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Tensor Product Model-based Robust Flutter Control Design for the FLEXOP Aircraft

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Abstract: This paper presents a flutter suppression control design methodology for the aircraft designed within the European research project, FLEXOP. The aim of the flutter suppression controller is to stabilize the aeroelastic modes and to extend the aircraft's flutter free envelope. The first step is to develop a low order control oriented linear parameter varying (LPV) model of the aircraft. This is based on the "bottom-up" modeling approach. The key idea is to reduce the order of the subsystems that form the nonlinear aeroservoelastic model. A critical requirement of the low order model is to capture the flutter modes accurately. The second step is the control design, which is based on polytopic LPV representation. The Tensor Product (TP) type polytopic model of the aircraft is obtained with TP model transformation. TP model transformation in a numerical method based on the higher order singular value decomposition (HOSVD). It can generate various types of convex representations for LPV systems and offers a trade-off between the accuracy and the complexity of the resulting TP model. The control structure is a parameter-varying state feedback and observer. The control design specifications include asymptotic stability, robustness against parameter variations influencing the flutter modes and constraint on the control values in order to keep the control signals low. The developed control system is validated by simulation using the high-fidelity, nonlinear model.

Keywords: Aeroservoelasticity, linear parameter-varying systems, polytopic and robust control.

1. INTRODUCTION

A critical goal of future aircraft design is the increased fuel efficiency. This can be achieved by increased wingspan and reduced weight and structure. Such design, however, leads to more flexible aircraft structure and increased aeroservoelastic (ASE) effects. Aeroelastic flutter is the adverse interaction of aerodynamics with structural dynamics and produces an unstable oscillation, Fung (1969). Therefore, an important characteristic of future flight control systems is the usage of active control to suppress ASE effects.

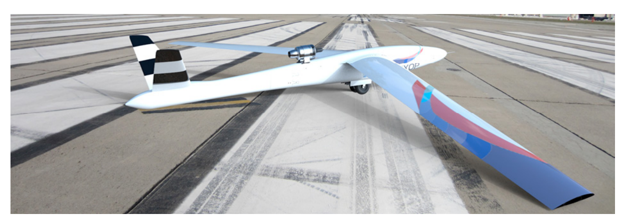

Fig. 1. FLEXOP demonstrator aircraft

The current paper focuses on the Flutter-Free Flight Envelope Extension for Economical Performance Improvement (FLEXOP) project, FLEXOP (2015-2018). FLEXOP is a European research project aiming to develop and demonstrate technological concepts to improve performance of flexible, high-aspect ratio, swept aircraft wings. A demonstrator Unmanned Areal Vehicle (UAV) is developed in the project (Section 2) that serves as a test bed for active flutter control techniques (see Figure 1.). The flutter suppression control law is designed based on an appropriate control oriented model, Theis et al. (2016); Luspay et al. (2019); Schmidt et al. (2019); PAAW (2014-2019). The linear parameter-varying (LPV) framework, Shamma (1988); Becker (1993) (Section 3.1), can serve as a good approach to model ASE systems for control design since it can capture the parameter varying dynamics of the aircraft. The ASE model is based on the integration of aerodynamics, structural dynamics and flight dynamics subsystems, Moreno et al. (2014a); Kotikalpudi (2017); Schmidt et al. (2016); Meddaikar et al. (2019), (Section 3.2). The resulting ASE model is usually highly coupled and nonlinear. In addition, the structural dynamics and aerodynamics make the dynamic order of the ASE models too large for control synthesis and implementation. The reduction of high dimensional LPV systems is still a challenging task, Wood (1995); Theis et al. (2015); Moreno et al. (2014b); Theis et al. (2017); Poussot-Vassal and Roos (2012); Luspay et al. (2017). Instead of LPV model reduction, the paper focuses on the "bottom-up" modeling approach, Takarics et al. (2018); Meddaikar et al. (2019), (Section 3.3). In such way the structural dynamics and aerodynamics models, which have simpler structure than then combined ASE model, are reduced before they are integrated into the ASE model.

