

COLOR MODELS OF SHADOW DETECTION IN VIDEO SCENES

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Abstract: In this paper we address the problem of appropriate modelling of shadows in color images. While previous works compared the different approaches regarding their model structure, a comparative study of color models has still missed. This paper attacks a continuous need for defining the appropriate color space for this main surveillance problem. We introduce a statistical and parametric shadow model-framework, which can work with different color spaces, and perform a detailed comparison with it. We show experimental results regarding the following questions: (1) What is the gain of using color images instead of grayscale ones? (2) What is the gain of using uncorrelated spaces instead of the standard RGB? (3) Chrominance (illumination invariant), luminance, or "mixed" spaces are more effective? (4) In which scenes are the differences significant? We qualified the metrics both in color based clustering of the individual pixels and in the case of Bayesian foreground-background-shadow segmentation. Experimental results on real-life videos show that CIE L*u*v* color space is the most efficient.

1 INTRODUCTION

Detection of foreground objects is a crucial task in visual surveillance systems. If we can retrieve the accurate silhouettes of the objects, or object groups, their high-level description becomes much easier, so it is favorable e.g. in detection of people (Havasi et al., 2006) or vehicles (Rittscher et al., 2000), respectively in activity analysis (Stauffer and Grimson, 2000).

The presence of moving shadows on the background makes difficult to estimate shape or behaviour of the objects, therefore, shadow detection is an important issue in the applications. However, we do not need to search for shadows cast on the foreground objects, since these 'self-shadowed' scenario parts consist to the foreground.

We find a thematic overview on several shadow detectors in (Prati et al., 2003). The methods are classified in groups based on their *model structures*, and the performance of the different model-groups are compared via test sequences. The authors note that the methods work in different color spaces, like RGB (Mi-

lic et al., 2000) and HSV (Cucchiara et al., 2001), however, it remains open-ended, how important is the appropriate color space selection, and which color space is the most effective regarding shadow detection. Moreover, we find also further examples: (Rittscher et al., 2000) used only gray levels for shadow segmentation, other approaches were dealing with the CIE L*u*v* (Martel-Brisson and Zaccarin, 2005), respectively CIE L*a*b* (Rautiainen et al., 2001) spaces. For the above reasons, the main issue of this paper is to give an experimental comparison of different color models regarding cast shadow detection on the video frames. For the comparison, we propose a general model framework, which can work with different color spaces. During the development of this framework, we have carefully considered the main approaches in the state-of-the art. Our presented model is the generalization of our previous work (Benedek and Szirányi, 2006).

2 BASIC NOTES

In (Prati et al., 2003), the authors distinguished *deterministic* methods (e.g. (Cucchiara et al., 2001)), which use on/off decision processes at each pixel, and *statistical* approaches (see (Mikic et al., 2000)) which contain probability density functions to describe the shadow-membership of a give image point. The classification of the methods whether they are deterministic or statistical depends often only on interpretation, since deterministic decisions can be done using probabilistic functions also. However, statistical methods have been widely distributed recently, since they can be used together with Markov Random Fields (MRF) to enhance the quality of the segmentation significantly (Wang et al., 2006).

First, we developed a deterministic method which classifies the pixels independently, since that way, we could perform a relevant quantitative comparison of the different color spaces. After that we gave a probabilistic interpretation to this model and we inserted it into a MRF framework which we developed earlier (Benedek and Szirányi, 2006). We compared the different results after MRF optimization qualitatively and observed similar relative performance of the color spaces to the deterministic model.

Another important point of view regarding the categorization of the algorithms in (Prati et al., 2003) is the discrimination of the *non parametric* and *parametric* cases. Non parametric, or 'shadow invariant' methods convert the video images into an illuminant invariant feature space: they remove shadows instead of detecting them. This task is often performed by a color space transformation, widely used illumination-invariant color spaces are e.g. the normalized rgb (Cavallaro et al., 2004), (Paragios and Ramesh, 2001) and $C_1C_2C_3$ spaces (Salvador et al., 2004). (We refer later to the normalized rgb as *rg* space, since the third color component is determined by the first and second.) In (Salvador et al., 2004) we find an overview on these approaches indicating that several assumptions are needed regarding the reflecting surfaces and the lightings. We have found in our experiments that these assumptions are usually not fulfilled in an outdoor environment, and these methods fail several times. Moreover, we show later that the *rg* and $C_1C_2C_3$ spaces are less effective also in the parametric case.

