenished over time, it is not obvious when to start the jobs when some optimization criteria are involved. In the basic problem (where all data is publicly known, and

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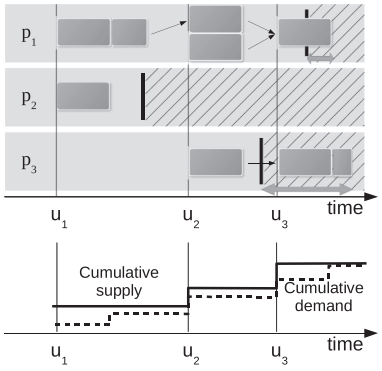


Fig. 1. A sample schedule with 3 projects and a single resource.

there are no selfish agents) a feasible schedule is sought in which the maximal tardiness among the projects is as small as possible. This problem can be efficiently solved optimally by the method of Carlier et al. (Apr. 1982).

In a multi-agent environment, projects are owned by self-interested rational agents which act autonomously to achieve their own goals. The due date of a project is only known by the corresponding agent. Further on, there is a *central inventory*, which allocates the materials to the jobs of the projects over time. However, there is also a conflict of interests: while the central inventory still aims at minimizing the maximum tardiness over all projects[1], each agent is interested in minimizing only its own tardiness. Therefore, the agents are competing for the resources, and they are inclined to be untruthful about their due dates in hope of achieving a more advantageous resource allocation for themselves. It is the central inventory, who can inspire the agents to tell their true due dates by using a *truthful allocation mechanism*, which ensures that reporting the true due dates yields the best outcome for each agent.

Main results of this paper. We investigate truthful mechanisms without payments for the above project scheduling problem. We will show that there exists no truthful mechanism that always finds an optimal[2] solution. After this, we describe the *Serial Dictatorship Mechanism* (SDM), which is truthful, and always finds a Pareto-optimal solution. Our SDM is based on the polynomial time procedure of Carlier et al. (Apr. 1982) for solving the project scheduling problem (without agents). We will investigate the properties of the SDM, and among others, we will show that it is not able to find all Pareto-optimal solutions for the problem. We will also summarize computational results. Further on, we define a randomized SDM which can find any Pareto-optimal solution with positive probability.

The motivation for this research comes from real world industrial production environments, where project leaders (the agents in the model) want to reserve the necessary resources greedily, in many cases too early, and in larger than necessary quantities, in order to finish their projects on time. Practical approaches, like prioritizing the most important products or customers, can help to alleviate the problem, but cannot guarantee any optimality criteria. This situation also resembles the coordination problem in supply chains, but a crucial difference is that in the latter appropriate payments can ensure truthfulness (see e.g., Egri & Váncza, 2013).

The paper is organized as follows. In Section 3, we review the classical scheduling model and its solution that will be the basis of the resource allocation problem. In Section 4, the mechanism design model is introduced, the impossibility of truthful and optimal mechanisms is proven, then the SDM for this problem is described and analyzed. Next, we present a randomized version of the SDM in Section 6. Finally, in Section 7, we conclude the results and mention some future research directions.

2. Literature review

Planning assembly operations including precedence constraints and material supply is a relevant and frequently studied problem in production systems (see e.g., Györgyi & Kis, 2018; Leyman & Vanhoucke, 2016; Nudtasomboon & Randhawa, 1997). Most of these studies, like traditional optimization problems, usually assume a central decision maker and do not consider the conflict of interests. The most natural situation where multiple non-cooperative parties are involved occurs in supply chain inventory control problems (see e.g., Aminzadegan, Tamannaei, & Rasti-Barzoki, 2019; Wang, Guo, & Wang, 2017). However, the game theoretical analysis and design is not limited to supply chains. Scheduling problems involving self-interested agents were already studied in the seminal work on algorithmic mechanism design (Nisan & Ronen, 2001), and even earlier (e.g., Váncza & Márkus, 2000). Since then, several authors have combined scheduling and mechanism design (e.g. Christodoulou & Koutsoupias, 2009; Heydenreich, Müller, & Uetz, 2007), but most scheduling papers consider *renewable resources*—such as machines—as agents. A large number of mechanisms involve *payments* for incentivizing the agents in scheduling or allocation settings (e.g., Chen et al., May 2016; Krysta, Telelis, & Ventre, 2015; Robu et al., Oct. 2013). Recently, mechanisms without money are also studied for scheduling problems (Giannakopoulos, Koutsoupias, & Kyropoulou, 2016), but to the best of our knowledge, scheduling mechanisms with non-renewable resources and without payments are not yet investigated.

Our *resource allocation problem* is related to one-sided matching problems without money, such as *house allocation* and *course allocation* (see e.g., Manlove, 2013). These models consist of two different sets of objects, where the elements of one set (called *applicants*) have privately known preference orderings over the elements of the other set (Kurata, Hamada, Iwasaki, & Yokoo, 2017). This is in contrast with two-sided matching problems, where the elements of both sets have preferences over the elements of the other set. The goal of the mechanism design for these problems is to give a matching between the two sets that satisfy certain properties, such as *truthfulness* and stability (e.g., *Pareto-optimality*)[3].

For these matching problems, a frequently used mechanism is the Serial Dictatorship Mechanism (SDM), which, in several cases, is the only mechanism satisfying the required properties, and furthermore, it is straightforward to implement (e.g., Abizada & Chen, 2016; Aziz & Mestre, 2014). An SDM considers a—random or pre-existing—priority ordering of the applicants and works as follows. First, it determines the set of optimal allocations with regard to the preferences of the applicant with the highest priority. Then in each consecutive step, it takes the set from the previous step, and determines a subset of the best allocations considering the preferences of the next applicant. After the last applicant, it yields an allocation from the final set. Note that if the preferences always imply a unique preferred allocation, then only the preferences of the applicant with the highest priority (the dictator) matters, which is the classical dictatorship.

The house allocation problem is the one-to-one version of one-sided matching, where each applicant can be paired with at most one house, and conversely, each house can be assigned to at most one applicant. For this problem the SDM is truthful, and in addition, it can generate every Pareto-optimal matching—with different priority orderings—, and it is the only Pareto-optimal mechanism (Abdulkadroğlu & Sönmez, 1998).

Dughmi and Ghosh (2010) study a one-sided, one-to-many General Assignment Problem (GAP) without money, and some of its special

[1] In the mechanism design literature this is referred to as *egalitarian social welfare*, which is considered to be more *fair* than minimizing the total tardiness, the *utilitarian social welfare* (see e.g., Rothe, 2015). However, most of the results presented in this paper remains valid when this latter objective is considered instead (see remarks).

[2] Throughout the paper we refer to optimality with respect to the objective of the central inventory.

[3] These properties will be formally defined in Section 4.

cases. In their model, job agents should be matched with capacitated machines. The problem is formulated as an integer program, and by relaxing the integrality constraints, an LP-based technique is shown to provide truthful approximate mechanisms.