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The main goal of the paper is to propose an LPV control design methodology for active flutter suppression of the FLEXOP aircraft. The focus is on polytopic LPV systems, Apkarian et al. (1995), specifically Tensor Product (TP) type representation. TP model transformation is a numerical method capable of transforming LPV systems into convex polytopic forms, Baranyi et al. (2013). It generates a canonical form of the LPV models based on the higher-order singular value decomposition (HOSVD), De Lathauwer et al. (2000). The higher-order singular values give a trade-off possibility between the complexity and accuracy of the resulting TP type polytopic model, Baranyi et al. (2013). The proposed control structure is an LPV observer and state feedback design. The main criteria of the control design is robustness against parameter variations affecting the flutter modes and constraints on the control signal (Section 4). An additional contribution is the validation of the "bottom-up"-based low order model of the aircraft. The controller is validated by the high-fidelity, nonlinear model of the FLEXOP aircraft in Matlab/Simulink (Section 5).

2. FLEXOP DEMONSTRATOR AIRCRAFT

The aircraft has a wingspan of 7 m and aspect ratio of 20. The empennage is configured as a V-tail and each wing has 4 control surfaces, Roessler et al. (2019). The outer control surfaces are used for flutter suppression, see Figure 2. The aircraft has two unstable aeroelastic modes. The

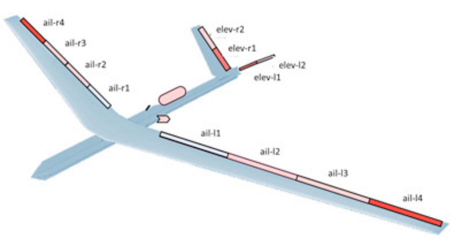


Fig. 2. FLEXOP aircraft control surface configuration

first aeroelastic mode (symmetric) goes unstable at 52 m/s and 50.2 rad/s and the second (asymmetric) at 55 m/s and 45.8 rad/s. In addition to the GPS and air data probe, the aircraft has inertial measurement units (IMUs) at the center of gravity and in the wings as shown in Figure 3.

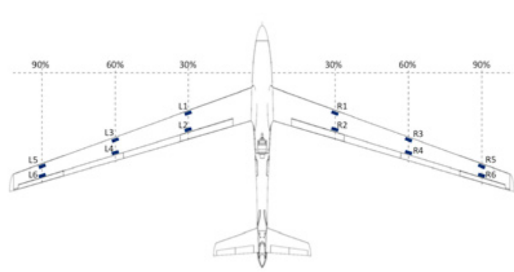


Fig. 3. FLEXOP aircraft sensor configuration

3. CONTROL ORIENTED BOTTOM-UP LPV MODEL

3.1 Linear Parameter Varying Models

Grid-based, Wu (1995) and polytopic LPV frameworks are in the focus of the paper. An LPV system is described by the state space model

$$\dot{x}(t) = A(\rho(t))\, x(t) + B(\rho(t))\, u(t) \tag{1a}$$

$$y(t) = C(\rho(t))\, x(t) + D(\rho(t))\, u(t) \tag{1b}$$

with the continuous matrix functions $A\colon \mathcal{P} \to \mathbb{R}^{n_x \times n_x}$, $B\colon \mathcal{P} \to \mathbb{R}^{n_x \times n_u}$, $C\colon \mathcal{P} \to \mathbb{R}^{n_y \times n_x}$, $D\colon \mathcal{P} \to \mathbb{R}^{n_y \times n_u}$, the state $x\colon \mathbb{R} \to \mathbb{R}^{n_x}$, input $u\colon \mathbb{R} \to \mathbb{R}^{n_u}$, output $y\colon \mathbb{R} \to \mathbb{R}^{n_y}$ and a time-varying scheduling signal $\rho\colon \mathbb{R} \to \mathcal{P}$, where $\mathcal{P}$ is a compact subset of $\mathbb{R}^{n_\rho}$. The parameter vector ρ may include elements of the state vector x, in this case the system belongs to the class of quasi LPV models. The system matrix $S(\rho(t))$ consists of:

$$S(\rho(t)) = \begin{bmatrix} A(\rho(t)) & B(\rho(t)) \\ C(\rho(t)) & D(\rho(t)) \end{bmatrix} \tag{2}$$