For the above reasons, we developed a *parametric* model: we extracted feature vectors from the actual and mean background values of the pixels and applied shadow detection as solving a classification problem in that feature space. This approach is widespread in the literature, and the key points are the way of *feature*

extraction, the *color space selection* and the *shadow-domain description* in the feature space. In Section 3, we introduce the feature vector which characterizes the shadowed pixels effectively. In Section 4, we describe the chosen shadow domain in the feature space, and define the deterministic pixel classification method. We show the quantitative classification results with the deterministic model regarding five real-world video sequences in Section 5. Finally, we introduce the MRF framework and analyse the segmentation results in Section 6.

We use three assumptions in the paper: (1) The camera stands in place and has no significant ego-motion. (2) The background objects are static (e.g. there is no waving river in the background), and the topically valid 'background image' is available in each moment (e.g. by the method of (Stauffer and Grimson, 2000)). (3) There is one *emissive light source* in the scene (the sun or an artificial source), but we consider the presence of additional effects (e.g. reflection), which may change the spectrum of illumination locally.

3 FEATURE VECTOR

Here, we define features for a parametric case where a shadow model can be constructed including some challenging environmental conditions. First, we introduce a well-known physical approach on shadow detection with marking that its model assumptions may not be fulfilled in real-world video scenes. Instead of constructing a more difficult illumination model, we overcome the appearing artifacts with a statistical description. Finally, the efficiency of the proposed model is validated by experiments.

3.1 Physical approach on shadow detection

According to the illumination model (Forsyth, 1990) the response $g(s)$ of a given image sensor placed at pixel s can be written as

$$g(s) = \int e(\lambda, s) \rho(\lambda, s) v(\lambda) d\lambda, \quad (1)$$

where $e(\lambda, s)$ is the illumination function, $\rho(s)$ depends on the surface albedo and geometric, $v(\lambda)$ is the sensor sensitivity. Accordingly, the difference in the shadowed and illuminated background values of a given surface point is caused by the different local value of $e(\lambda, s)$ only. For example, outdoors, the illumination function is the composition of the direct (sun), diffused (sky) and reflected (from other non-emissive objects) light components in the illuminated

background, while in the shadow the effect of the direct component is missing.

Although the validity of eq. (1) is already limited by several scene assumptions (Forsyth, 1990), in general, it is still too difficult to exploit appropriate information about the corresponding background-shadow values, since the components of the illumination function are unknown. With further strong simplifications (Forsyth, 1990) eq.(1) implies the well-known 'constant ratio' rule. Namely, the ratio of the shadowed $g_{sh}(s)$ and illuminated value $g_{bg}(s)$ of a given surface point is considered to be constant over the image:

$$\frac{g_{sh}(s)}{g_{bg}(s)} = A, \quad (2)$$

where A is the shadow 'darkening factor', and it does not depend on s .

In the CCD camera model (Forsyth, 1990) the RGB sensors are narrow banded and the constant ratio rule is valid for each color channel independently. Accordingly, the shadow descriptor is a triple $[A_r, A_g, A_b]$ containing the ratios of the shadowed and illuminated background values for the red, green and blue channels. Due to the deviation of the scene properties from the model assumptions (Forsyth, 1990), imprecise estimation of the background values (Stauffer and Grimson, 2000) and further artifacts caused by video compression and quantification, the ratio of the shadowed and estimated background values is never constant. However, to prescribe a *domain* instead of a single value for the ratios results a powerful detector (Siala et al., 2004). In this way, shadow detection is a one-class-classification problem in the three dimensional color ratio space.

3.2 Constant ratio rule in different color spaces

In this section, we examine, how can we use the previous physical approach in different color systems. We begin the description with some notes. We assume that the video images are originally available in the RGB color space, and for the different color space conversions, we use the equations in (Tkalcic and Tasic, 2003). The ITU D65 standard is used for calibration of the CIE $L^*u^*v^*$ and $L^*a^*b^*$ spaces. In the HSV, CIE $L^*u^*v^*$ and $L^*a^*b^*$ spaces we should discriminate two types of color components. One component is related to the brightness of the pixel (V , respectively L^* ; we refer them later as 'luminance' components), while the other components correspond to the 'chrominances'. We classify the color spaces also: since the normalized rg and $C_1C_2C_3$ spaces contain only chrominance components we will call them

'chrominance spaces', while grayscale and RGB are purely 'luminance spaces'. In this terminology, HSV, CIE $L^*u^*v^*$ and $L^*a^*b^*$ are 'mixed spaces'.