The many-to-many extension of one-sided matching is the course allocation problem, where both the applicants and the courses have quotas for their connections. The SDM can generate every Pareto-optimal matching, however, it is truthful only in special cases (Cechlárová et al., 2016; Cechlárová & Fleiner, 2017). Kash et al. (2014) present a dynamic version of the matching problem, where the agents are not present simultaneously, but can arrive any time, and their demands are not known in advance. They regard renewable computational resources (such as CPU and memory), but they consider them *consumable*, i.e., once allocated, it is irrevocable, thus they are actually non-renewable.

Our resource allocation model is different from the above mentioned matching problems in several aspects. First of all, the preferences are not ordinal but cardinal, and not arbitrary: there is a scheduling problem in the background with a predefined structure that influences the preferences. For example, having a resource earlier is (weakly) preferred compared to having it later—if the goal is to minimize the tardiness. The matching also cannot be arbitrary, each job should be matched exactly with the required resources, only the timing can vary. Furthermore, contrary to the house and course allocation problems, the incoming batches of resources are divisible: they can be shared among several jobs. However, since satisfying only a part of the resource requirements has no value for the jobs, the problem resembles more to the matching than the *cake-cutting* models (see e.g., Brandt, Conitzer, Endriss, Lang, & Procaccia, 2016; Rothe, 2015).

Finally, we mention that if, in addition to non-renewable resources, the processing of jobs also require some renewable resources, such as machines, then quite a few results are known. Carlier (1984) was the first who studies machine scheduling problems with non-renewable resources, and further complexity results can be found in e.g., Grigoriev et al. (2005), Gafarov et al. (2011). The approximability of machine scheduling problems is thoroughly studied for the makespan objective in single as well as parallel machine environments by Györgyi and Kis (2015, 2015, 2017), for the maximum lateness objective by Györgyi and Kis (2017), and for the total weighted completion time objective by Kis (2015) and Györgyi and Kis (2019).

3. The scheduling model

3.1. The project scheduling problem with non-renewable resources

Let us consider a set of projects P. Each project $p \in P$ has a due date d_p and a set of jobs J_p. Each job $j \in J_p$ has a processing time t_j. We assume that the J_p are disjoint and let J denote the union of all the J_p, containing altogether n jobs. Each project has a set of precedence relations $A_p \subset J_p \times J_p$, and if $(j, k) \in A_p$ then job j must be finished before job k starts. We assume that the precedence relations induce a directed acyclic graph.

There is a set of non-renewable resources R, where each $\rho \in R$ has an initial supply of $b_{\rho,1}$ at time $u_1 = 0$, and additional supplies of $b_{\rho\ell}$ at times u_ℓ for $\ell = 2, \ldots, q$, where we assume that $u_1 < u_2 < \ldots < u_q$. Each job j requires a quantity of $a_{\rho j} \geqslant 0$ of resource $\rho \in R$ at its start.

We assume that for each resource $\rho \in R$ the demand does not exceed the supply, i.e., $\sum_{j\in J} a_{\rho j} \leqslant \sum_{\ell=1}^{q} b_{\rho\ell}$, otherwise the scheduling problem has no solution. For simplicity, we assume equality without loss of generality.

Let I denote an instance of the scheduling problem defined by the above introduced parameters.

Let $\mu_{\rho j\ell}$ denote the quantity of resource ρ allocated to job j at time u_ℓ. We call $\mu = \{\mu_{\rho j\ell}\}$ an *allocation* of the supplied resources to the jobs, if for each resource ρ and time u_ℓ the supply $b_{\rho\ell}$ is divided among the jobs: $\sum_{j\in J} \mu_{\rho j\ell} = b_{\rho\ell}$. We call an allocation *feasible*, if every job j has enough resources allocated, i.e., $\forall\, \rho \in R$: $a_{\rho j} \leqslant \sum_{\ell=1}^{q} \mu_{\rho j\ell}$.[4] Henceforward we consider only feasible allocations and refer to them simply as allocations.

A schedule s is a function mapping each job j to its start time s_j, along with an allocation μ. In order to have the schedule uniquely determined by an allocation, we assume that each job starts as early as possible, i.e., when (i) all the required resources are allocated to it, and (ii) every one of its predecessors defined by the precedence constraints are finished. Let us denote therefore the start time of job $j \in J_p$ w.r.t. allocation μ by

$$s_j^{(\mu)} = \min\left\{ s \geqslant 0 \mid \forall\, \rho\colon \sum_{u_\ell \leqslant s} \mu_{\rho j\ell} \geqslant a_{\rho j} \text{ and } \forall\, (k, j) \in A_p\colon e_k^{(\mu)} \leqslant s \right\}, \tag{1}$$

where $e_k^{(\mu)}$ denotes the finish time of job k: $e_k^{(\mu)} = s_k^{(\mu)} + t_k$.

Finally, let $T_p^{(\mu)}$ denote the *tardiness* of project p as the non-negative difference between its due date and the maximal finish time of its jobs:

$$T_p^{(\mu)} = \max\{\max_{j\in J_p} e_j^{(\mu)} - d_p,\, 0\}. \tag{2}$$

If allocation μ determines a schedule s, then the tardiness of the schedule is the maximal tardiness of the projects, i.e., $T_s = \max_{p\in P} T_p^{(\mu)}$.

3.2. The Carlier–Rinnooy Kan algorithm

Carlier et al. (Apr. 1982) gave a polynomial time algorithm for solving the above defined problem. We briefly recapitulate the main ideas of their solution here, since we are going to use a modified version of it in the SDM. Let us consider the graph defined by the jobs as nodes and precedence relations as edges, where the weight of edge (j, k) is t_j. Let $U(j)$ denote the set of all (direct or indirect) successors of job j, and W_{jk} the weight of the maximal path length between jobs j and $k \in U(j)$. For each project p, we define the cost function for each job $j \in J_p$ as $f_j(t) = \max\{t - d_p, 0\}$, i.e., the tardiness of the job j finishing at time t, with regard to the project's due date. In addition, let $B_\rho(u_\ell) = \sum_{\tau=1}^{\ell} b_{\rho\tau}$, the cumulative supply of resource ρ until time u_ℓ.

Then one can define a lower bound on the maximal tardiness in case job j starts at time u_ℓ:

$$\gamma_{\ell j} = \max\{f_j(u_\ell + t_j), \max\{f_k(u_\ell + t_k + W_{jk}) \mid k \in U(j)\}\}. \tag{3}$$

The algorithm seeks the smallest γ (denoting the maximal tardiness), such that $\forall\, \rho, \ell$: $\sum_{j\in J}\{a_{\rho j} \mid \gamma < \gamma_{\ell j}\} \leqslant B_\rho(u_{\ell-1})$, where $B_\rho(u_0) = 0$ (see A). For a fixed ℓ the smallest γ_ℓ^* can be found with a median search procedure, and the optimal $\gamma^* = \max_\ell \gamma_\ell^*$, for more details, see Carlier et al. (Apr. 1982).

Having the γ^*, the allocation μ can be computed by Algorithm 1.