In a grid representation, the LPV system is described as a collection of LTI models $(A_k,\ B_k,\ C_k,\ D_k) = (A(\rho_k), B(\rho_k), C(\rho_k), D(\rho_k))$ obtained from evaluating the LPV model at a finite number of parameter values $\{\rho_k\}_1^{n_{\text{grid}}} = \mathcal{P}_{\text{grid}} \subset \mathcal{P}$. In polytopic representation the LPV model takes the following form

$$S(\rho(t)) = \sum_{r=1}^{R} w_r(\rho(t)) S_r \tag{3}$$

$S(\rho(t))$ is given as the parameter varying combinations of LTI system matrices $S_r \in \mathbb{R}^{(n_x+n_u)\times(n_x+n_y)}$ called LTI vertex systems. The combination is defined by the weighting functions $w_r(\rho(t)) \in [0,1]$. The dependence on time t is suppressed in the remainder.

3.2 High fidelity nonlinear model of the FLEXOP aircraft

The ASE model of the FLEXOP aircraft is developed based on a subsystem approach as seen in Figure 4. Each of the subsystems are developed separately. The structural dynamics model is obtained from a `Nastran` finite element (FE) model. The aerodynamics is modeled using the vortex lattice method (VLM) for steady and doublet lattice method (DLM) for unsteady models. The fidelity of the aerodynamics can be further improved by computational fluid dynamics (CFD) methods. Dynamic models for flight systems such as engines, for external disturbances, for sensors and actuators are added to form the full-order nonlinear ASE model. The nonlinear equations of motions are derived based on a mean axes reference frame, Schmidt (2012). The details of the ASE model are given in Wuestenhagen et al. (2018); Meddaikar et al. (2019). The model has 12 rigid body states, 100 flexible mode states and 1040 aerodynamic lag states in addition to the actuator dynamics. This model is considered as the high-fidelity, full order model (FOM). The LPV model of such system is of too high order for control design.

3.3 Bottom-up modeling

The bottom-up modeling is pursued in order to obtain an LPV model of the FLEXOP aircraft that is of sufficiently low order for control design. The key idea is to reduce the subsystems before the integration into the nonlinear model. The reason behind this is that the structural dynamics and aerodynamics subsystems have simpler structure than the combined ASE model. Thus, the order of these subsystems can be reduced by simpler and more tractable reduction techniques. Such approach leads to a low order ASE model (LOM).

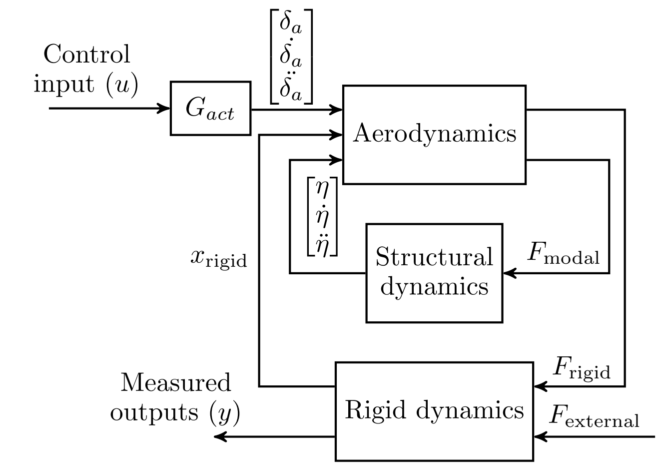


Fig. 4. ASE subsystem interconnection

The LPV model of the resulting LOM is compared to the LPV model of the FOM to verify its accuracy. The grid based LPV models of the LOM and FOM are derived in the following way. The nonlinear ASE model is first trimmed for straight and level flights at various airspeeds after which Jacobian linearization is carried out. The scheduling parameter is defined as $\rho = V_s$ in the interval $[30, 65]$ m/s over a grid of 71 equidistant points.