As we stated in the last section, the ratios of the shadowed and illuminated values of the R, G, B color channels regarding a given pixel are near to a global reference value $[A_r, A_g, A_b]$. In the following, we show by experiments that the 'constant ratio rule' is a reasonable approximation regarding the 'luminance' components of other color spaces also.

While shadow may darken the 'luminance' values of the pixels significantly the changes in the 'chrominances' is usually small. In (Cucchiara et al., 2001), the hue difference was considered as a zero-mean noise factor. This approach is sometimes inaccurate, for example outdoors, due to the ambient light of the blue sky, the shadow shifts to the 'blue' color domain (Fig 1, third column). We show that modelling the offset between the shadowed and illuminated 'chrominance' values of the pixels with a Gaussian additive term is appropriate.

To sum it up, if the current value of a given pixel in a given color space is $[x_0, x_1, x_2]$ (the indices 0, 1, 2 correspond to the different color components), the estimated background value is there $[m_0, m_1, m_2]$, we define the shadow descriptor $\bar{\Psi} = [\psi_0, \psi_1, \psi_2]$ by the following. For $i = \{0, 1, 2\}$:

- If i is the index of a 'luminance' component:

$$\psi_i(s) = \frac{x_i(s)}{m_i(s)}. \quad (3)$$

- If i is the index of a 'chrominance' component:

$$\psi_i(s) = x_i(s) - m_i(s). \quad (4)$$

We classify s as shadowed point, if its $\bar{\Psi}(s)$ value lies in a prescribed domain.

The efficiency of this feature selection can be observed in Fig. 1, where we plot the one dimensional marginal histograms of the occurring ψ_0 , ψ_1 and ψ_2 values for manually marked shadowed and foreground points of a 100-frames long outdoor surveillance video sequence ('Entrance pm'). Apart from some outliers, the shadowed ψ_i values lie for each color space and each color component in a 'short' interval, while the difference of the upper and lower bounds of the foreground values is usually greater. However, there is significant overlap between the one dimensional foreground and shadow histograms, therefore, as we examine in the next section, an effective shadow domain description is needed.

We define the descriptor in grayscale and in the rg space similarly to eq. (3) and (4) considering that $\bar{\Psi}$ will be a one, respectively two dimensional vector in these cases.

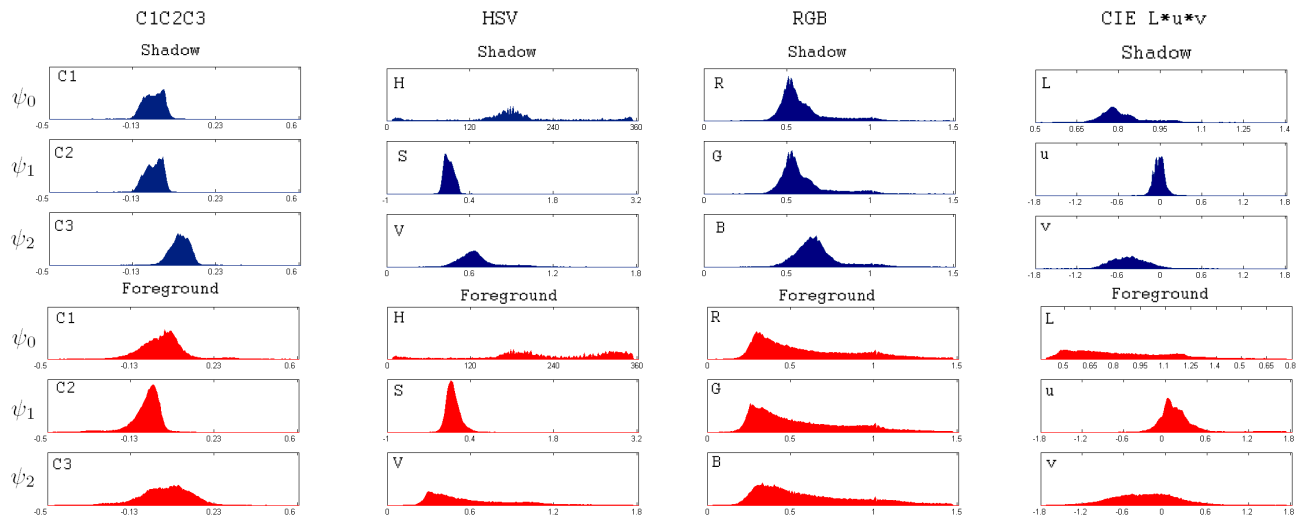


Figure 1: One dimensional projection of histograms of foreground (red) and shadow (blue) $\bar{\psi}$ values in the 'Entrance pm' test sequence.

