ECG Feature Learning by Using Rational Variable Projection Autoencoders

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Abstract

In this paper, we propose a model-based shallow autoencoder structure to automatically extract features from electrocardiogram (ECG) data. The encoding path in our model employs parametrized orthogonal transformations by means of rational function systems, and utilizes Variable Projections (VP) to compute low-dimensional representations of individual heartbeats. After the global training of this rational VP autoencoder, we used the linear coefficients of the projections in the encoding as ECG heartbeat features. We evaluated the performance of the proposed feature learning scheme on the standard 5-class AAMI heartbeat classification problem using the benchmark MIT-BIH Arrhythmia Database, training separate support vector machine and random forest classifier models on the extracted features. Employing the subject-oriented (inter-patient) evaluation scheme, we achieved an accuracy exceeding 94%. This performance is comparable to other state-of-the-art ECG classification approaches, while providing a computationally simple and explainable method for learning features from raw ECG data.

1. Introduction

State-of-the-art deep learning methodologies offer highly effective ways for extracting optimal representations from medical data. However, these are typically model-agnostic, end-to-end methods that demand substantial computational power. In this paper, we focus on lightweight model-based learning approaches to automatically extract features from electrocardiogram (ECG) data, providing interpretable and explainable parameters besides optimal representation.

Model performance is evaluated in the context of heartbeat classification for arrhythmia detection, employing the standard 5-class AAMI classification problem, following the subject-oriented (inter-patient) evaluation scheme proposed by de Chazal [1]. The recent developments in this field is predominated by end-to-end deep learning approaches (like deep, convolutional, and recurrent neural networks). For an overview we refer to [2, 3], but we note that the results are challenging to objectively compare, because the involved number and type of arrhythmia classes and also the evaluation schemes vary in the papers. Traditional machine learning approaches involve feature extraction methods combined with separate classifiers. Here, hand-crafted features might include both morphological (waveform related) and dynamic (rhythm related) descriptors of ECG heartbeats. Morphological features are typically extracted in a model-based manner, involving dimension reduction by means of mathematical transformations (like statistical and shape descriptors, principal and independent component analysis, wavelets, or variable projections), and dynamic descriptors are usually RR interval features. We refer to [4] for an earlier survey.

We developed a model-based shallow autoencoder structure, where the encoder employs parametrized orthogonal transformations by means of rational function systems: the so-called real valued Malmquist–Takenaka (MT) basis. Utilizing the Variable Projections (VP) in the encoding path, our approach computes low-dimensional representations of individual heartbeats, incorporating both linear and non-linear parameters. The trainable non-linear parameters comprise the poles of rational basis functions, represented in hyperbolic geodetic polar coordinates. MT systems are widely used in signal processing and control theory (see e.g. [5]). Our approach is also inspired by the recent success of adaptive orthogonal transformations and VP in biomedical signal processing applications: as model-driven methods [6, 7], and also in traditional machine learning [8–10] and model-based deep learning settings [11–13]. To investigate the generalization ability of the proposed feature learning method, the training was not patient-specific; instead, non-linear parameters were globally trained and tested on patient-wise distinct subsets of the entire dataset. After training the rational VP autoencoder, we used the linear coefficients of the projections in the encoding as the features extracted from the ECG measurements. Subsequently, separate support vector machine (SVM) and random forest (RF) classifier models were trained on the extracted features to distinguish normal and abnormal heartbeat signals. The proposed method provides an efficient and compact feature representation,

 Page 1 ISSN: 2325-887X DOI: 10.22489/CinC.2024.165

where both the non-linear parameters and the linear coefficients hold explainable interpretation related to heartbeat morphology.

2. Rational variable projection

In the following, we briefly introduce the rational Malmquist–Takenaka (MT) system and the corresponding VP, along with its advantages for ECG signal processing. The MT basis functions can be expressed as

$$\Phi_n(z) := \frac{\sqrt{1-|b_n|^2}}{1-\overline{b_n}z}\prod_{k=0}^{n-1} B_{b_k}(z) \quad (z \in \mathbb{T},\ n \in \mathbb{N}),$$

where $b_n \in \mathbb{D}$ ($n \in \mathbb{N}$) are the so-called inverse poles that serve as non-linear system parameters, $\mathbb{T}$ and $\mathbb{D}$ denotes the complex unit circle and unit disk, respectively, and

$$B_b(z) := \frac{z-b}{1-\overline{b}z} \quad (z \in \mathbb{T},\ b \in \mathbb{D})$$

denotes the Blaschke function of parameter b. The MT functions forms an orthonormal system on the unit circle, with respect to the usual scalar product

$$\langle F, G\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(e^{it})\overline{G(e^{it})}\,dt \quad (F, G \in L^2(\mathbb{T})).$$