The ν-gap metric $\delta_\nu(\cdot,\cdot)$ is used as a measure to compare the LOM and FOM LPV models. It takes into account the feedback control objective. It takes values between zero and one, where zero is attained for two identical systems. A system P_1 that is within a distance ϵ to another system P_2 in the ν-gap metric, i. e. $\delta_\nu(P_1, P_2) < \epsilon$, will be stabilized by any feedback controller that stabilizes P_2 with a stability margin of at least ϵ, Vinnicombe (1993). A plant at a distance greater than ϵ from the P_2, on the other hand, will in general not be stabilized by the same controller. It can be calculated frequency by frequency as

$$\delta_\nu(P_1(\mathrm{j}\,\omega), P_2(\mathrm{j}\,\omega)) = \|\,(I + P_2(\mathrm{j}\,\omega)\; P_2^*(\mathrm{j}\,\omega))^{-1/2}\; (P_1(\mathrm{j}\,\omega) - P_2(\mathrm{j}\,\omega))\; (I + P_1^*(\mathrm{j}\,\omega)\; P_1(\mathrm{j}\,\omega))^{-1/2}\,\|_\infty \quad (4)$$

The ν-gap metric is a linear time invariant (LTI) technique and the goal is to evaluate it at each LPV grid point. Since the LOM is aimed for flutter suppression control design, the ν-gap metric is investigated for an input/output set that is relevant for the control design. These are $L4$, $R4$ inputs and vertical acceleration (a_z) and pitch rate (q) measurements at the c.g. and at the 12 IMUs. The goal of the control design is flutter suppression. The flutter frequency determines the frequency range for which an accurate model is required. Therefore, the frequency range of interest is defined up to 100 rad/s.

Reduction of the structural dynamics model The structural dynamics model is an LTI system, therefore, state truncation can be applied. Retaining the first 6 structural modes and modes 19, 20, 21 results in acceptable accuracy. This way the reduced order structural dynamics model is of 18 states as opposed to the 100 states of the FOM.

Reduction of the DLM aerodynamics The aerodynamic lag terms can be given in the following state space form

$$\begin{aligned} \dot{x}_{aero} &= \frac{2V}{\bar{c}} A_{lag} x_{aero} + B_{lag} \left[\dot{x}_{rigid}\;\; \dot{\eta}\;\; \dot{u}\right]^T \\ y_{aero} &= C_{lag} x_{aero} \end{aligned} \quad (5)$$

where V is the airspeed, $\bar{c}$ is the reference chord, $\dot{x}_{rigid}$ are the rigid body states, η represent the structural dynamics states and u is the control surface deflection. A linear balancing transformation matrix T is computed for the aerodynamics model given by A_{lag}, B_{lag} and C_{lag} in (5). The reduced model is obtained by rezidualizing the states with the smallest Hankel singular values. Keeping 2 lag states results in acceptable accuracy. The ν-gap plot of the FOM and LOM is shown in Figure 5. The resulting

