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Structure Selection and LPV Model Identification of a Car Steering Dynamics ⋆

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Abstract: A Linear Parameter-Varying (LPV), discrete-time black box model of an electric power assisted steering system of a passenger car is identified from open-loop step response measurement data. The goal is to provide a nominal model for control design and analysis that is able to describe the principal characteristics of the system in the whole region of steering angle and speed range of 3 to 30 km/h. Examining a set of experimental data by using classical linear time-invariant black box modeling and validation techniques, the structure of the LPV model is determined. The parameters of the model are identified based on minimizing a quadratic error criterion by nonlinear optimization algorithms.

Keywords: System identification, black box modeling, linear parameter varying models, steering system dynamics, electric power steering, automotive control

1. INTRODUCTION

The ultimate goal of modeling presented in this paper is to support the model based control design for the steering system of an electric passenger car. The vehicle is adapted to support research activities in the field of autonomous vehicle control. In the current stage of the project low speed navigation tasks can be performed. Steering maneuvers are carried out by directly controlling the electric power assist unit – a permanent magnet synchronous (PMS) motor – by a real-time board computer. In normal operation mode the electric power steering (EPS) unit receives its input from a torque sensor mounted in the steering column to measure the driver's torque. In the autonomous control mode, this torque sensor signal is replaced by the control signal of the navigation controllers, and the hand-wheel is released by the pilot.

The low speed navigation control algorithms have a hierarchical structure of two levels. Based on a kinematic vehicle model, the upper level controller computes a steering angle that must be realized to follow a specified trajectory. The lower level controller receives this steering angle command and manipulates the EPS unit to realize the required steering angle as accurate as possible.

The goal of modeling in this paper is to describe the dynamic system including the EPS unit and the lateral dynamics of the vehicle. Available measurements are the vehicle speed as scheduling variable of the dynamics and the steering angle (a constant multiple of the measured hand-wheel angle). The manipulated control signal which is supplied to the power assist unit is also available and noise free.

It can be observed from the experiments taken in the autonomous control mode that the EPS unit introduces a dead-zone, i.e., no torque is applied when the control input is smaller than a certain threshold. In general, EPS control units have also a speed dependent characteristics, and may also depend on other inertial variables of the vehicle. The lateral vehicle dynamics is known to heavily depend on the vehicle speed. In this identification problem, the EPS and the steering dynamics are modeled jointly as a single dynamical system.

In the literature, the steering dynamics is often linearized for control purposes where the starting point of the design is an LPV vehicle model, Poussot-Vassal et al. (2011); Németh and Gáspár (2012). The steering dynamics is usually scheduled by the vehicle speed. A physically parameterized continuous-time velocity-scheduled LPV state-space model of a heavy-truck is identified in the paper by Rödönyi and Bokor (2005).

Proca and Keyhani (1998) identified the unknown subset of parameters of a continuous-time first principle state-space model of a power steering system. A detailed nonlinear model is also presented by Żardecki (2011).

In contrast to the above works, a control oriented low order model is derived in this paper merely from experimental data.

The contribution of the paper is the following. Structure and parameters of a black box steering model are identified from data. The model is valid over the whole region of operation (speed from 3 to 30km/h and steering angle

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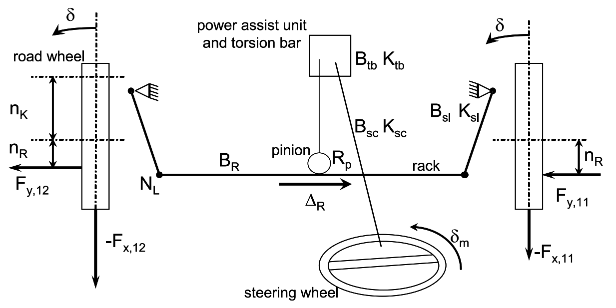

Fig. 1. Electronic Power System architecture

in [±30°]). The quality of the resulted LPV model is evaluated by comparing it with a set of local output-error models, which are identified at a single point of the region of operation.