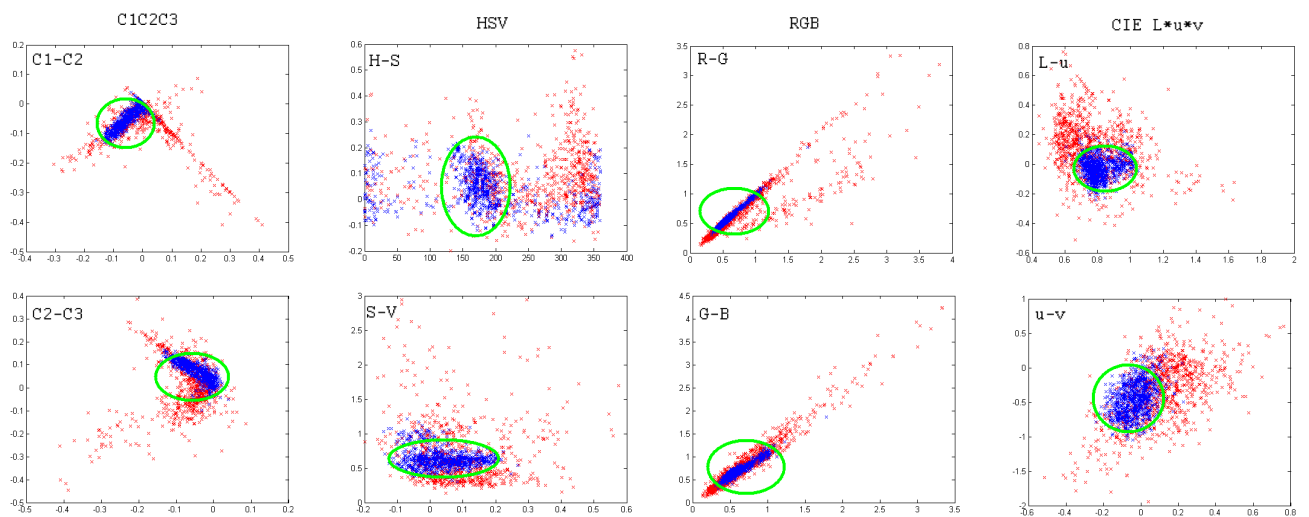


Figure 2: Two dimensional projection of foreground (red) and shadow (blue) $\bar{\psi}$ values in the 'Entrance pm' test sequence. Green ellipse is the projection of the optimized shadow boundary.

4 SHAPE OF THE SHADOW DOMAIN

The shadow domain is usually defined by a manifold having a prescribed number of free parameters, which fit the model to a given scene/situation. Previous methods used different approaches. The *domain of shadows* in the feature space is usually an interval for grayscale images (Wang et al., 2006). Regarding color scenes, this domain could be a three dimensional rectangular bin (Cucchiara et al., 2001): ratio/difference values for each channel lie between defined threshold; an ellipsoid (Mikic et al., 2000), or it may have general shape, like in (Siala et al., 2004). In the last case a Support Vector Domain Description is proposed in the RGB color ratios' space.

By each domain-selection we must consider overlap between the classes, e.g. there may be foreground points whose feature values are in the shadow domain. Therefore, the chosen shadow-domain should be not only large enough, containing 'almost all' the feature values corresponding to the occurring shadowed points, but also 'narrow' to decrease the number of the background or foreground points which are erroneously classified as shadows.

Accordingly, if we 'only' prescribe that a shadow descriptor should be accurate, the most general domain shape seems to be the most appropriate. However, in practise, a corresponding problem appears: the shadow domain may alter significantly (and often rapidly) in time due to the changes in the illumination conditions, and adaptive models are needed to follow these changes. It is sometimes not possible to train a model with supervision regarding each forthcoming case of illumination. Therefore, those domains are preferred, which have less free parameters, and we can construct an update strategy regarding them.