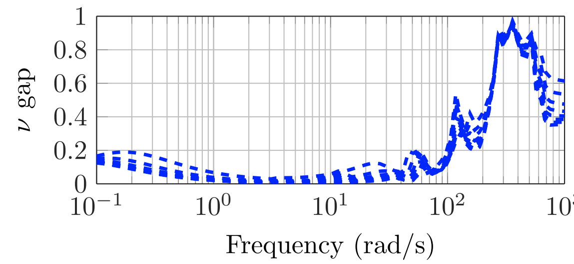


Fig. 5. ν-gap values between the FOM and LOM

bottom-up LOM if of 56 states, that consists of 12 rigid body states, 18 structural dynamic states, 2 aerodynamic lag states and 24 actuator dynamics states. In addition to the ν-gap plots, the pole migration, Bode plots and numerical simulation responses of the LOM and FOM are compared. Further details of the bottom-up modeling of the FLEXOP aircraft can be found in Meddaikar et al. (2019). Figure 6. shows the pole migrations of the LOM and FOM LPV models. The FOM LPV model predicts flutter at 52 and 55 m/s at frequencies of 50.2 rad/s and 45.8 rad/s. The LOM LPV model predicts flutter at 52.5 and 56.5 m/s at 50.3 rad/s and 46 rad/s.

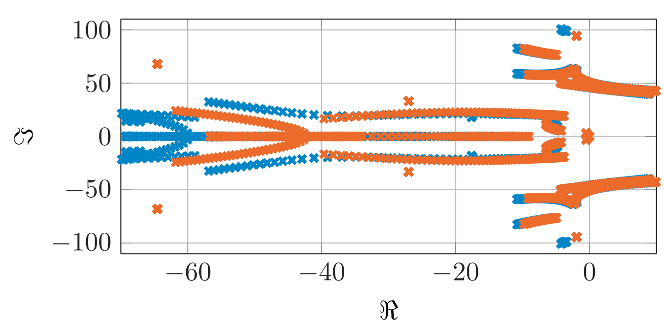


Fig. 6. Pole migration of the LOM (×) and FOM (×)

4. PROPOSED CONTROL DESIGN

The control design for flutter suppression of the FLEXOP aircraft is based on the TP type polytopic LPV model. Such representation can be obtained from the grid based LPV model via TP model transformation, Baranyi et al. (2013).

4.1 TP type polytopic model

The TP type polytopic form of an LPV system can be defined in the following way

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \mathcal{S} \underset{n\in \mathrm{N}}{\boxtimes} w_n(\rho_n) \begin{bmatrix} x \\ u \end{bmatrix} \quad (6)$$

The core tensor $\mathcal{S} \in \mathbb{R}^{I_1 \times \cdots \times I_N \times n_x + n_u \times n_x + n_y}$, that is of N dimension, is created from the LTI system matrices $S_{i_1,\ldots,i_N} \in \mathbb{R}^{n_x+n_u \times n_x+n_y}$. A convex combination of the vertexes is defined by the weighting functions for all n

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \mathcal{S} \underset{n \in \mathrm{N}}{\boxtimes} w_n^{Co}(\rho_n) \begin{bmatrix} x \\ u \end{bmatrix} \quad (7)$$

The TP model is a higher structured polytopic representation since it can always be given as:

$$S(\rho) = \sum_{r=1}^{R} w_r^{Co}(\rho) S_r \quad (8)$$

Vertexes S_r are equivalent to the vertexes stored in tensor $\mathcal{S}$, as $S_r = S_{i_1,i_2,\ldots,i_n}$ and $w_r(\rho) = \Pi_{n=1}^{N} w_{n,i_n}(\rho_n)$. The finite index r is a linear equivalent of multidimensional indexes $i_1, i_2, \ldots, i_N$.

4.2 Uncertainty structure

The uncertainty structure is based on Tanaka and Wang (2001) and takes the following form

$$\dot{x} = \left[A(\rho) + D_a(\rho)\Delta_a(t)E_a(\rho)\right] x + B(\rho) u \quad (9)$$

where the uncertain block $\Delta_a(t)$ satisfies

$$\|\Delta_a(t)\| \leq \frac{1}{\gamma_a}, \quad \Delta_a(t) = \Delta_a^T(t), \quad (10)$$

and $D_a(\rho)$ and $E_a(\rho)$ are known scaling matrices.

4.3 Control design structure

The paper considers a state feedback based control and observer design that satisfies $x(t) - \hat{x}(t) \to 0 \quad$ as $\quad t \to \infty$, Scherer and Weiland (2000); Tanaka and Wang (2001).

