Robust Suspension Fault Detection♦,■

Sébastien VARRIER, Balázs NEMETH,
Damien KOENIG, John J. MARTINEZ and Péter GÁSPÁR
*** Gipsa-Lab, 38402 Saint Martin d' Hères, Cedex, France (e-mail:**
Sebastien.Varrier@grenoble-inp.fr,
Damien.Koenig@esisar.grenoble-inp.fr,
John-Jairo.Martinez-Molina@grenoble-inp.fr).

**** Systems and Control Laboratory, Computer and Automation Research Institute Hungarian Academy of Sciences, Kende u. 1317, Budapest, H-1111, Hungary**

Received: November 7, 2012

ABSTRACT

The work presented in this paper focuses on robust fault detection for suspension system. Most of suspension system - classical, active or semi active ones - contains non linear parameters within their reference model such as dampers or springs. The approach taken relies in developing fault indicator for such system. The non-linear model is rewritten as an LPV one, where the scheduling parameters are linked with the non-linear parameters. Then fault indicators are synthesized for the discrete-time LPV model based on the extended parity-space approach. Validation of the approach is performed with CarSim.
Keywords: suspension system, LPV, parity space, residual, robust fault detection.

♦ This work is partially supported by the French National Project ANR 0308 Blanc.

■ This work is partially supported by collaborative project PICS CROTALE in collaboration between France and Hungary.

1. INTRODUCTION

The main objective of a diagnosis is to characterize the state of a system : healthy or faulty. The analytical redundancy is comparing data provided by sensors with a reference model [3]. The output, called residual, gives an information on the matching between sensor data and the model. If the residual is null, the system is healthy. Otherwise, it is faulty.

Due to the increasing complexity of modern processes, faults on systems can lead to extremely serious conse- quences. Therefore, several fault detection and isolation (FDI) strategies for LPV systems have been used to en-force security in systems. Analytical redundancy-based methods are quite the most handled strategies, including parity space techniques initiated in [9] and well recalled in [12], some statistical and geometrical methods as proposed by [5,7,4], and some observer-based approaches as in [10,8,13].

In this paper, it is propoed 2 methodologies to detect fault on all kind of suspension systems : passive, active and semi-active. The parity-space methodology has been used as it proposed an easy implementable solution, compatible with on-line computations on embeeded systems.

The classical parity-space approach designed for LTI system is recalled in the next subsection 1.1 while its extention to LPV ones, presented in [16], is recalled in section 2. Then section 3 presents the modeling of the different suspension systems, and section 4 the result of fault detection on semi-active suspension system. Finally, section 5 concludes the paper.

1.1 Parity space-based diagnosis on LTI systems

The main work about Parity Space-based diagnosis was applied on Linear Time Invariant (LTI) systems and proposed by Chow and Willsky in 1984 [3]. The key is to consider a linear dynamic model :

$$x(k+1) = Ax(k) + Bu(k) + Sf(k) \tag{1}$$

$$y(k) = Cx(k) + Du(k) + Ff(k) \tag{2}$$

where $y \in \mathbb{R}^m$ is the vector of measurements, $f \in \mathbb{R}^m$ is the vector of faults, $x \in \mathbb{R}^n$ is the vector of unknown variables and $u \in \mathbb{R}^l$ is the vector of the inputs of the system.

The methodology of the parity space approach to detect faults on the system (1.1) is to express the outputs on a horizon s. It can be written on a matrix form as :

$$Y_s = G_{A,B,D}U_s + Hx(k) + G_{A,S,F}F \tag{3}$$

with $G_{A,B,D} = \begin{bmatrix} D & 0 & \cdots & \cdots & 0 \\ CB & D & 0 & \cdots & 0 \\ CAB & CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ CA^{s-1}B & CA^{s-2}B & \cdots & CB & D \end{bmatrix}$,

$Y_s = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+s) \end{bmatrix}$, $U_s = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+s) \end{bmatrix}$ and $H = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix}$.