For these reasons, we used an elliptical shadow domain descriptor having *parallel axes* with the xyz coordinate axes:

$$\text{Pixel } s \text{ is shadowed} \Leftrightarrow \sum_{i=0}^2 \left(\frac{\Psi_i(s) - a_i}{b_i} \right)^2 \leq 1, \quad (5)$$

where $\{a_i, b_i \mid i = 0, 1, 2\}$ are the shadow domain parameters. For these parameters, a similar update procedure can be constructed to that we introduced in (Benedek and Szirányi, 2006). We found in the experiments, that the parameters of the 'chrominance' components are approximately constant in time. Although the mean darkening ratio of the 'luminance' components may change significantly, it can be estimated by finding the peak of the joint foreground-shadow Ψ histograms, which can be constructed without supervision, with an effective background subtraction algo-

rithm (e.g. (Stauffer and Grimson, 2000)).

We note that with the SVM method (Siala et al., 2004), the number of free parameters is related to the number of the support vectors, which can be much greater than the six scalars of our model. Moreover, for each situation, a novel SVM should be trained. For these reasons, we preferred the ellipsoid model, and in the following we examine its limits. For the sake of completeness, we note that the domain defined by eq. (5) becomes an interval if we work with grayscale images, and a two dimensional ellipse in the rg space. We visualize the shadow domain of the 'Entrance p-m' test sequence in Fig. 2, where the two dimensional projection of the occurring foreground and shadow $\bar{\Psi}$ values are shown corresponding to different color space selections. We can observe that the components of vector $\bar{\Psi}$ are strongly correlated in the RGB space (and also in $C_1C_2C_3$), and the previously defined ellipse cannot present a narrow boundary. (It would be better to fit an ellipse with arbitrary axes, but that choice would cause more free parameters in the system.) In the HSV space, the shadowed values are not within a convex hull, even if we considered that the hue component is actually periodical (hue = $k * 2\pi$ means the same color for each $k = 0, 1, \dots$). Based on the above facts, the CIE $L^*u^*v^*$ space seems to be a good choice. In the next section, we support this statement by experimental results.

5 COMPARATIVE EVALUATION WITH THE ELLIPSE MODEL

In this section, we show the tentative limits of the elliptical shadow domain defined by eq. (5). The evaluations were done through manually generated ground truth sequences regarding the following five videos:

- 'Laboratory' test sequence from the benchmark set (Prati et al., 2003). This shot contains a simple indoor environment.
- 'Highway' video (from the same benchmark set). This sequence contains dark shadows but homogenous background without illumination artifacts.
- 'Entrance am', 'Entrance noon' and 'Entrance p-m' sequences captured by the 'Entrance' (outdoor) camera of our university campus in different parts of the day. These sequences contain difficult illumination and reflection effects and suffer from sensor saturation (dark objects and shadows).

The evaluation metrics was the foreground-shadow discrimination rate. Denote the number of correct-

Video	Scene	Dark	Worst	Best
Laboratory	indoor	0.73	gray, RGB	Luv, Lab
Entrance am	outdoor	0.50	gray, RGB	Luv, Lab
Entrance pm	outdoor	0.39	$C_1C_2C_3$, rg	Luv, Lab
Entrance noon	outdoor	0.35	$C_1C_2C_3$, rg	Luv, Lab
Highway	outdoor	0.23	$C_1C_2C_3$, rg	Luv, RGB

Table 1: Indicating the two most successful and the two less effective color spaces regarding each test sequence. We also denote the mean of the darkening factor for shadows in the third column.

ly identified foreground pixels of the evaluation sequence by T_F . Similarly, we introduce T_S for the number of well classified shadowed points, A_F and A_S is the number of all the foreground, respectively shadowed ground truth points. The discrimination rate is defined by:

$$D = \frac{T_F + T_S}{A_F + A_S}. \quad (6)$$

Since the goal is to compare the limits of the discrimination regarding different color systems, we optimized the parameters in eq. (5) through the training data with respect to maximize the discrimination rate. We summarized the discrimination rates in Fig. 3, regarding the test sequences. We can observe that the CIE $L^*a^*b^*$ and $L^*u^*v^*$ produce the best results. However, the relative performance of the other color systems is strongly different in different videos (see also Table 1). In sequences containing dark shadows (Entrance pm, highway), the 'chrominance spaces' produce poor results, while the gray, RGB and Lab/Luv results are similarly effective. If shadow is brighter (Entrance am, Laboratory), the performance of the 'chrominance spaces' becomes reasonable, but the 'luminance spaces' are relatively poor. (In this case, shadow is characterized better by the illuminant invariant features than the luminance darkening domain). Since the hue coordinate in HSV is very sensitive to the illumination artifacts (Section 3), the HSV space is effective only in case of light-shadow.