Algorithm 1. Computing the allocation

Require γ^*
  for $\ell = 2$ **to** q **do**
    {Allocate resources to jobs that would be late starting at u_ℓ}
    for j: $\gamma_{\ell j} > \gamma^* \wedge \gamma_{\ell-1 j} \leqslant \gamma^*$ **do**
      Allocate the necessary resources to job j arbitrarily from the resources arriving earlier than time u_ℓ and not yet allocated. (Due to the construction of γ^*, there always exist enough free resources.)
    end for
  end for
  for j: $\gamma_{qj} \leqslant \gamma^*$ **do**
    Allocate the necessary resources to job j arbitrarily from the resources not yet-allocated.
  end for

[4] Since we assumed that the total supply equals the total demand of the resources, it is easy to see that equality holds in the definition of feasibility.

Table 1
The $\gamma_{\ell j}$ values for the example

	j				
	1	2	3	4	
ℓ	1	0	0	0	0
	2	2	1	2	1

3.3. An example

In order to demonstrate the algorithm, let us consider a simple example with only one resource, two projects, four jobs and two supply times. Each job j ($1 \leqslant j \leqslant 4$) has equal processing times $t_j = 1$ and requires one unit of the resource: $a_j = 1$, where we have omitted the index for the single resource. The precedences of the projects are as follows: $(j_1, j_2) \in A_{p_1}$ and $(j_3, j_4) \in A_{p_2}$, while their due dates are $d_p = 2$ ($1 \leqslant p \leqslant 2$). There are two supply times, $u_1 = 0$ and $u_2 = 2$, both with two units of supplied materials: $b_\ell = 2$ ($1 \leqslant \ell \leqslant 2$)—again with omitted index for the resource. The resulted $\gamma_{\ell j}$ values are shown in Table 1.

Then the algorithm will compute $\gamma_1^* = 0$ and $\gamma_2^* = 1$, from which $\gamma^* = 1$, i.e., the optimal schedule will result in one time unit tardiness. This schedule is when jobs j_1 and j_3 receive their required resources and start at time u_1, while the rest at time u_2.

4. Mechanism design for project scheduling

In the mechanism design problem we consider project agents with their due dates as private information. All other information is assumed to be public knowledge.[5] We examine the problem of a central inventory, which has to allocate the resources supplied over time to the jobs.

We seek a *direct revelation* mechanism that consists of two steps: (i) collecting due date information from the projects, and (ii) allocating resources to the jobs. Since the project agents are interested in their own tardiness, they might report false due dates to the central inventory in order to influence the allocation to their advantage. We refer to the reported due dates as $d_p{}'$. For practical reasons, we restrict our study to mechanisms without money, i.e., it is not allowed to offer resources at different prices based on their arrival time.

Definition 1 (*Mechanism*). Let I denote a scheduling problem instance. A deterministic resource allocation mechanism is a function mapping the problem instance to an allocation: $\Phi(I) = \mu$.

Definition 2 (*Preference*). Project p prefers allocation μ to μ' ($\mu \succ_p \mu'$), if $T_p^{(\mu)} < T_p^{(\mu')}$, and weakly prefers μ to μ' ($\mu \succeq_p \mu'$), if $T_p^{(\mu)} \leqslant T_p^{(\mu')}$.

An important property of a mechanism is *truthfulness*, when the agents cannot decrease their resulted tardiness by misreporting the due dates.

Definition 3 (*Truthfulness*). Let I denote an arbitrary scheduling problem instance and I'_p the same problem, but with due date $d_p{}'$ of project p instead of d_p. A mechanism Φ is *truthful*, if for each instance I, project p, and due date $d_p{}'$: $\Phi(I) \succeq_p \Phi(I'_p)$.

Note that the definition uses weak preference, thus reporting a false due date does not necessarily worsen the tardiness of a project. Requiring strict preference would be problematic for the existence of truthful mechanisms. For example, if a project has an appropriately late due date, any feasible allocation results in no tardiness for that project, thus reporting any due date results in the same zero tardiness for the agent. However, we assume *benevolent* agents henceforward, i.e., they report truthfully, if they cannot decrease their tardiness by misreporting.

Definition 4 (*Optimality*). We consider an allocation μ *optimal* for a scheduling problem instance, if μ determines a schedule that minimizes the maximal tardiness of the projects with respect to the true due dates d_p.

Note that an optimal allocation μ may not be optimal for the reported due dates d'_p.

Since it is often impossible to guarantee an optimal solution, frequently weaker criteria are considered instead. A widely used property for characterizing an acceptable solution is the *Pareto-optimality*, when the resulted allocation cannot be improved for any agent without damaging the others.

Definition 5 (*Pareto-optimality*). An allocation μ *Pareto-dominates* μ', if $\forall\, p$: $\mu \succeq_p \mu'$ and $\exists\, p$: $\mu \succ_p \mu'$. An allocation μ is *Pareto-optimal*, if no other allocation Pareto-dominates it. A mechanism is Pareto-optimal, if for all inputs it yields a Pareto-optimal allocation.

Note that in case of maximal tardiness minimization, not every optimal allocation is Pareto-optimal. However, if an allocation μ *Pareto-dominates* μ', then the maximal tardiness implied of μ cannot be greater than that of μ'. This property guarantees that there is at least one optimal allocation among the Pareto-optimal ones.

4.1. Impossibility of truthful and optimal mechanisms

Firstly, it is obvious that if a mechanism is not truthful—hence it does not always receive the real due dates of the projects—then it is impossible to guarantee the optimality of the solution. Therefore, we restrict our investigation to truthful mechanisms in the sequel. The first fundamental question is whether there exists a truthful mechanism that can always find an optimal solution. The next proposition shows that unfortunately this is not the case.

Proposition 1. *If a mechanism is truthful, it cannot determine an optimal solution for all scheduling problem instances.*

Proof. The mechanism is assumed to be truthful, therefore it is informed about the real due dates. In this setting two different scheduling problems are considered. In addition, the optimality of the mechanism is also assumed, thus it results in the optimal schedule for both problems. It is then shown that these two assumptions contradict each other, therefore no mechanism can be truthful and optimal at the same time.

By contradiction, suppose we have a mechanism that is truthful, and on all inputs returns an optimal solution to the scheduling problem. Now we examine how it works on the following problem instance I. There are only two projects, p_1 and p_2, consisting of one job each, j_1 and j_2, respectively, and a single resource ρ with an initial supply of $b_{\rho 1} = 1$ at $u_1 = 0$, and a second supply of $b_{\rho 2} = 1$ at $u_2 = 4$. The two jobs are identical, i.e., $t_{j_1} = t_{j_2} = 3$, and $a_{\rho j_1} = a_{\rho j_2} = 1$, but project p_1 has a due-date of $d_{p_1} = 4$, and project p_2 has a due-date of $d_{p_2} = 5$. Notice that in any feasible schedule at most one job may start at $u_1 = 0$, the other must wait for the second supply at u_2. Since the mechanism is truthful, both projects report their true due dates. Then the mechanism must find the unique optimum in which j_1 starts at u_1, and j_2 starts at u_2. The tardiness of p_1 is then 0, and that of p_2 is 2 time units. This is depicted in Fig. 2a.