From the structure of equation (3), analytical redundancy exists if and only if the orthogonal complement space of the columns of the matrix H is not empty. To ensure this property, s has to be chosen larger than the index of observability of the system (1). Under such conditions, a parity matrix W can always be formulated eliminating the unknown variables x and satisfying :

$$W \perp H \Leftrightarrow WH = 0 \tag{4}$$

$r_c = WH$ is called the construction residual. The corresponding parity vector is :

$$r = W(Y_s - G_{A,B,D}U_s) \tag{5}$$

and is also called implementation residual. If the system is healthy, $f = 0$, and the residual is null $r = 0$.

Nevertheless, in a faulty behavior, faults are non null $f \neq 0$. The residual is consequently sensitive to the fault f as $r = WG_{A,S,F}f$.

1.2 Robustness against unknown inputs

In some applications, it can be interesting no to be sensitive to unknown inputs. The system (1,2) can be rewritten as :

$$x(k+1) = Ax(k) + Bu(k) + Sf(k) + B_d d(k) \tag{1'}$$

$$y(k) = Cx(k) + Du(k) + Ff(k) + B_d d(k) \tag{2'}$$

where $d(k)$ stands for unknown inputs. The previous parity space approach lead to the following equation :

$$Y_s = G_{A,B,D}U_s + Hx(k) + G_{A,B_d,D_d}D_s + G_{A,S,F}F \tag{3'}$$

In order to guarantee the robustness against the unknown inputs, the parity matrix W is synthesized as :

$$(W \perp H)\,\&\,(W \perp G_{A,B_d,D_d}) \Leftrightarrow W[H \mid G_{A,B_d,D_d}] = 0$$

Same conclusions than in section 1.1 can be made.

1.3 Parity space-based diagnosis on uncertain systems

Recently, the interest of researchers for taking into account modeling uncertainties in fault detection is growing up. First results were proposed by Adrot in his Ph.D thesis [1], where the parity matrix where not obtained by a perfect orthogonality, but by an optimization process. In Idrissi *et al* [4] a perfect orthogonality is obtained for static uncertain systems.

In the following study, a parity space approach is used to tackle the varying parameter, proposing a scheduled parity matrix with a perfect orthogonality on LPV dynamic systems.

2. RESIDUAL GENERATION ON LPV SYSTEMS

2.1 System modeling

Consider an LPV system defined by :

$$x(k+1) = A(\rho(k))x(k) + B(\rho(k))u(k) \quad (6)$$

$$y(k) = C(\rho(k))x(k) + D(\rho(k))u(k) \quad (7)$$

where ρ is the vector of the N scheduling parameters $\rho = [\rho_0(k) \quad \rho_1(k) \quad \cdots \quad \rho_N(k)]^t$.

Equation (6) can be rewritten on a matrix form :

$$[A(\rho(k)) \quad -\mathbb{I}_n]\begin{bmatrix} x(k) \\ x(k+1) \end{bmatrix} = -B(\rho(k))u(k) \quad (8)$$

According to the method presented in section 1.1, equation (8) can be expressed along the horizon s in a matrix form :

$$\mathcal{A}(\rho)\underbrace{\begin{bmatrix} x(k) \\ x(k+1) \\ \vdots \\ x(k+s) \end{bmatrix}}_{\mathcal{X}(s)} = \mathcal{B}(\rho)\underbrace{\begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+s-1) \\ u(k+s) \end{bmatrix}}_{\mathcal{U}(s)} \quad (9)$$

with

$$\mathcal{A}(\rho) = \begin{bmatrix} A(\rho(k)) & -\mathbb{I}_n & 0 & \cdots & 0 \\ 0 & A(\rho(k+1)) & -\mathbb{I}_n & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & A(\rho(k+s-1)) & -\mathbb{I}_n \end{bmatrix}$$

$$\mathcal{B}(\rho) = \begin{bmatrix} B(\rho(k)) & 0 & \cdots & 0 & 0 \\ 0 & B(\rho(k+1)) & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \cdots & 0 & B(\rho(k+s-1)) & 0 \end{bmatrix}$$