6 SEGMENTATION BY USING BAYESIAN OPTIMIZATION

For practical use, the above color model should be inserted into a background-foreground-shadow segmentation process. Since we want to test the color

model itself, we use a Markov Random Field (MRF) optimization procedure (Geman and Geman, 1984) to get the globally optimal segmentation upon the above model.

The results in the previous section confirm that using the defined elliptical shadow domain, the CIE $L^*u^*v^*$ color space is the most effective to separate shadowed and foreground pixels only considering their colors, if we have enough training data. However, in several applications, we should consider the following facts:

1. Representative ground truth foreground-shadow points are not available, the optimal ellipse parameters should be estimated somehow.
2. The classification of a given pixel is usually done considering not only its color, but also the class of the neighbors (MRF).

Here we suit the proposed model to an adaptive Bayesian model-framework, and show that the advantage of using the appropriate color space can be measured directly in the applications.

The segmentation framework is a Markov Random Field (Geman and Geman, 1984), more specifically a Potts model (Potts, 1952). An image S is considered to be a two-dimensional grid of pixels (sites), with a neighborhood system on the lattice. The procedure assigns a label ω_s to each pixel $s \in S$ from the label-set: $L = \{bg, sh, fg\}$ corresponding to the three classes: foreground (fg), background (bg) and shadow (sh). Therefore, the segmentation is equivalent with a global labeling $\Omega = \{\omega_s \mid s \in S\}$. Each class at each pixel position is characterized by a conditional density function: $p_k(s) = P(x_s | \omega_s = k)$, $k \in L, s \in S$. Eg. $p_{bg}(s)$ is the probability of the fact that the background process generates the observed color value x_s at pixel s .

Following the Potts model, the optimal segmentation corresponds to the labeling which minimizes:

$$\hat{\Omega} = \operatorname{argmin}_{\Omega} \sum_{s \in S} -p_{\omega_s}(s) + \sum_{r, q \in S} \Phi(\omega_r, \omega_q), \quad (7)$$

where the Φ term is responsible for getting smooth, connected regions in the segmented image. $\Phi(\omega_r, \omega_q) = 0$ if q and r are not neighboring pixels, otherwise:

$$\Phi(\omega_r, \omega_q) = \begin{cases} -\beta & \text{if } \omega_r = \omega_q \\ +\beta & \text{if } \omega_r \neq \omega_q \end{cases}$$

The definition of the density functions $p_{bg}(s)$ and $p_{fg}(s)$ $s \in S$ is the same, as we defined in (Benedek and Szirányi, 2006). We use a mixture of Gaussian model for the pixel values in the background, where the parameters are determined using (Stauffer and Grimson, 2000). The foreground probabilities come from spatial pixel value statistics (Benedek and

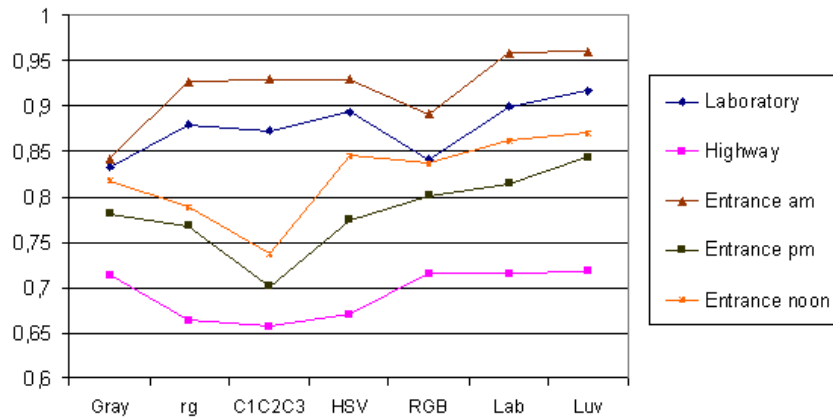


Figure 3: Foreground-shadow discrimination coefficient (D in eq. (6)) regarding different sequences

Szirányi, 2006).