2. MODELING APPROACH AND CHARACTERIZATION OF EXPERIMENTS

A general configuration of a steering system is illustrated in Fig. 1. The power assist unit is mounted on either the steering column, or the rack or the pinion. Based on first principle physical models of the steering system (see, e.g., Acarman et al. (2002)) and the assumed nonlinear characteristics of the power assist unit, it is expected that the system can be described as a series interconnection of a, possibly static, nonlinear system (the assist unit) and a speed scheduled LPV system.

Output error-models identified from experiments taken at constant speed may capture the dominant modes of the dynamics, even in the present of nonessential nonlinearities, Ljung (1999). Step response measurements provide a first idea about the static gain characteristics of the system. It is supposed that a set of local linear OE models provide hints for the choice of an appropriate model structure. Each local OE model is identified from a single step response experiment, and all experiments are taken over the whole region of operation, i.e., over a set of fixed different speed and a set of different constant control inputs. In this way every experiment is associated with a pair of parameters, $(v_{mean,i}, u_i)$: the average speed during the experiment and the constant control value of the step input. The distribution of available experiments on this (v_{mean}, u) coordinate system can be seen in Fig. 2.

Some representative experiments are shown in Figs. 3 and 4. Experiments are numbered from 1 to 127. The sampling time for the measurements is 20ms.

Measured steering angle and speed in Experiments 77 and 95 are plotted in Fig. 3. Experiment 77 shows a stair-like behavior, an example for the experiments indicated by purple squares in Fig. 2. There are multiple constant and rising segments in the steering angle in these experiments.

Experiments 21 and 27 present a typical phenomena when disturbances or variation of speed drive the steering angle in a position where, due to the steering system's geo-

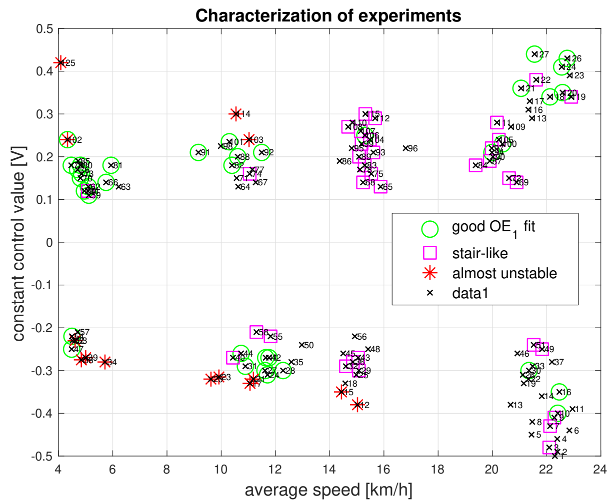

Fig. 2. Characterization of step response experiments

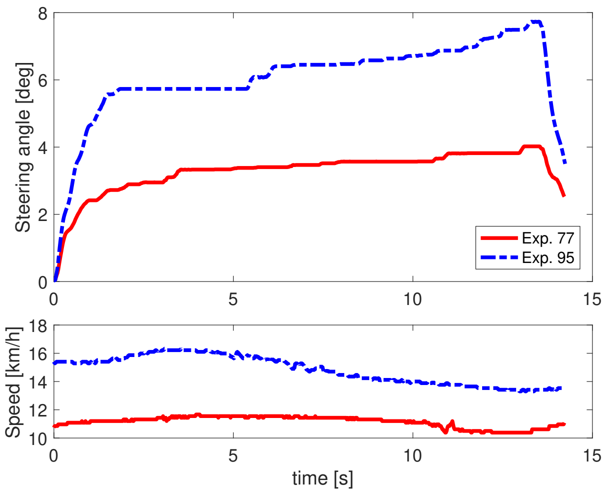

Fig. 3. Some representative experiments

metrical characteristics, less torque is required to increase the steering angle. Exp. 21 illustrates this situation: with constant control input, which is bigger than that of Exp. 27 only by a small amount, the steering angle increases up to the saturation position where the control input is set to zero.