Now consider the problem instance I' which differs from I only in the due date of p_2, which is $d_{p_2}{}' = 2$. Then the mechanism must return the unique optimum for this instance, in which job j_1 starts at u_2, and job j_2 starts at u_1, giving a tardiness of 3 for project p_1, and 1 for project p_2, see Fig. 2b.

Considering again the problem instance I, this latter schedule—which is also feasible, but not optimal for instance I—results in 0

[5] This restriction is not necessary, only assumed for keeping the model simple. The set of private information can be extended to every parameter related to the projects. In this case, one does not have to use a *direct* mechanism—i.e., where the agents should report the full private information—only the $a_{\rho j}$ and $\gamma_{\ell j}$ values are required by the mechanism.

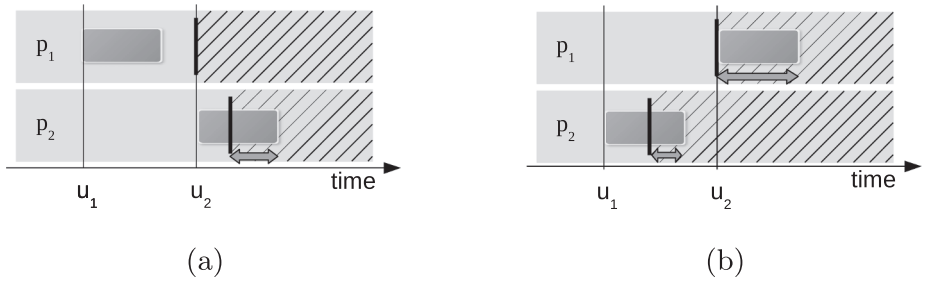


Fig. 2. The optimal schedules in two different problem instances.

tardiness for project p_2, thus $\Phi(I') \succeq_{p_2} \Phi(I)$. This means that even in case of instance I, project p_2 would be better off reporting the due dates of instance I', which contradicts the assumption of truthfulness of the mechanism. Assuming both truthfulness and optimality has led to a contradiction, therefore both of them cannot be true at the same time. □

Note that the claim of the proposition remains valid if we change the optimality criterion to the total tardiness instead of the maximal tardiness.

Corollary 1. The naï ve mechanism in which the central inventory computes an optimal allocation with the Carlier–Rinnooy Kan algorithm is not truthful.

This corollary means that if the central inventory uses the naï ve mechanism, it is beneficial for the projects to report an earlier due date than the true one. This corresponds to the industrial practice where everyone requests the resources as soon as possible.

4.2. Serial Dictatorship Mechanism

Instead of minimizing the maximal tardiness as the naï ve mechanism does, the well-known SDM considers a multi-objective optimization problem. The basic idea is having the agents fixed in some priority ordering, and the set of possible outcomes are restricted iteratively according to the preferences of the agents, respecting the ordering. That is, the mechanism chooses from the set of all schedules a subset minimizing the tardiness of the agent with the highest priority. Afterwards, the mechanism chooses a subset of this subset containing schedules minimizing the tardiness of the next agent in the order, and this process continues iteratively. Finally, the output is chosen from the remaining set. For the sake of simplicity we assume that the priority ordering is a fixed, commonly known input of the mechanism.

More formally, the mechanism takes the projects in decreasing order of priority, i.e., the higher the priority of a project, the lower its index is. The mechanism executes an optimization step for each project. In step 1, it takes the project p_1 with the highest priority, and creates an allocation $\mu^{(1)}$ that minimizes $T'^{(\mu^{(1)})}_{p_1}$, where T' denotes the tardiness function of (2) considering the reported due dates instead of the real ones. Then in each subsequent step k, a new allocation $\mu^{(k)}$ is computed that minimizes $T'^{(\mu^{(k)})}_{p_k}$, with the constraints that it cannot increase the tardinesses of the projects with higher priorities, i.e., $\forall\, k' \in \{1, \ldots, k-1\}$: $T'^{(\mu^{(k)})}_{p_{k'}} \leqslant T'^{(\mu^{(k-1)})}_{p_{k'}}$. The resulted tardinesses of projects with lower priorities than p_k are completely disregarded in step k. An allocation is said to be optimal in step k, if it minimizes $T'^{(\mu^{(k)})}_{p_k}$ with respect to the above mentioned tardiness constraints.

The allocation $\mu^{(k)}$ can be computed with a modified version of the Carlier–Rinnooy Kan algorithm. In step k, instead of $\gamma_{\ell j}$, we use the following $\gamma^{(k)}_{\ell j}$:

$$\gamma^{(k)}_{\ell j} = \begin{cases} \gamma_{\ell j}', & j \in J_{p_k} \\ \infty, & j \in J_{p_{k'}} \text{ and } k' < k \text{ and } \gamma_{\ell j}' > T'^{(\mu^{(k-1)})}_{p_{k'}}, \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $\gamma_{\ell j}'$ is defined by (3), but considering the reported d_p' due dates in the cost function f instead of the real ones. For the jobs of project p_k, this involves the lower bounds $\gamma_{\ell j}'$ for the tardiness, while for any other job it is either zero or infinity. For projects considered before p_k, any allocation that would result in larger tardiness for them than in the previous step, infinite tardiness is used, these are therefore cannot start at u_ℓ or later. For the remaining case (when $\gamma^{(k)}_{\ell j}$ is defined as 0), the jobs may start at u_ℓ without increasing the tardiness of the corresponding project.

Similarly to the original algorithm, we are looking for the smallest $\gamma^{(k)}$, such that $\forall\, \rho, \ell$: $\sum_{j \in J} \{a_{\rho j} \mid \gamma^{(k)} < \gamma^{(k)}_{\ell j}\} \leqslant B_\rho(u_{\ell-1})$, see Algorithm 2.

Algorithm 2. Serial Dictatorship Mechanism

Require $p_1, \ldots, p_n$: an arbitrary priority ordering of the projects
The projects announce their due dates to the central inventory
for $k = 1$ **to** n **do**
  for $\ell = 1$ **to** q **do**
    Compute the $\gamma^{(k)}_{\ell j}$ values
$\gamma^{(k)*}_{\ell} := \min\left\{\gamma^{(k)} \mid \forall\, \rho: \sum_{j \in J} \{a_{\rho j} \mid \gamma^{(k)} < \gamma^{(k)}_{\ell j}\} \leqslant B_\rho(u_{\ell-1})\right\}$
  end for
  $\gamma^{(k)*} := \max_\ell \gamma^{(k)*}_{\ell}$
  Compute $\mu^{(k)}$[6]
end for
Allocate the resources according to $\mu^{(n)}$.

[6] This is not necessary in steps $k < n$, since only the tardinesses of projects $p_1, \ldots, p_k$ are used in the next step. For project p_k this will be equal to $\gamma^{(k)*}$, while for the other projects the tardinesses remain the same as in the previous step. When $k = n$, the allocation can be computed by Algorithm 1 using $\gamma^{(k')*}$ instead of γ^* whenever $j \in J_{p_{k'}}$.

Theorem 1. The SDM is truthful.