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}} \\ \hat{y} \end{bmatrix} &= S(\rho) \begin{bmatrix} \hat{x} \\ u \end{bmatrix} + \begin{bmatrix} K(\rho) \\ 0 \end{bmatrix} (y - \hat{y}) \\ u &= -F(\rho)\hat{x} \end{aligned} \quad (11)$$

The system $S(\rho)$, controller $F(\rho)$ and observer $K(\rho)$ take the following TP model structure:

$$\begin{aligned} S(\rho) &= \mathcal{S} \underset{n \in N}{\boxtimes} w_n^{Co}(\rho_n) \\ F(\rho) &= \mathcal{F} \underset{n \in N}{\boxtimes} w_n^{Co}(\rho_n) \\ K(\rho) &= \mathcal{K} \underset{n \in N}{\boxtimes} w_n^{Co}(\rho_n) \end{aligned} \quad (12)$$

4.4 Linear matrix inequality (LMI) based control design

The control design is based on specifications formulated in terms of LMIs. The control performance objectives are:

- Asymptotically stable controller and observer;
- Constraint on the control value;
- Robust stability against parameter uncertainties.

LMI theorems derived in Tanaka and Wang (2001); Scherer and Weiland (2000) are used for the control design.

Theorem 1. **Globally and asymptotically stable controller for uncertain LPV systems**: A controller stabilizing the uncertain LPV system (9) can be obtained by solving the following LMIs for $P = P^T > 0$ and M_r $(r = 1, \ldots, R)$

$$S_{rr} < 0, \quad T_{rs} < 0,$$

where

$$S_{rr} = \begin{bmatrix} PA_r^T + A_rP - B_rM_r - M_r^TB_r^T & D_{ar} & PE_{ar}^T \\ D_{ar}^T & -I & 0 \\ E_{ar}P & 0 & -\gamma_a^2 I \end{bmatrix}$$

and

$$T_{rs} = \begin{bmatrix} \begin{bmatrix} PA_r^T + A_rP \\ -B_rM_s - M_s^TB_r^T \\ +PA_s^T + A_sP \\ -B_sM_r - M_r^TB_s^T \end{bmatrix} & D_{ar} & D_{as} & PE_{ar}^T & PE_{as}^T \\ D_{ar}^T & -I & 0 & 0 & 0 \\ D_{as}^T & 0 & -I & 0 & 0 \\ E_{ar}P & 0 & 0 & -\gamma_a^2 I & 0 \\ E_{as}P & 0 & 0 & 0 & -\gamma_a^2 I \end{bmatrix}$$

for $r < s \leq R$, except the pairs (r, s) such that $\forall \rho(t)$: $w_r(\rho(t))w_s(\rho(t)) = 0$ and where $M_r = F_rP$. The feedback gains can be obtained as $F_r = M_rP^{-1}$.

Theorem 2. **Constraint on the control value**: Assume, that $\|x(0)\| \leq \phi$, where $x(0)$ is unknown, but the upper bound ϕ is known. The constrain $\|u(t)\| \leq \mu$ is enforced at all times $t > 0$ if the following LMIs hold

$$\phi^2 I \leq X, \quad \begin{bmatrix} X & {M_r}^T \\ M_r & \mu^2 I \end{bmatrix} \geq 0.$$

The observer vertex gains $K_{i_1,\, i_2,\, \ldots,\, i_N}$ stored in observer core tensor $\mathcal{K}$ are calculated in a similar fashion by applying the duality between the observer and controller.

4.5 Results of the control design

TP model of the FLEXOP aircraft TP model transformation was applied to the grid based LOM LPV model The airspeed domain under consideration is defined as $\rho = V_s$ in the interval $[45, 65]$ m/s over a grid of 41 equidistant points. The magnitude of the singular values drop significantly after the third singular value. Therefore, a 3 vertex system representation provides a good approximation for the grid based LPV model. The CNO type weighting functions are given in Figure 7. The the following signals are measured: $y = [a_{y_{CG}}\ Z_E\ q_{CG}\ L_{3az}\ L_{5az}\ L_{6az}\ R_{3az}\ R_{5az}\ R_{6az}]^T$.