Before inserting our model in the previously defined MRF framework, we give to the shadow-classification step defined in Section 4 a probabilistic interpretation. We rewrite eq. (5): we match the current $\overline{\Psi}(s)$ value of pixel s to a probability density function $f(\overline{\Psi}(s))$, and decide its class:

$$\text{pixel } s \text{ is shadowed} \Leftrightarrow f(\overline{\Psi}(s)) \geq t. \quad (8)$$

The domains defined by eq. (5) and eq. (8) are equivalent, if f is a Gaussian density function (η):

$$\begin{aligned} f(\overline{\Psi}(s)) &= \eta(\overline{\Psi}(s), \overline{\mu}_\Psi, \overline{\Sigma}_\Psi) = \\ &= \frac{1}{(2\pi)^{\frac{3}{2}} \sqrt{\det \overline{\Sigma}_\Psi}} \exp \left[-\frac{1}{2} (\overline{\Psi}(s) - \overline{\mu}_\Psi)^T \overline{\Sigma}_\Psi^{-1} (\overline{\Psi}(s) - \overline{\mu}_\Psi) \right] \end{aligned}$$

with the following parameters:

$$\overline{\mu}_\Psi = [a_0, a_1, a_2]^T, \quad \overline{\Sigma}_\Psi = \text{diag}\{b_0^2, b_1^2, b_2^2\},$$

while

$$t = (2\pi b_0 b_1 b_2)^{-\frac{3}{2}} e^{-\frac{1}{2}}.$$

Using Gaussian distribution for the occurring feature values is supported also by the one dimensional marginal histograms in Fig. 1.

In the following, the use of the previously defined probability density functions in the MRF model is straightforward:

$$p_{\text{sh}}(s) = f(\overline{\Psi}(s)).$$

The flexibility of this MRF model comes from the fact that we defined $\overline{\Psi}(s)$ shadow descriptors for different color spaces differently in Section 3. The method sets the parameters of the class models adaptively, similarly to (Benedek and Szirányi, 2006).

We compared the segmentation results using different color spaces in the MRF model (Fig. 4), and observed that the quality of the segmentation depends on the used color space similarly to that we measured in Section 5 .

7 CONCLUSION

This paper examined the color modelling problem of shadow detection. We developed a model framework for this task, which can work with different color spaces. Meanwhile, the model can detect shadows under significantly different scene conditions and it has a few free parameters which is advantageous in practical point of view. In our case, the transition between the background and shadow domains is described by statistical distributions. With this model, we compared several well known color spaces, and observed that the appropriate color space selection is an important issue regarding the segmentation results. We validated our method on five video shots, including well-known benchmark videos and real-life surveillance sequences, indoor and outdoor shots, which contain both dark and light shadows. Experimental results show that CIE L*u*v* color space is the most efficient both in the color based clustering of the individual pixels and in the case of Bayesian foreground-background-shadow segmentation.

REFERENCES

- Benedek, C. and Szirányi, T. (2006). Markovian framework for foreground-background-shadow separation of real world video scenes. In *Proc. Asian Conference on Computer Vision*. LNCS, 3851.
- Cavallaro, A., Salvador, E., and Ebrahimi, T. (2004). Detecting shadows in image sequences. In *Proc. of IEEE Conference on Visual Media Production*.
- Cucchiara, R., Grana, C., Neri, G., Piccardi, M., and Prati, A. (2001). The saktbot system for moving object detection and tracking. In *Video-Based Surveillance Systems-Computer Vision and Distributed Processing*.