The outputs of the system are given by equation (7). It yields :

$$\mathcal{P}(\rho)\mathcal{X}(s) = \mathcal{Q}(\rho)\underbrace{\begin{bmatrix} \mathcal{Y}(s) \\ \mathcal{U}(s) \end{bmatrix}}_{\mathcal{Z}(s)} \quad (10)$$

with $\mathcal{Y}(s) = [y(k)^t \quad y(k+1)^t \quad \cdots \quad y(k+s)^t]^t$,

$$\mathcal{P}(\rho) = \begin{bmatrix} C(\rho(k)) & 0 & \cdots & 0 \\ 0 & C(\rho(k+1)) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & C(\rho(k+s)) \\ A(\rho(k)) & -\mathbb{I}_n & 0 & \cdots 0 \\ 0 & A(\rho(k+1)) & -\mathbb{I}_n & \ddots \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots 0 & A(\rho(k+s-1)) & -\mathbb{I}_n \end{bmatrix}$$

$$\mathcal{Q}(\rho) = \begin{bmatrix} \mathbb{I}_n & -\mathcal{D}(\rho) \\ 0 & -\mathcal{B}(\rho) \end{bmatrix}$$

$$\mathcal{D}(\rho) = \begin{bmatrix} D(\rho(k)) & 0 & \cdots & 0 \\ 0 & D(\rho(k+1)) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & D(\rho(k+s)) \end{bmatrix}$$

The main advantage of considering the above structure of the LPV systems lies in the fact that matrices $A(\rho(k))$ and $C(\rho(k))$ are linear in the scheduling parameters and can be expressed as :

- $A(\rho(k)) = A_0 + \sum_{i=1}^{N} \rho_i(k) A_i$
- $C(\rho(k)) = C_0 + \sum_{i=1}^{N} \rho_i(k) C_i$

As a consequence, $\mathcal{P}(\rho)$ can be decomposed following each scheduling parameter $\rho_i(k+z)$ ($i \in [1, N]$ and $z \in [0, s]$) as :

$$\mathcal{P}(\rho) = P_0 + \sum_{z=0}^{s} \sum_{i=1}^{N} \rho_i(k+z) P_{zi} \qquad (11)$$

with $P_0 = \begin{bmatrix} C_0 & 0 & \cdots & 0 \\ 0 & C_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & C_0 \\ A_0 & -\mathbb{I}_n & 0 & 0 \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & A_0 & -\mathbb{I}_n \end{bmatrix}$ $\quad P_{zi} = \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \ddots & C_i & \vdots & \vdots \\ \vdots & \ddots & \ddots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 \\ 0 & \ddots & A_i & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix} \begin{matrix} \\ \\ \leftarrow z^{th} \\ \\ \\ \\ \\ \\ \\ \end{matrix}$ $\quad \forall z < s$

If $z = s$, the lower part of the $\mathcal{P}_{zi}$ matrix (corresponding to the part of the matrix A) is null.

2.2 PARITY MATRIX $\mathcal{W}$

The objective in this subsection is to find the so called parity matrix $\mathcal{W}(\rho)$ verifying

$$\mathcal{W}(\rho) \cdot \mathcal{P}(\rho) = 0 \qquad (12)$$

One can observe that the parity matrix is in our case dependent on the scheduling parameters. The objective is to decrease the conservatism, and to obtain a perfect decoupling face to the scheduled $\mathcal{P}$ matrix.

In [4], a structure for the parity matrix has been proposed to solve the previous equation.