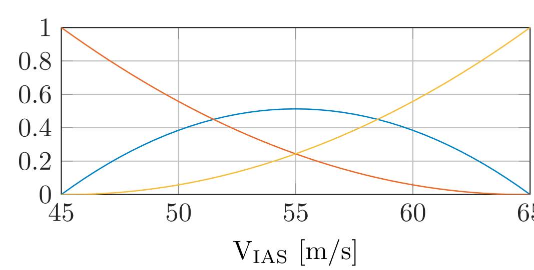


Fig. 7. CNO type weighting functions

Uncertainty model of the FLEXOP aircraft Since the aim of the control design is flutter suppression, it desirable to have robust stability in case of uncertainty in the flutter modes. 10% uncertainty is assumed in 2 elements of $A(\rho)$ that strongly influence the flutter modes. The pole migration of the flutter modes of the nominal and uncertain models are given in Figure 8.

Constraint on the control value The values for constraints ϕ and μ are determined based on physical considerations. Part of the states of the LOM model of the FLEXOP aircraft have physical meaning, thus a reasonably accurate upper bound on their values can be assumed.

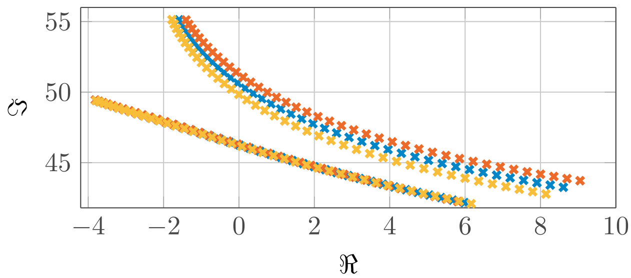


Fig. 8. Uncertainty of the flutter modes: nominal model (×), +10% uncertainty (×), -10% uncertainty (×)

In case of the structural dynamics modes and the aerodynamic lag states, open loop simulations are run to get a bound on $\|x\|$. This approach leads to $\|x\| = \phi = 45$. In order to keep the control signal u low, the lowest bound on μ that leads to a feasible design was found to be $\mu = 2$.

The resulting control structure is capable of stabilizing the LOM model of the FLEXOP aircraft (Figure 9.) and does not introduce any undesired fast poles.

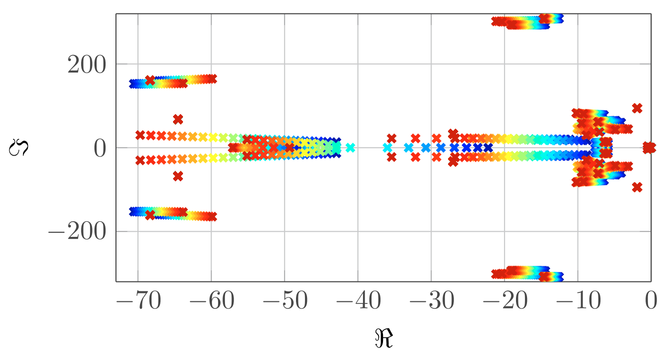
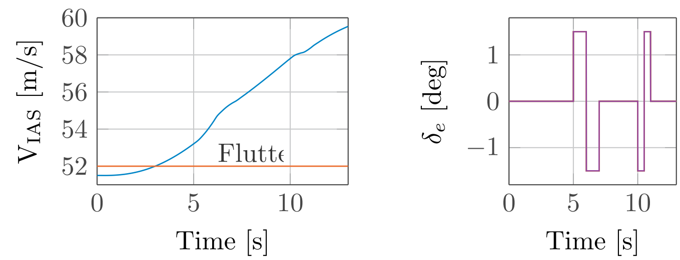