Proof. Let us consider an arbitrary project p_k. In steps $1, \ldots, k-1$, the due date d_{p_k}' is disregarded by the mechanism, therefore reporting it falsely cannot decrease the tardiness of p_k. In step k, the mechanism minimizes the tardiness of p_k with respect to the constraints derived from the previous steps, and using the reported due date of p_k. We claim that reporting a false due date cannot decrease the tardiness of p_k. Suppose, p_k reports a false due date $d_{p_k}' < d_{p_k}$ and let $\mu'^{(k)}$ and $\mu^{(k)}$ denote the corresponding allocations. If the tardiness of p_k is smaller with respect to $\mu'^{(k)}$ than that for $\mu^{(k)}$, then $\mu'^{(k)}$ would be a better allocation for p_k even when reporting its true due date, which is a contradiction. In steps $k' = k+1, \ldots, n$, the tardiness $T'^{(\mu^{(k')})}_{p_k} = T'^{(\mu^{(k)})}_{p_k}$ remains constant: it cannot increase due to the construction of the mechanism, but it also cannot decrease, otherwise $\mu^{(k)}$ is not optimal in step k, which is a contradiction. □

Note that the proof of truthfulness requires that the agents cannot influence the priority ordering. From now on, we take advantage of its truthfulness, and assume that the SDM possesses the real due dates. Let

us prove the Pareto-optimality of the mechanism.

Theorem 2. The SDM is Pareto-optimal.

Proof. Let's indirectly assume that $\exists\ \mu'$ Pareto-dominating $\mu^{(n)}$, i.e., $\forall\ p \in P$: $\mu' \geq_p \mu^{(n)}$ and $\exists\ p_k$: $\mu' \succ_{p_k} \mu^{(n)}$. This contradicts optimality of $\mu^{(k)}$ in step k, thus such μ' cannot exist. □

Corollary 2. *If there is a schedule where no project is tardy, then the SDM returns such a schedule.*

For several matching problems, every Pareto-optimal solution can be generated by an SDM by using different priority orderings. Unfortunately, this does not hold for our resource allocation problem. As a consequence, although there exists at least one optimal allocation among the Pareto-optimal ones, it is possible that such allocations cannot be found by the SDM with any permutation of priority ordering.

Proposition 2. *There might be Pareto-optimal solutions of the scheduling problem that cannot be found using an SDM with any permutation of priority ordering.*

Proof. Let us consider a simple scheduling problem with two projects of two jobs each, one resource and two supply times. Let $d_{p_1} = d_{p_2} = u_2$, $a_{\rho j_1} = 1$, $a_{\rho j_2} = 1$, $a_{\rho j_3} = 1$, $a_{\rho j_4} = 1$, $A_{p_1} = \{(j_1, j_2)\}$, $A_{p_2} = \{(j_3, j_4)\}$, $b_{\rho 1} = b_{\rho 2} = 2$, and $t_{j_1} = t_{j_2} = t_{j_3} = t_{j_4} = (u_2 - u_1)/2$.

There are only two priority orderings for two agents, but 3 Pareto-optimal solutions shown in Fig. 3. The two possible orderings of the projects for the SDM result in (maximal) tardiness $t_{j_1} + t_{j_2} = t_{j_3} + t_{j_4}$, illustrated in Fig. 3a and b. However, the allocation shown on Fig. 3c is also Pareto-optimal and its maximal tardiness is the half of what is achievable with an SDM. □

Proposition 2 implies that using SDMs may exclude the possibility of generating an optimal solution—despite always being Pareto-optimal. Unfortunately, there is an even more serious drawback of the SDMs. As the next theorem shows, the difference between the optimal and the maximal tardiness generated by an SDM is unbounded.

Proposition 3. *The maximal tardiness found by the SDM can be arbitrary larger than the optimal one.*

Proof. Let us consider a simple scheduling problem with two projects of one job each, one resource and two supply times. Let $d_{p_1} = u_2$, $d_{p_2} = u_1$, $a_{\rho j_1} = a_{\rho j_2} = 1$, $b_{\rho 1} = b_{\rho 2} = 1$ and $t_{j_1} = t_{j_2}$ (a fixed constant).

Fig. 4a illustrates the optimal schedule for this case, when the job of the second project gets the resource at u_1 and the other job at u_2. This result in tardinesses for both projects equal to their processing times. The solution on Fig. 4b is resulted by an SDM where p_1 has the higher priority. In order to avoid (or minimize) its tardiness, the job of p_1 must get the resource arriving at u_1. This results in $u_2 - u_1 + t_{j_2}$ maximal tardiness at project p_2.

As $u_2 \to \infty$, the maximal tardiness resulted by the optimal allocation does not change, but with the SDM it grows infinitely. □

Note that the claim of Proposition 3 remains valid if we change the optimality criterion to the total tardiness instead of the maximal tardiness.

In order to compare the optimum of the scheduling problem with the one obtained by SDM, we shift the tardiness values of the schedules, which is a common technique in scheduling theory (see e.g., Grigoriev et al., 2005). That is, the *shifted tardiness value* of a schedule s is

$$T_s^{\Delta} := T_s + u_q.$$

The shifted tardiness of any feasible schedule is u_q or more. Let $T_{\text{opt}}^{\Delta} := T_{\text{opt}} + u_q$ denote the tardiness of an optimal schedule increased by u_q. The *relative error* of some schedule s is

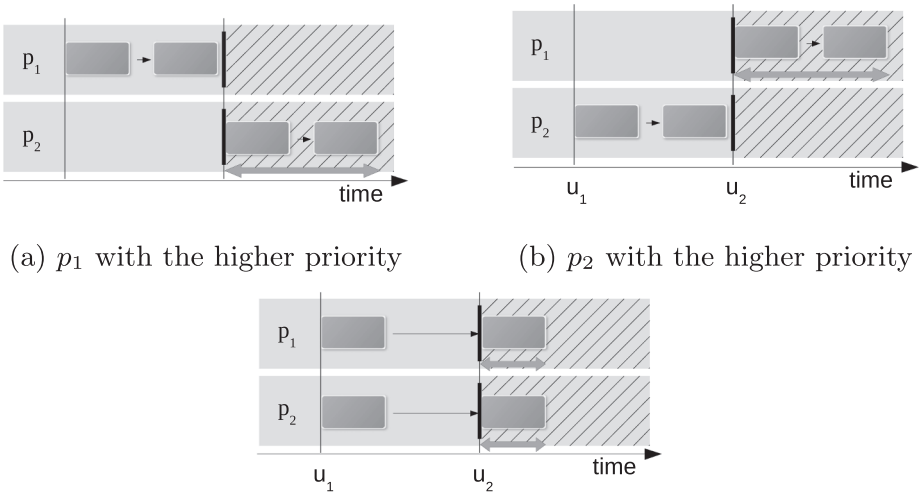
$$\text{Rel}(s) := \frac{T_s^{\Delta}}{T_{\text{opt}}^{\Delta}}. \tag{5}$$

By this formula, the relative error of an optimal schedule is 1. The following easy observation shows that with this normalized objective function, the relative error of those schedules obtained by SDM is at most 2.