Adapted to this case and after some algebraic manipulations, it follows :

$$\begin{aligned}
\mathcal{W}(\rho) &= \omega_0 \\
&+ \sum_{z=0}^{s} \sum_{i=1}^{N} \omega_{zi}\rho_i(k+z) \\
&+ \sum_{z=0}^{s} \sum_{i=1}^{N} \sum_{w>z}^{s} \sum_{j=1}^{N} \omega_{zi,wj}\rho_j(k+w)\rho_i(k+z) \\
&+ \sum_{z=0}^{s} \sum_{i=1}^{N} \sum_{j\geq i}^{N} \omega_{zi,zj}\rho_j(k+z)\rho_i(k+z) \\
&+ \cdots \\
&+ \underbrace{\sum_{z=0}^{s} \sum_{i=1}^{N} \cdots \sum_{t>r}^{s} \sum_{m=1}^{N}}_{q\text{productsof2sums}} \omega_{zi,\cdots,tm}\rho_m(k+t)\cdots\rho_i(k+z) \\
&+ \sum_{z=0}^{s} \sum_{i=1}^{N} \cdots \sum_{m\geq l}^{N} \omega_{zi,\cdots,zm}\rho_m(k+z)\cdots\rho_i(k+z)
\end{aligned} \tag{13}$$

This hard expression is simply expressing that there are as many submatrices ω_i as $\tilde{\rho}$, where $\tilde{\rho}$ denotes the products of $\rho_i(k+z)$.

It has to be pointed out that this parity matrix depends on the parameter q which reflects the maximal number of products of scheduling parameters. For instance, if $q=2$, $\mathcal{W}$ is defined by the four first lines of equation (24), and there do not have more than 2 product of ρ_i.

Each binomial zi characterizes a given parameter $\rho_i(k+z)$, and as each ρ_i is a scalar, all the products are commutable $\rho_i(k+z)\rho_j(k+y) = \rho_j(k+y)\rho_i(k+z)$. In order to avoid confusions, it will further be denoted respecting $z<y$ in that way $\rho_i(k+z)\rho_j(k+y)$.

Finally, the product $\mathcal{W}(\rho)\mathcal{P}(\rho)$ can be expressed as :

$$\begin{aligned}
\mathcal{W}(\rho)\mathcal{P}(\rho) &= \Gamma_0 + \rho_0(k)\Gamma_{00} + \cdots + \rho_N(k)\Gamma_{0N} \\
+ &\quad \rho_0(k+s)\Gamma_{s0} + \cdots + \rho_N(k+s)\Gamma_{sN} \\
+ &\quad \rho_0(k)^2\Gamma_{00,00} + \rho_0(k)\rho_0(k+1)\Gamma_{10,00} \\
&\quad \vdots \\
+ &\quad \rho_N(k+s)\rho_{N-1}(k+s)\cdots\rho_0(k+s)\Gamma_{s0,\cdots sN}
\end{aligned}$$

with every parameters Γ expressed as :

- $\Gamma_0 = \omega_0 P_0$
- $\Gamma_{zi} = \omega_{zi}P_0 + \omega_0 P_{zi}$
- $\Gamma_{zi,yj} = \omega_{zi,yj}P_0 + \omega_{zi}P_{yj} + \omega_{yj}P_{zi}$
- $\Gamma_{zi,yj,xk} = \omega_{zi,yj,xk}P_0 + \omega_{zi,yj}P_{xk} + \omega_{zi,xk}P_{yj} + \omega_{yj,xk}P_{zi}$

Note that in every evaluation of each Γ, matrices ω appear in a left product while P appear on the right one.