Dead-zone, asymmetry and nonlinearity in the steady state gain can be observed from the step response experiments. In order to quantify these first impressions, OE models are fitted for each experiment, and their parameters are analyzed.

3. LOCAL OUTPUT ERROR MODELS

In the following, y_t, u_t, v_t and e_t denote respectively the steering angle, control input, vehicle speed measured at time t and measurement noise. In order to see the dominant modes of the system at various operating points, output error models

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} + \ldots + a_n y_{t-n}$$
$$+ b_0 u_{t-\tau} + b_1 u_{t-1-\tau} + \ldots + b_n u_{t-n-\tau} + e_t \quad (1)$$

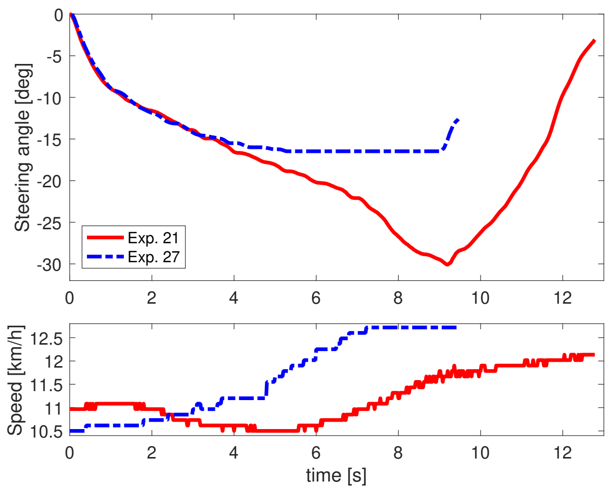

Fig. 4. Motivation for nonlinear output feedback

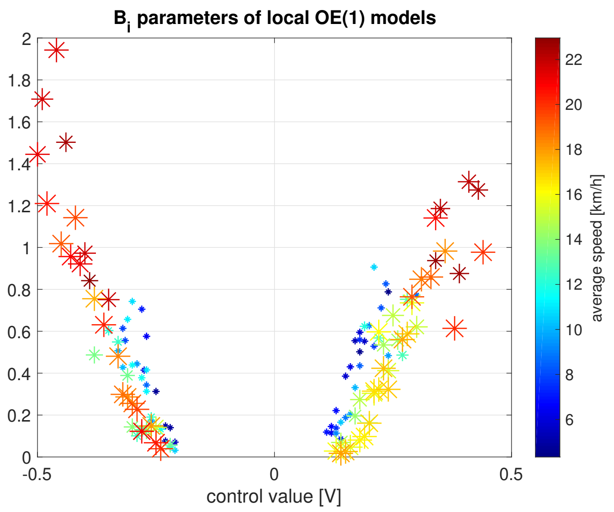

Fig. 5. Relation among parameters B_i of local OE(1) models, average speed and control value

of order $n \in \{1, 2, 3, 4, 5\}$ are identified for every experiment. The input delay is denoted by τ. Based on inspecting Akaike's information criterion, confidence ellipsoids of the identified poles, and variations of the poles for repeated experiments (or experiments with similar conditions), see Ljung (1999), it can be concluded that a first order model is sufficient to describe well the data at every point of the region of operation. The best choice for the input delay is at least two, it may vary for every experiment. In the following analysis the parameters of the first order OE models with $\tau = 2$ are examined in terms of the vehicle speed and the applied control input,

$$y_t = Ay_{t-1} + Bu_{t-\tau} + e_t. \tag{2}$$

That is, for each experiment an OE(1) model is fitted that minimizes a quadratic prediction error criterion. With every experiment indexed by i a corresponding parameter pair (A_i, B_i) is associated.