Proposition 4. *The relative error of any schedule computed by SDM is at most 2.*

Proof. In order to prove the statement, we define a trivial feasible schedule with a relative error of at most 2, and argue that no job in a schedule obtained by SDM starts later than the same job in the trivial schedule.

In the trivial schedule s_{trivial} all the jobs of all the projects are started at time u_q or later if they have some predecessors. More precisely, in the trivial schedule first we schedule all the jobs without any predecessors at time u_q, then we schedule their immediate successors at the earliest possible time without violating the precedence constraints, etc. (or in other words, we schedule the jobs in topological order from time u_q on without any unnecessary delays). The trivial schedule satisfies all the precedence constraints by construction, and all the resource constraints as well, since by time u_q, all the resources are supplied, and the total



(a) p_1 with the higher priority (b) p_2 with the higher priority

(c) nondictatorial optimal solution

Fig. 3. Pareto-optimal solutions of the problem.

(a) The optimal solution (b) Solution of the (p_1, p_2)-SDM.

Fig. 4. Two possible allocations for the problem.

supply equals the total demand for each resource by assumption.

Now consider the allocation computed by the SDM. In this allocation each job gets its required resources not later than u_q. In the schedule determined by the allocation each job starts as early as possible, therefore no job can start later than the same job in the trivial schedule.

Finally, we claim that the relative error of the trivial schedule is at most 2. On the one hand, in any feasible schedule, the shifted tardiness of any project is at least u_q as we have already noted. Now consider an optimal schedule s_{opt}, and increase the start time of each job by u_q. In the resulting schedule s', every job starts after u_q. Notice that in s', no job starts before the same job in the trivial schedule. Since the tardiness of each project is increased by u_q in s', we conclude that

$$\text{Rel}(s_{\text{trivial}}) = \frac{T^{\Delta}_{\text{trivial}}}{T^{\Delta}_{\text{opt}}} \leqslant \frac{T^{\Delta}_{s'}}{T^{\Delta}_{\text{opt}}} = \frac{T^{\Delta}_{\text{opt}} + u_q}{T^{\Delta}_{\text{opt}}} \leqslant \frac{T_{\text{opt}} + 2u_q}{T_{\text{opt}} + u_q} \leqslant 2. \qquad \square$$

Note that the tardiness value is shifted in order to avoid zero in the denominator of the relative error. Another possibility to do this is to compare the resulted error to the optimal tardiness with the following formula:

$$\frac{T_{\text{sdm}} - T_{\text{opt}}}{\max\{T_{\text{opt}}, 1\}}, \tag{6}$$

where T_{sdm} denotes the tardiness of the schedule produced by an SDM. Due to Corollary 2, if $T_{\text{opt}} = 0$ then this equals zero, otherwise—assuming integer parameters—it reduces to $(T_{\text{sdm}} - T_{\text{opt}})/T_{\text{opt}}$. However, we will consider the relative error defined by (5) hereafter.

We can also get an upper bound on the absolute error of the schedule resulted by the SDM. In order to do this, let us define a relaxed problem without resource constraints. The optimal solution for this problem, s_{relaxed}, is when each job starts as early as possible: those jobs that do not have predecessors start at $u_1 = 0$, while others start as soon as their predecessors are finished. Note that in s_{relaxed} every job starts exactly u_q time unit earlier than in s_{trivial}. Thus, if T_{relaxed} and T_{sdm} denote the maximal tardiness of s_{relaxed} and a schedule resulted by an SDM, respectively, we have $T_{\text{relaxed}} \leqslant T_{\text{opt}} \leqslant T_{\text{sdm}} \leqslant T_{\text{trivial}} \leqslant T_{\text{relaxed}} + u_q$. By rearranging these inequalities, we get an upper bound for the absolute error:

$$T_{\text{sdm}} - T_{\text{opt}} \leqslant u_q. \tag{7}$$

This can be used to measure the relation of the absolute error and its upper bound by $(T_{\text{sdm}} - T_{\text{opt}})/u_q$, which yields a value between 0 and 1.

5. Numerical study

In order to asses the performance of SDM in practice, we have conducted a series of computational experiments. To this end, we have generated several problem instances with various characteristics, and compared the maximal tardinesses obtained by the Carlier–Rinnooy Kan algorithm and by the SDM with a random priority ordering. For comparison, we used the relative error defined by formula (5). Due to the efficient polynomial-time algorithms used, solving one problem instance takes only a few milliseconds on a standard laptop computer, including the execution of the Carlier–Rinnooy Kan algorithm, the SDM and the input/output operations.

5.1. Illustration of the performance of the SDM

We have generated problem instances with $|P| \in \{10, 50, 100\}$ projects and $q \in \{5, 10, 15\}$ supply dates. In all instances the number of jobs in each project was $|J_p| = 5$. The project parameters were random numbers, i.e., $d_p \sim U(1, 50)$ for each project p, $t_j \sim U(1, 5)$ and $a_{\rho j} \sim U(0, 5)$ for all the jobs j and resources ρ, where $U(a, b)$ denotes the discrete uniform distribution on the interval $[a, b]$. The density of the precedence graph of each project was 0.2, i.e., each project p contained $0.2|J_p|(|J_p| - 1) = 4$ directed edges. These edges were generated between random jobs of the project, but without adding multiple edges or cycles to the graph. The supplies were generated with $u_1 = 1$, $(u_\ell - u_{\ell-1}) \sim U(1, 50/q)$, $b_{\rho\ell} \sim U(0, \sum_j a_{\rho j} - B_\rho(u_{\ell-1}))$, and $b_{\rho q} = \sum_j a_{\rho j} - B_\rho(u_{q-1})$. The results show how the relative error varies depending on $|P|$, $|R|$ and q. Each value in Table 2 represents the average (or maximum) of the relative errors over 1000 problem instances.

Tables 2,a c and e suggest that there are two ways to decrease the expected error: with more frequent supplies or with less resources. When the number of supplies increases, there are usually more opportunities to schedule the non-tardy projects closer to their due dates, thus freeing some resources for the low priority projects. Decreasing the number of resources seems to be difficult in practice, but only the scarce resources are relevant for the problem. If the inventory keeps enough *safety stock*, then that resource does not constrain the schedule, thus it can be omitted from the model. Of course, both approaches come at a price which should be considered and balanced with the estimated cost of the tardiness.

Tables 2,b d and f present the maximum error considering the same instances as for the average. Similarly to the average case, the maximum error also decreases when q increases. Furthermore, it can be observed that the maximum error tends to decrease with more projects. Since these values are the extreme cases, it is more difficult finding trends in these tables, but they can be used for estimating the worst case scenarios.

5.2. Experiments with varying the number of supplies

One may presume that when the number of supplies increases while the total amount supplied remains the same, the resources are available earlier, thus the error decreases. As the following example shows, not only the error does not always decrease, but the resources can even arrive later, thus it is possible that the tardiness increases. In contrast to the previous subsection, here we consider an evenly distributed supply. Fig. 5 shows that increasing the number of supplies results almost always in delayed availability. If evenly distributed supply is not assumed, even less relationship can be said between the number of supplies and the availability.