As the product $\mathcal{W}(\rho)\mathcal{P}(\rho)$ has to be null whatever the parameters ρ are, one solution is to ensure that each term Γ is null. Hence, the product can be rewritten in a matrix form:

$$\mathcal{W}(\rho)\cdot\mathcal{P}(\rho) = 0 \Leftrightarrow$$

$$\underbrace{\begin{bmatrix} {\omega_0}^t \\ {\omega_{01}}^t \\ \vdots \\ {\omega_{sN}}^t \\ \vdots \\ \end{bmatrix}^t}_{\Omega} \cdot \underbrace{\begin{bmatrix} P_0 & xxx & \cdots & P_{zi} & xxx \\ 0 & xxx & \cdots & 0 & xxx \\ \vdots & xxx & \cdots & P_0 & xxx \\ 0 & xxx & \cdots & 0 & xxx \end{bmatrix}}_{\mathcal{M}} \begin{matrix} \\ \\ \leftarrow zi \\ \\ \end{matrix} = 0 \tag{14}$$

3 APPLICATION ON SUSPENSIONS SYSTEMS

3.1 Modeling of suspensions

Different ways of modeling suspension systems has been proposed in the literature. The following equation recalls the modeling of a classical passive suspension.

$$\underbrace{\begin{bmatrix}\ddot{z}_s\\ \dot{z}_s\\ \ddot{z}_{us}\\ \dot{z}_{us}\end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix}-\frac{c}{m_s} & -\frac{k_s}{m_s} & \frac{c}{m_s} & \frac{k_s}{m_s}\\ 1 & 0 & 0 & 0\\ \frac{c}{m_{us}} & \frac{ks}{m_{us}} & -\frac{c}{m_{us}} & -\frac{k_s+k_t}{m_{us}}\\ 0 & 0 & 1 & 0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\dot{z}_s\\ z_s\\ \dot{z}_{us}\\ z_{us}\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}0\\0\\ \frac{k_t}{m_{us}}\\0\end{bmatrix}}_{B_u} z_r + B_F F_o$$

$$y = \begin{bmatrix}\ddot{z}_s\\ \ddot{z}_{us}\\ z_{def}\end{bmatrix} = \underbrace{\begin{bmatrix}-\frac{c}{m_s} & -\frac{ks}{m_s} & \frac{c}{m_s} & \frac{ks}{m_s}\\ \frac{c}{m_{us}} & \frac{ks}{m_{us}} & -\frac{c}{m_{us}} & -\frac{ks+k_t}{m_{us}}\\ 0 & 1 & 0 & -1\end{bmatrix}}_{C}\begin{bmatrix}\dot{z}_s\\ z_s\\ \dot{z}_{us}\\ z_{us}\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ \frac{k_t}{m_{us}}\\0\end{bmatrix}}_{D_u} z_r + B_F F_o$$

So, the model under consideration is an LTI model.

In [14], an LPV model of active suspension is depicted. The modeling can be rewritten in the following LPV state space form :

$$\begin{aligned}\dot{x} &= A(\rho)x + B_u z_r + B_c F_c\\ y &= C(\rho)x + D_u z_r + D_c F_c\end{aligned}$$

From [15], a non linear model of semi-active dampers is proposed. It results the following LPV model of the system :

$$\underbrace{\begin{bmatrix}\ddot{z}_s\\ \dot{z}_s\\ \ddot{z}_{us}\\ \dot{z}_{us}\end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix}-\frac{\tilde{c}}{m_s} & -\frac{\tilde{k}_s}{m_s} & \frac{\tilde{c}}{m_s} & \frac{\tilde{k}_s}{m_s}\\ 1 & 0 & 0 & 0\\ \frac{\tilde{c}}{m_{us}} & \frac{ks}{m_{us}} & -\frac{\tilde{c}}{m_{us}} & -\frac{\tilde{k}_s+k_t}{m_{us}}\\ 0 & 0 & 1 & 0\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\dot{z}_s\\ z_s\\ \dot{z}_{us}\\ z_{us}\end{bmatrix}}_{x} + \underbrace{\begin{bmatrix}-\frac{f_c\rho_1}{m_s}\\ 0\\ \frac{f_c\rho_1}{m_{us}}\\ 0\end{bmatrix}}_{B_I(\rho_1)} I + \underbrace{\begin{bmatrix}0\\0\\ \frac{k_t}{m_{us}}\\0\end{bmatrix}}_{B_u} z_r + B_F F_o$$