Fig. 9. Pole migration of the closed loop

5. SIMULATION RESULTS

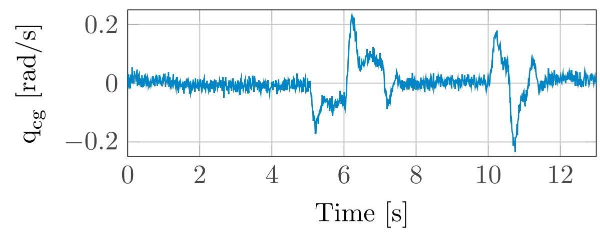
The effectiveness of the control design is validated via time domain simulations. The TP type observer and state feedback controller is connected to the high-fidelity, nonlinear ASE model of the FLEXOP aircraft. The simulation starts from trim condition, straight and level flight at 51.5 m/s, slightly under the flutter speed. Ramp signal is added to the trim value of the throttle signal in order to push the airspeed beyond flutter, see Figure 10. The control surfaces are kept in trim condition and are scheduled by the airspeed. Disturbance is injected through the elevators by 1.5° doublets. The actuators have 1 ms delay and sensors have noise. The response and the control signals of the FLEXOP aircraft are given in Figures 11 and 12.

It can be concluded that the TP type polytopic control structure designed based on the "bottom-up" model of the FLEXOP aircraft is successful in the flutter suppression of the high-fidelity, nonlinear ASE model. The control system is designed for airspeed up to 65 m/s and on the high-fidelity model it works well up to approximately 60-61 m/s airspeed. Thus, it can expand the flutter free envelope of the aircraft by 15%. The control commands are in realistic interval of ±2°. The time delay of the actuators and the sensor noise of the measurements do not have a significant effect on the control performance.

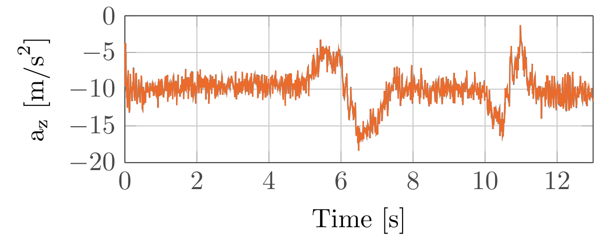


(a) Airspeed V_{IAS} (b) Elevator disturbance

Fig. 10. Simulation conditions



(a) q_{cg}



(b) $a_{z_{L6}}$ (——), $a_{z_{R6}}$ (——)

Fig. 11. Response of the FLEXOP aircraft

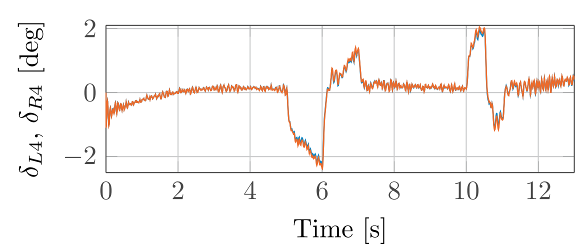


Fig. 12. Control signals: δ_{L4} (——), δ_{R4} (——);

6. CONCLUSION

The paper proposes a flutter suppression control design methodology for the FLEXOP demonstrator aircraft. The control oriented low order LPV model is obtained via the "bottom-up" modeling approach. The frequency range of interest in which the low order model is expected to be accurate is defined based on the flutter frequencies and on the actuator bandwidth. The ν-gap metric in the frequency rage of interest between the high-fidelity and low order models is below 0.2. The proposed control structure consist of a robust LPV observer and state feedback controller. The control design is based on the TP type polytopic model of the aircraft. Such 3 vertex system model is obtained via TP model transformation. The effectiveness of the resulting control system is validated by simulations using the high-fidelity, nonlinear model of the FLEXOP aircraft. The results show that the proposed control system extends the flutter free envelope of the aircraft from 52 m/s to approximately 60-61 ms. The control signal values are in a realistic interval and the controller is not overly sensitive to the time delay of the actuators and noise of the sensors.

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