Two parameters, A_i and B_i, of all identified OE(1) models are plotted in Fig. 5 and 6 in terms of control and average speed of the actual experiment.

Concerning the distribution of parameters B_i (Fig. 5), a linear relation with both speed and control can be

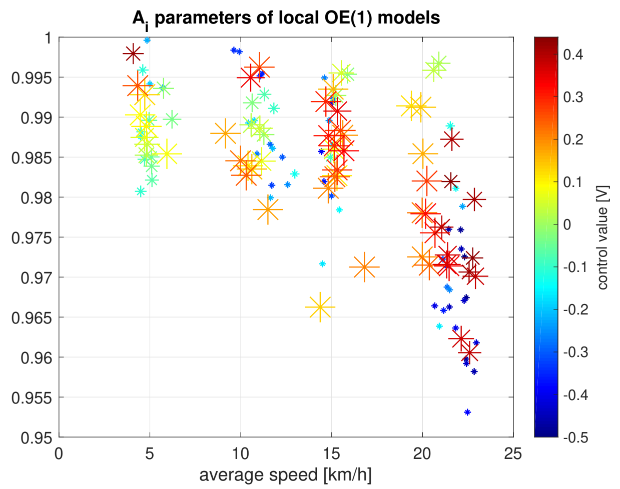

Fig. 6. Relation among parameters A_i of local OE(1) models, average speed and control value

assumed. Every point in the figure corresponds to a point in Fig. 2. Average speed is illustrated by colors and an increasing marker size. An asymmetric dead-zone can be observed which complies with our experiences.

Parameter A_i, the pole of linear OE(1) dynamics, shows a falling characteristics with speed, but dependence on the control value, which is closely related to the steady state steering angle values, is rather uncertain, see Fig. 6. Local OE(1) models are clearly fit to the effect of disturbances. Unfortunately, we have only one experiment at a given average speed and control value pair, thus no clear relation between this pole and control signal/steering angle can be concluded. Based on some experiments, however, such as those presented in Fig. 4, it can be assumed that a steering angle dependent pole shifting is present. This can be represented in the model by adding a static nonlinear output-feedback in the form

$$y_t = a(v_{t-1})y_{t-1} + b(v_{t-1})g_{t-1} + B(u_{t-1}, v_{t-1})u_{t-1-\tau} + e_t$$
$$g_t = c(y_t)y_t \tag{3}$$

where the new input gain $b(v_t)$ for the feedback effect is a nonlinear function of speed, and $c(y_t)$ is a steering angle dependent nonlinear feedback gain. The resulted LPV model has the following form

$$y_{t+1} = A(v_t, y_t)y_t + B(u_t, v_t)u_{t-\tau}, \tag{4}$$

where $A(v_t, y_t) = a(v_t) + b(v_t)c(y_t)$.

4. IDENTIFICATION OF LPV MODELS

According to the asymmetry of the steering dynamics, two LPV models are identified: one for the positive and one for the negative steering angle values. The final model can be considered as a hybrid LPV system. In the rest of the paper only the positive side is presented.

The LPV model is parametrized as follows. Based on the discussion on input gains B_i in the preceding section, $B(u, v)$ is assumed to be a piecewise linear function of its arguments. Let u_1 and u_2 denote the largest control values for which the ESP does not generate any torque, i.e., the borders of the dead-zone, at the lowest speed, v_{min} and the highest speed v_{max}, respectively. Let B_1 and B_2 the value of function $B(u, v)$ at a maximal control value, u_{max}, at v_{min} and v_{max}, respectively. These four parameters uniquely determine the piecewise linear function