$$y = \begin{bmatrix}\ddot{z}_s\\ \ddot{z}_{us}\\ z_{def}\end{bmatrix} = \underbrace{\begin{bmatrix}-\frac{\tilde{c}}{m_s} & -\frac{ks}{m_s} & \frac{\tilde{c}}{m_s} & \frac{ks}{m_s}\\ \frac{\tilde{c}}{m_{us}} & \frac{ks}{m_{us}} & -\frac{\tilde{c}}{m_{us}} & -\frac{ks+k_t}{m_{us}}\\ 0 & 1 & 0 & -1\end{bmatrix}}_{C}\begin{bmatrix}\dot{z}_s\\ z_s\\ \dot{z}_{us}\\ z_{us}\end{bmatrix} + \underbrace{\begin{bmatrix}-\frac{f_c\rho_1}{m_s}\\ \frac{f_c\rho_1}{m_{us}}\\ 0\end{bmatrix}}_{D_I(\rho_1)} I + \underbrace{\begin{bmatrix}0\\ \frac{k_t}{m_{us}}\\0\end{bmatrix}}_{D_u} z_r + B_F F_o$$

It has to be noticed that this LPV system contains the scheduling parameters only in matrices B and D, on a multiplicative way. As a consequence, it can be rewritten as an LTI system with an external input as follows :

$$\begin{aligned}\dot{x} &= Ax + B_u z_r + \widetilde{B_I}\mathrm{I}_{\rho_1}\\ y &= Cx + D_u z_r + \widetilde{D_c}\mathrm{I}_{\rho_1}\end{aligned}$$

where $\mathrm{I}_\rho = \mathrm{I}\,\mathrm{x}\rho_1$.

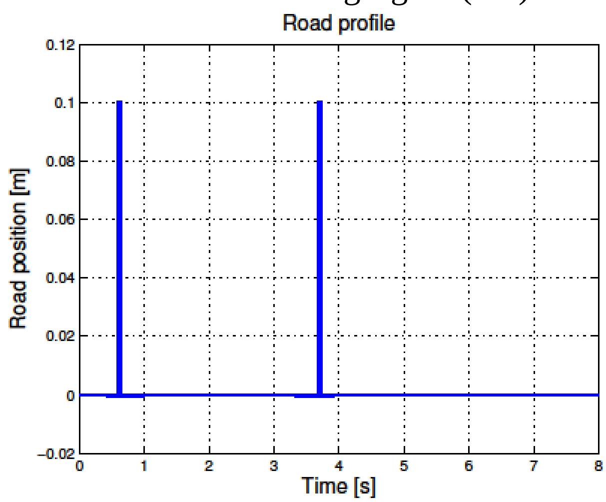
3.1 Fault detection procedure

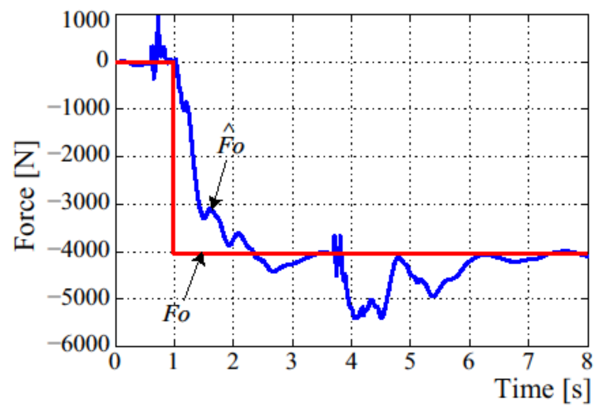
According to the 3 proposed model, the approaches presented in sections I and II can be applied. The following table recaps its application :

Passive damper :	1 – Discretize the system. 2 – Apply the LTI approach guarantying robustness against the road profile.
Active dampers :	1 – Discretize the system. 2 – Apply the LPV approach guarantying robustness against the road profile.
Semi-active dampers :	1 – Discretize the system. 2 – Apply the LTI approach according to the extern input I_ρ guarantying robustness against the road profile.