Using the above supply patterns, we considered a simple scheduling problem with $|P| = 10$ and $|J_p| = 5$. The jobs of each project had

Table 2
Average and maximum relative errors

		Resources ($\|R\|$)					
		1	2	4	6	8	10
q	5	1.03	1.06	1.08	1.10	1.10	1.12
	10	1.01	1.01	1.02	1.03	1.02	1.03
	15	1.00	1.01	1.01	1.01	1.01	1.01

(a) Average with $|P| = 10$ projects

		Resources ($\|R\|$)					
		1	2	4	6	8	10
q	5	1.81	1.81	1.81	1.74	1.82	1.71
	10	1.68	1.67	1.51	1.51	1.52	1.61
	15	1.54	1.49	1.34	1.55	1.29	1.56

(b) Maximum with $|P| = 10$ projects

		Resources ($\|R\|$)					
		1	2	4	6	8	10
q	5	1.04	1.07	1.11	1.14	1.17	1.18
	10	1.01	1.02	1.03	1.04	1.04	1.04
	15	1.00	1.01	1.01	1.01	1.02	1.02

(c) Average with $|P| = 50$ projects

		Resources ($\|R\|$)					
		1	2	4	6	8	10
q	5	1.72	1.73	1.75	1.83	1.76	1.71
	10	1.31	1.32	1.49	1.35	1.49	1.50
	15	1.16	1.20	1.25	1.18	1.17	1.23

(d) Maximum with $|P| = 50$ projects

		Resources ($\|R\|$)					
		1	2	4	6	8	10
q	5	1.05	1.08	1.12	1.15	1.19	1.19
	10	1.01	1.02	1.03	1.04	1.04	1.06
	15	1.00	1.01	1.01	1.01	1.02	1.02

(e) Average with $|P| = 100$ projects

		Resources ($\|R\|$)					
		1	2	4	6	8	10
q	5	1.66	1.76	1.68	1.70	1.75	1.71
	10	1.36	1.68	1.35	1.39	1.38	1.40
	15	1.25	1.20	1.23	1.15	1.20	1.26

(f) Maximum with $|P| = 100$ projects

sequential precedence relations, the due dates for each project were $d_p = 0$, while the processing times of the jobs and the demands for the resource were generated according to $t_j \sim U(1, 5)$ and $a_{1j} \sim U(0, 5)$. In both $q = 5$ and $q = 15$ cases the optimal schedule resulted in $T_{\text{opt}} = 20$, while in the former $T_{\text{sdm}} = 24$ and in the latter $T_{\text{sdm}} = 25$. Therefore the error increased together with the number of supplies, due to the delayed availability shown in Fig. 5.

In order to investigate further the relationship between the number of supplies and the relative error considering only one resource and evenly distributed supply, we have performed further simulations. Table 3 shows the resulted average and maximum errors based on 100 simulation runs.

6. SDM with random endowments

In order to remedy the negative consequences of Proposition 2, we introduce a randomized extension of the SDM in this section.

Definition 6 (*Randomized mechanism (Nisan & Ronen, 2001)*). A randomized mechanism is a probability distribution over a family

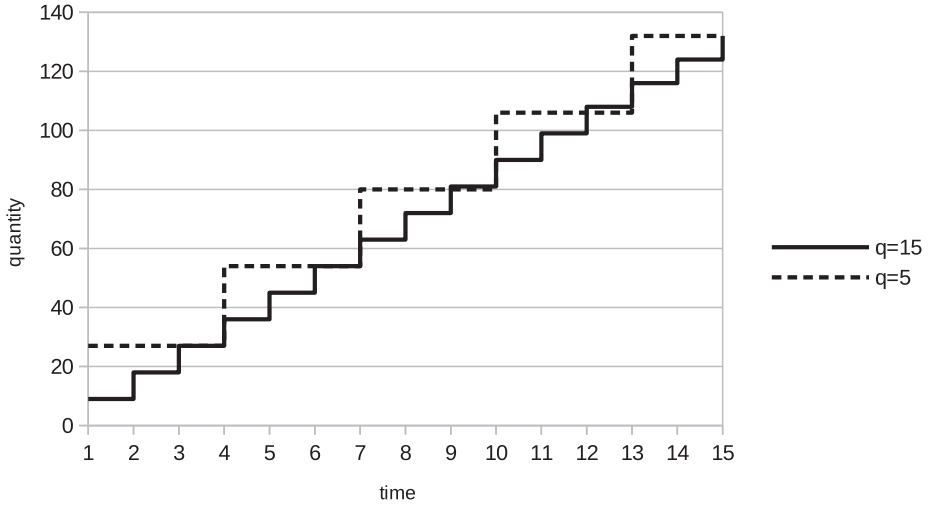


Fig. 5. Cumulative supply of a resource.

Table 3
Relative errors considering one resource and evenly distributed supply.

		Projects ($\|P\|$)		
		10	50	100
q	5	1.02	1.00	1.00
	15	1.03	1.01	1.01

(a) Average relative errors.

		Projects ($\|P\|$)		
		10	50	100
q	5	1.23	1.00	1.00
	15	1.32	1.07	1.07

(b) Maximum relative errors.

$\{\Phi_r\}$ of deterministic mechanisms. A randomized mechanism is called *truthful* (*Pareto-optimal*), if each deterministic mechanism in its support is truthful (Pareto-optimal).

Let us modify the SDM such that it starts with a random (feasible) allocation $\mu^{(0)}$, and in each step k it makes a Pareto-improvement on it, resulting in allocation $\mu^{(k)}$. This randomized mechanism can be interpreted as follows: given a random allocation r, the mechanism Φ_r is a deterministic mechanism that executes Pareto-improvements on the allocation r according to the given priority ordering. Then the SDM with random endowments (SDMRE) is a probability distribution over $\{\Phi_r\}$.

Algorithm 3. Computing a random allocation

for $\ell = 1$ **to** q **do**
  for $\rho \in R$ **do**
    while $b_{\rho\ell} > 0$ **do**
      Let j be a random job such that $a_{\rho j} > 0$
      Let $\mu_{\rho j\ell} = \min\{a_{\rho j}, b_{\rho\ell}\}$
      Decrease $a_{\rho j}$ and $b_{\rho\ell}$ with the allocated quantity $\min\{a_{\rho j}, b_{\rho\ell}\}$
    end while
  end for
end for

The initial allocation can be computed for example with Algorithm 3. Note that not every feasible allocation can be produced by this algorithm: whenever a supply and a demand is chosen, either the whole demand will be covered with the allocation or the whole supply will be allocated for that demand. It is easy to see however, that any allocation can be transformed into an allocation that can be the output of Algorithm 3 and they both imply the same schedule. Therefore this method does not exclude any significant solutions.