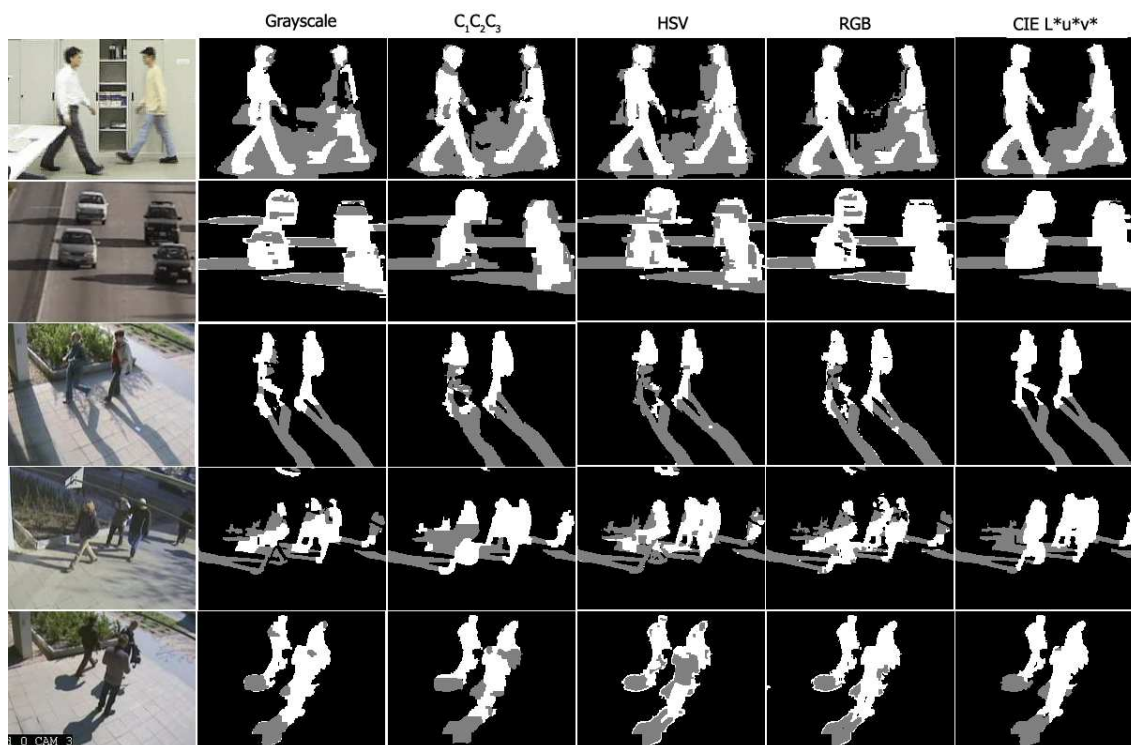


Figure 4: MRF segmentation results with different color models. Test sequences (up to down): 'Laboratory', 'Highway', 'Entrance am', 'Entrance pm', 'Entrance noon'.

- Forsyth, D. A. (1990). A novel algorithm for color constancy. In *International Journal of Computer Vision*.
- Geman, S. and Geman, D. (1984). Stochastic relaxation, gibbs distributions and the bayesian restoration of images. *IEEE Trans. Pattern Analysis and Machine Intelligence*, pages 721–741.
- Havasi, L., Szilávik, Z., and Szirányi, T. (2006). Higher order symmetry for non-linear classification of human walk detection. *Pattern Recognition Letters*, 27:822 – 829.
- Martel-Brisson, N. and Zaccarin, A. (2005). Moving cast shadow detection from a gaussian mixture shadow model. In *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*.
- Mikic, I., Cosman, P., Kogut, G., and Trivedi, M. M. (2000). Moving shadow and object detection in traffic scenes. In *Proceedings of International Conference on Pattern Recognition*.
- Paragios, N. and Ramesh, V. (2001). A mrf-based real-time approach for subway monitoring. In *Proc. IEEE Conference in Computer Vision and Pattern Recognition*.
- Potts, R. (1952). Some generalized order-disorder transformation. In *Proceedings of the Cambridge Philosophical Society*.
- Prati, A., Mikic, I., Trivedi, M. M., and Cucchiara, R. (2003). Detecting moving shadows: algorithms and evaluation. *IEEE Trans. Pattern Analysis and Machine Intelligence*, (7):918–923.
- Rautiainen, M., Ojala, T., and Kauniskangas, H. (2001). Detecting perceptual color changes from sequential images for scene surveillance. *IEICE Transactions on Information and Systems*, pages 1676 – 1683.
- Rittscher, J., Kato, J., Joga, S., and Blake, A. (2000). A probabilistic background model for tracking. In *Proc. European Conf. on Computer Vision*.
- Salvador, E., Cavallaro, A., and Ebrahimi, T. (2004). Cast shadow segmentation using invariant color features. *Comput. Vis. Image Underst.*, (2):238–259.
- Siala, K., Chakchouk, M., Chaieb, F., and O.Besbes (2004). Moving shadow detection with support vector domain description in the color ratios space. In *Proceedings of the 17th International Conference on Pattern Recognition*.
- Stauffer, C. and Grimson, W. E. L. (2000). Learning patterns of activity using real-time tracking. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 22(8):747–757.
- Tkalcic, M. and Tasic, J. (2003). Colour spaces - perceptual, historical and applicational background. In *Proc. Eurocon 2003*.
- Wang, Y., Loe, K.-F., and Wu, J.-K. (2006). A dynamic conditional random field model for foreground and shadow segmentation. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 28(2):279–289.