3.2 Results

The approach has been applied on a vehicle suspension system. The vehicle is equipped with semi-active suspensions. The road is composed of two successive bumps of 10cm as illustrated in the following figure (left) :





Simulations are performed in CarSim software. An actuator fault of $F_C = -4000\,N$ appeared at $t = 1s$. The fault detection methodology brought one residual illustrated in the front figure (right). It can be seen that the effect of road profile has well been attenuated, and the residual is well sensitive to the fault.

4 CONCLUSION

In this paper, it is proposed to detect faults on different kinds of suspension systems. It is recalled methodologies to detect fault on LTI systems and its extension to LPV ones. According to those theories, various kinds of faults on suspensions can be detected as sensor faults or actuator ones. An application on a semi-active suspension system is presented. A fault on the semi-active shock absorber has been detected, highlighting the effectiveness of the approach. As a perspective, applications on real vehicle are being handled.

REFERENCES

[1] J. Ackermann and T. Bunte. Yaw disturbance attenuation by robust decoupling of car steering. *Control Engineering Practice*, 5:1131–1136, 1997.

[2] O. Adrot. *Diagnostic à base de modèles incertains utilisant l'analyse par intervalles : l'approche bornante*. PhD thesis, Institut National Polytechnique de Lorraine, Centre de Recherche en Automatique de Nancy, 2000.

[3] P. Apkarian, P. Gahinet, and G. Becker. Self-scheduled $\mathcal{H}_\infty$ control of linear parameter-varying systems : A design example. *Automatica*, 31:1251–1261, 1995.

[4] Gary Balas, József Bokor, and Zolt'an Szabó. Invariant subspace for lpv systems and their applications. *IEEE Transactions on Automatic Control*, 48:2065–2069, 2003.

[5] Michèle Basseville and Igor V. Nikiforov. *Detection of Abrupt Changes - Theory and Application*. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1993.

[6] József Bokor and Gary Balas. Linear parameter varying systems : a geometric theory and applications. *Proceedings of the 16th IFAC World Congress*, 16, 2005.

[7] J. Cao and J. Gertler. Fault diagnosis in a class of nonlinear systems using identification and glr testing. *Proceedings of the American Control Conference*, 6:5052–5057, 2004.

[8] Jie Chen and Ron Patton. *Robust Model-Based Fault Diagnosis for Dynamic Systems*. Kluwer Academic Publishers, 1999.

[9] E. Chow and A. Willsky. Analytical redundancy and the design of robust failure detection systems. *IEEE Transactions on Automatic Control*, 1:603–619, 1984.

[10] S.X. Ding. *Model-based Fault Diagnosis Techniques*. Springer, Berlin, 2008.

[11] P.M. Frank. Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy : a survey and some new results. *Automatica*, 26:459–474, 1990.

[12] Janos Gertler. *Fault detection and diagnosis in engineering systems*. New York : Marcel Dekker, 1998.

[13] Silvio Simani, Cesare Fantuzzi, and Ron J. Patton. *Model-based Fault Diagnosis in Dynamic Systems Using Identification Techniques*. Springer, Berlin, 2002.

[14] Péter Gáspár, Zoltán Szabó, Gábor, Szederkényi and József Bokor. Two-level controller design for an active suspension system. *16th Mediterranean Conference on Control and Automation*

[15] Anh-Lam Do, Olivier Sename and Luc Dugard. *An LPV Control Approach for Semi-active Suspension Control with Actuator Constraints*. ACC 2012.

[16] Sébastien Varrier, Damien Koenig and John-J Martinez. *A Parity Space-Based Fault Detection On LPV Systems : Approach For Vehicle Lateral Dynamics Control System*, Safeprocess 2012, Mexico.