The allocation $\mu^{(k)}$ can be computed with a modified version of the SDM algorithm, where in step k we use the following $\gamma_{\ell j}^{(k)}$:

$$\gamma_{\ell j}^{(k)} = \begin{cases} \gamma_{\ell j}', & j \in J_{p_k} \\ \infty, & j \in J_{p_{k'}} \text{ and } k' \neq k \text{ and } \gamma_{\ell j}' > T'^{(\mu^{(k-1)})}_{p_{k'}}. \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

For project p_k, this takes the lower bounds of the tardiness, while for any other job it is either zero or infinity. For projects other than p_k, any allocation that would result in larger tardiness for them than in the previous step, infinite tardiness is considered, these are therefore excluded from an optimal solution. Any other allocation is allowed, thus they cause no tardiness in this step. This means that the algorithm minimizes the tardiness of project p_k, while it enforces upper bounds on the other projects' tardinesses.

Similarly to the SDM, we are looking for the smallest $\gamma^{(k)}$, such that $\forall\, \rho, \ell$: $\sum_{j\in J} \{a_{\rho j} \,|\, \gamma^{(k)} < \gamma_{\ell j}^{(k)}\} \leqslant B_\rho(u_{\ell-1})$, see Algorithm 4.

Algorithm 4. SDM with Random Endowments (SDMRE)

Require $p_1, \ldots, p_n$: an arbitrary ordering of the projects
  Let $\mu^{(0)}$ be a random (feasible) allocation
  The projects announce their due dates to the central inventory
  for $k = 1$ **to** n **do**
    for $\ell = 1$ **to** q **do**
      Compute the $\gamma_{\ell j}^{(k)}$ values
      $\gamma_\ell^{(k)*} := \min\left\{\gamma^{(k)} |\ \forall\, \rho\colon \sum_{j\in J} \{a_{\rho j} \,|\, \gamma^{(k)} < \gamma_{\ell j}^{(k)}\} \leqslant B_\rho(u_{\ell-1})\right\}$
    end for
    $\gamma^{(k)*} := \max_\ell \gamma_\ell^{(k)*}$
    Compute $\mu^{(k)}$
  end for
  Allocate the resources according to $\mu^{(n)}$

Theorem 3. *The SDMRE is truthful.*

Proof. Let us consider an arbitrary step k of the mechanism. Since the algorithm minimizes the tardiness of p_k in this step, it cannot benefit from a false $d_{p_k}{}'$. For any other $k' \neq k$, the $T'^{(\mu^{(k-1)})}_{p_{k'}}$ tardiness serves only as a constraint on the $T'^{(\mu^{(k)})}_{p_{k'}}$, which both change similarly depending on $d_{p_k}{}'$. Therefore also $p_{k'}$ cannot benefit from reporting a false due date. □

Note that the proof of truthfulness requires that the agents can influence neither the priority ordering nor the initial allocation.

Theorem 4. *An allocation is Pareto-optimal if and only if it can be the output of an SDMRE.*

Proof. The mechanism executes Pareto-improvements on the allocation until no more such improvement exists. When the algorithm stops, it results in an allocation which is not Pareto-dominated by any other allocation, therefore it is Pareto-optimal by definition. In addition, since the initial allocation is arbitrary (any Pareto-optimal allocation can be generated with positive probability). □

Note however, that since every Pareto-optimal solution can be the output of the SDMRE, the claim of Proposition 3 is still valid for this mechanism. Furthermore, numerical studies on the same problem instances have shown that the resulted relative errors of the SDMRE are almost the same as those of the SDM presented in Table 2: the average errors were exactly the same in case of 10 projects, in case of 50 projects 0.01 was the difference in only one case, while in case of 100 projects 0.01 was the difference in only three cases. The differences between the maximum errors were not greater than 0.05 on average. It seems that the randomization resulted only in a theoretical improvement compared to the SDM and does not provide any increase in efficiency.

7. Conclusions

In this paper the material allocation problem was introduced for project scheduling involving competing self-interested agents with privately known due dates. A novel resource allocation model was presented and studied in a mechanism design setting without using monetary transfers as incentives. A classical scheduling algorithm has been modified for implementing the Serial Dictatorship Mechanism, which is then proven to be truthful and Pareto-optimal.

It would be interesting to investigate realistic special cases for the scheduling problem. For example, the supply of resources is usually not random, but follows some pattern resulted from the applied ordering policy, such as the *fixed order quantity* or *fixed time period*. Another possibility is to consider similar projects, which occurs when the

products with different *features* define almost identical projects with slightly different resource requirements. The model also could be extended with renewable resource (e.g., machine) constraints, for which case the computational complexity introduces additional challenges.

CRediT authorship contribution statement

Peter Egri: Conceptualization, Methodology, Software, Writing - original draft. **Tamás Kis:** Conceptualization, Methodology, Writing - original draft.

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Appendix A. Modification of the Carlier–Rinnooy Kan algorithm

In Carlier et al. (Apr. 1982), the following inequalities can be found (considering only a single resource): $\forall\ \ell$: $\sum_{j\in J}\{a_j \mid \gamma \leqslant \gamma_{\ell j}\} \leqslant B(u_\ell)$, and “for fixed ℓ, the smallest value γ_ℓ^* for which [the inequality] is satisfied can be found by a *median* finding procedure […]”. However, such smallest value γ_ℓ^* may not exist. Consider the following simple example with a single job j and one resource only. There are two supplies at times $u_1 = 0$, and $u_2 = 1$ with supplied quantities $b_{1,1} = 1$ and $b_{1,2} = 1$, respectively, and demand $a_j = 2$. Hence, the cumulative supplies are $B(u_1) = 1$ and $B(u_2) = 2$, respectively, and job j can start only at u_2. Then for $\ell = 1$, with $\gamma = \gamma_{1,1}$ the inequality does not hold, but for any $\epsilon > 0$, with $\gamma = \gamma_{1,1} + \epsilon$ the inequality is satisfied, since the left-hand-side is 0.

Thus we use $\forall\ \rho$, ℓ: $\sum_{j\in J}\{a_{\rho j} \mid \gamma < \gamma_{\ell j}\} \leqslant B_\rho(u_{\ell-1})$ instead. We now show that if these inequalities are satisfied, then the resulted maximal tardiness cannot be greater than γ in an optimal schedule. Let us indirectly assume that for an optimal μ allocation there exists a project p with $T_p^{(\mu)} > \gamma$. This means that $\exists\ j^* \in J_p$: $f_{j^*}(e_{j^*}^{(\mu)}) > \gamma$. Let us consider a chain $(j_1, \ldots, j_{l_{\max}} = j^*)$, where $(j_l, j_{l+1}) \in A_p$ and $e_{j_l}^{(\mu)} = s_{j_{l+1}}^{(\mu)}$, but there exists no job k with $(k, j_1) \in A_p$ and $e_k^{(\mu)} = s_{j_1}^{(\mu)}$. Then $s_{j_1}^{(\mu)} = u_\ell$ must hold for some ℓ. But then, by definition, $\gamma_{\ell j_1} = f_{j^*}(e_{j^*}^{(\mu)}) > \gamma$, and neither the precedence constraints (by the choice of j_1), nor resource availability (since $\sum_{j\in J}\{a_{\rho j} \mid \gamma < \gamma_{\ell j}\} \leqslant B_\rho(u_{\ell-1})$ by assumption) blocks j_1, which contradicts the assumption that j_1 starts at the earliest start time permitted by the resource and the precedence constraints.

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