$$B(u, v) = B_3(v) \frac{u - u_3(v)}{u_{max} - u_3(v)} \tag{5}$$

where

$$B_3(v) = \frac{v}{v_{max}}(B_2 - B_1) + B_1 \tag{6}$$

$$u_3(v) = \frac{v}{v_{max}}(u_2 - u_1) + u_1 \tag{7}$$

This way the input gain $B(u, v)$ is parameterized by

$$\theta_B = [u_1,\ u_2,\ B_1,\ B_2]^T.$$

For the parametrization of $A(v, y)$, piecewise linear functions $a(v)$, $b(v)$ and $c(y)$ are chosen. The vector of variable parameters, θ_A, consists of the function values over a fixed grid of arguments, i.e,

$$\theta_A := [a_1, \ldots, a_{n_v}, b_1, \ldots, b_{n_v}, c_1, \ldots, c_{n_y}]^T,$$

where $a_1 = a(v_1), \ldots, c_1 = c(y_1), \ldots$ with $v_1, \ldots, v_{n_v} \in [v_{min}, v_{max}]$ and $y_1, \ldots, y_{n_y} \in [0, y_{max}]$ are fixed grid points in the admissible interval of speeds and steering angles, respectively.

The set of fixed parameters consists of the grid points corresponding to some selected speed and steering angle values. The optimal choice of these points is determined by the data: only those values are worth selecting where enough data is concentrated.

The goal of identification is to minimize the normalized quadratic error criterion

$$V(\theta) = \frac{1}{N_e} \sum_{i=1}^{N_e} \frac{\sum_{t=0}^{N_i - 1}(y_t - y_t(\theta))^2}{\sum_{t=0}^{N_i} y_t^2} \tag{8}$$

where N_e is the number of experiments used in the identification, N_i is the length of experiment i, $\theta = [\theta_B^T,\ \theta_A^T]^T$ is the vector of variable parameters of the LPV model (4), and $y_t(\theta)$ is the simulated response of the LPV model at time t.

5. EVALUATION OF IDENTIFIED LPV MODELS

The results of identification are summarized in this section. The piecewise linear functions $a(v), b(v), c(y)$ and $B(u, v)$ minimizing (locally) criterion function $V(\theta)$ are presented in Fig. 7. It can be seen from the functions $a(v), b(v)$ and $c(y)$ that $A(v, y)$ is very close to 1 at very low speed, and it is also close to 1 at about 12-15 km/h. This is in accordance with Fig. 2, where red stars represent experiments with saturating steering angle.

Criterion function (8) can be computed for the OE(1) models so that $y_t(\theta)$ is replaced with the simulated responses of the OE models. When comparing the local OE and the LPV models we have to keep in mind that both models have advantages. OE models are able to fit the effect of the actual disturbances in Experiment i, but LPV models are able to exploit speed information to improve the fit.

Figures 8-11 show the measured steering angle and the simulated model outputs of both the local OE model and the global LPV model for some selected experiments. It can be seen that in a wide range of speed and steering angle, the LPV model show better fit to the data than the OE models due to the scheduling speed information. The achieved criterion function values are $V(\{OE\}_{i=1}^{N_e} = 0.00585)$ for the local OE models and $V(\theta) = 0.0189$ for the LPV model, more than three times larger than that of the OE models.

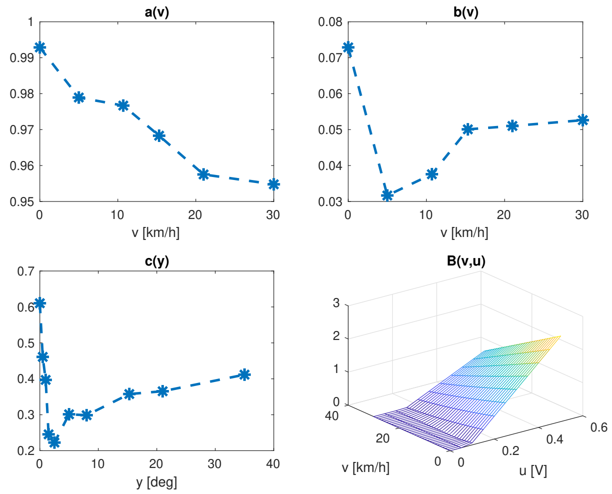

Fig. 7. Identified parameters of LPV steering model

The advantage of the nonlinear output feedback term defined by (3) can be seen if the identification is repeated without that term. In this case the achieved criterion was 1.74 times larger than that of the complete LPV model with feedback.

As can be seen in Fig. 2 the experiments do not represent the whole region of operation equally well. If we want to improve the LPV model denser grids of speed and steering angle should be chosen. According to the discussion in the preceding section the model should be trained by a sufficient amount of data at those grid points.

As a conclusion, it can be said that the presented approach for choosing a low order LPV model of the steering dynamics is promising, but to ensure that the model is really acceptable over the whole region of operation, specially where small disturbances may drive the system to the saturation boundary, more experiments are required.

6. CONCLUSION

The goal of the modeling is to provide a autonomous vehicle control oriented low order discrete-time model that captures the fundamental characteristics of an electrical power assisted steering system. The proposed approach to find an appropriate model structure is to identify a set of local linear models that cover the whole region of operations in terms of vehicle speed and control effort/steering angle, and then to analyze the dependence of the parameters on speed, control input and steering angle. Fortunately, all local linear models a found to be of first order on the whole region of operation. The analysis of the parameter dependencies and the experimental data suggested 1.) a hybrid system (different dynamics for the left and respectively the right directions); 2.) in

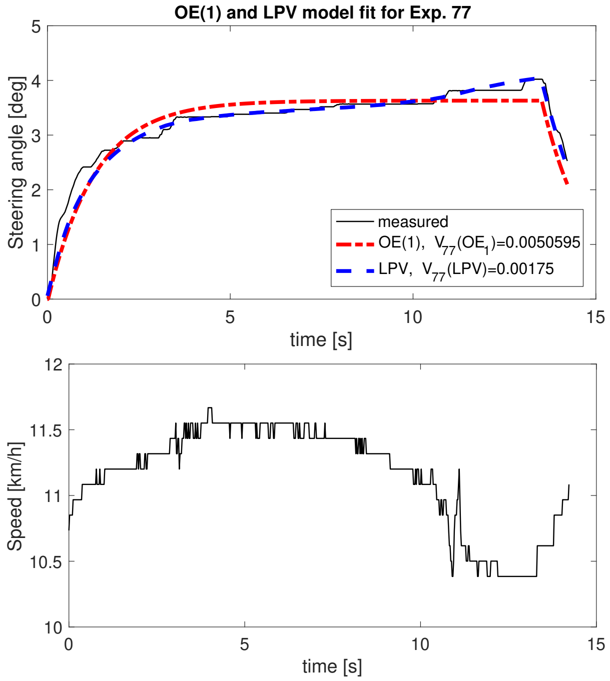

Fig. 8. Fit of local OE(1) and LPV model for Experiment 77

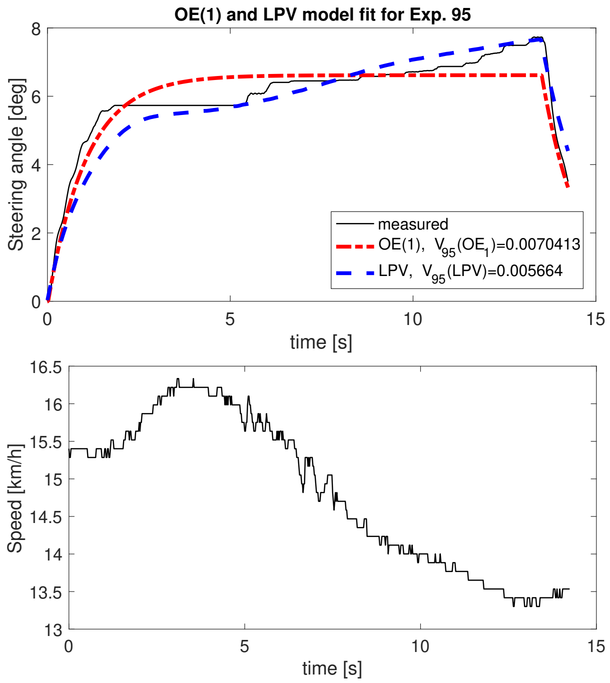

Fig. 9. Fit of local OE(1) and LPV model for Experiment 95

each direction, a first order LPV system where the state transition parameter A is scheduled by the speed and the steering angle, while the input gain B is scheduled by speed and control input. Both A and B are defined as special piecewise linear functions, whose parameters are optimized according to a normalized quadratic error criterion.

It can be concluded that, although the obtained LPV model outperformed the local linear models, validation of

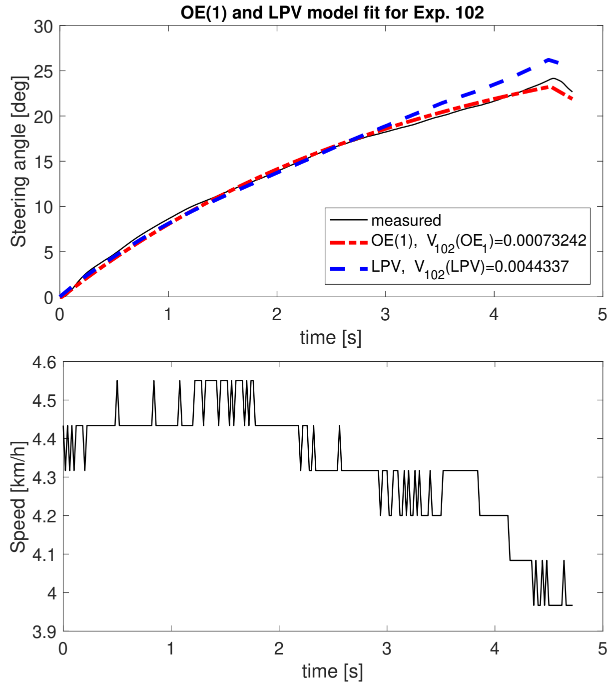

Fig. 10. Fit of local OE(1) and LPV model for Experiment 102

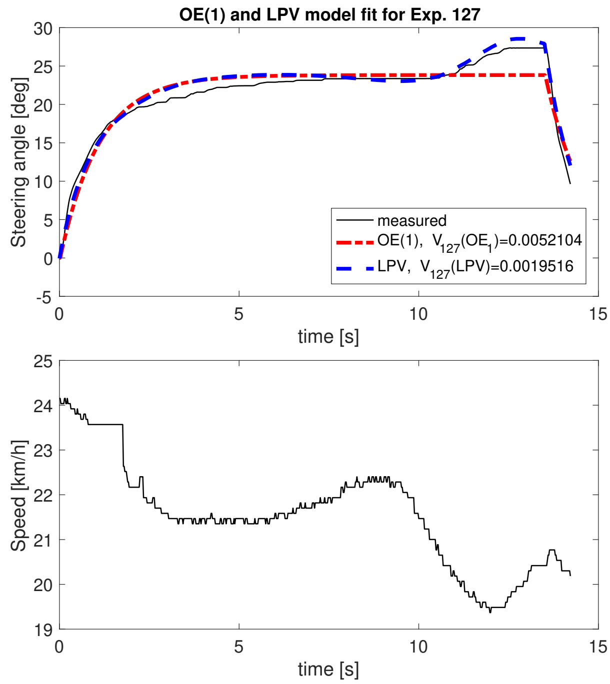

Fig. 11. Fit of local OE(1) and LPV model for Experiment 127

the model and further improvement require much more experiments.

An important future work is closed-loop validation of the model where the criterion of model assessment is related to the control objective.

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