If $b_0 = 0$, then a real valued orthonormal MT system can be constructed as $\Psi_0 := 1$ and

$$\Psi_{2n-1} := \sqrt{2}\,\mathrm{Re}\,\Phi_n,\ \Psi_{2n} := \sqrt{2}\,\mathrm{Im}\,\Phi_n \quad (n \in \mathbb{N}^+).$$

For ECG heartbeat modeling purposes, consider a discrete representation. Let M and $P \in \mathbb{N}^+$ be the number of sampling points and the number of non-trivial inverse poles, respectively, and $\theta := (0, b_1, \ldots, b_P) \in \mathbb{D}^{P+1}$. Denote the system matrix by $D_\theta \in \mathbb{R}^{M\times(2P+1)}$, whose columns consists of the MT basis functions uniformly sampled over the unit circle, i.e. for $j = 0, 1, \ldots, M-1$ and $k = 0, 1, \ldots, 2P$:

$$[D_\theta]_{jk} = \Psi_k\left(e^{i(2\pi j/M-\pi)}\right).$$

Then the input signal $x \in \mathbb{R}^M$ can be encoded by linear parameters $E_\theta x \in \mathbb{R}^{2P+1}$ for the approximation $x \approx D_\theta E_\theta x$. If θ is fixed, the ordinary least squares optimal encoding is given by $E_\theta x = D_\theta^+ x$, where D^+ denotes the Moore–Penrose pseudoinverse of operator D_θ. Applying this encoding-decoding scheme, the matrix product $P_\theta x = D_\theta E_\theta x$ is an orthogonal projection of the original signal x.

To address separable nonlinear least square problem, i.e. finding the optimal nonlinear θ parameters, the Variable Projection method – introduced in [14] – presents a general framework. Using their results the problem can be reduced to the optimization of only the nonlinear parameters:

$$\min_{\theta, E_\theta} \|x - D_\theta E_\theta x\|_2^2 = \min_\theta \|x - P_\theta x\|_2^2,$$

for which a general formula for the Jacobian was provided in [14].

Previous studies [6–9] have shown that rational functions and MT systems in particular are well-suited for ECG heartbeat modelling: the MT orthogonal projections provide an efficient low-dimensional representation of the signals, where both the linear and non-linear parameters have an explainable interpretation corresponding to ECG morphology. Namely, the inverse poles are related to the location and general shape of ECG waveforms, while the linear coefficients represent local variations.

3. Methodology

In this paper, we propose a shallow autoencoder involving rational VP to model ECG heartbeats. We considered 3 non-linear parameters $a_1, a_2, a_3 \in \mathbb{D}$ with multiplicity of $(2, 3, 1)$, i.e. they are repeated 2, 3, and 1 times in the inverse pole vector θ. We chose 3 parameters in order to properly represent the main waveforms of the ECG heartbeats (P, QRS, and T waves), similar to [6, 9]. The non-linear parameters can be optimized heartbeat-wise, patient-wise, or database-wise as well (see e.g. [6], [9], and [11], respectively). Here however a different approach has been chosen, where five different rational VP autoencoders were trained by the optimization

$$\min_\theta \sum_{x\in Y} \|x - D_\theta E_\theta x\|_2^2,$$

where $x \in Y \subset \mathbb{R}^M$ represents the heartbeat signals from a predefined subset Y of the training set. Motivated by the morphological differences between the 5 arrhythmia classes, for each class a distinct set of poles was optimized globally on the corresponding beats from the training set. Then for the heartbeats the encoding operator was constructed as the direct sum of the optimized projection operators, which can be formulated as follows:

$$F_1(x) = \big(E_N \oplus E_S \oplus E_V \oplus E_F \oplus E_Q\big)(x), \qquad (1)$$

where $x \in \mathbb{R}^M$ is signal and E_C is the encoding operator of the class $C \in \{N, S, V, F, Q\}$. Moreover this was extended with two other explicitly calculated feature maps. The first is the vector of distances from the subspaces optimized for the arrhythmia classes i.e.:

$$\Delta(x) = \big(\|P_N(x) - x\|_2, \ldots, \|P_Q(x) - x\|_2\big),$$

and the second, denoted as $RR(x)$, consists of the RR interval length before and after the beat. Thus the extended

Method	Feature vectors	Classifier	Accuracy
de Chazal et al. 2004 [1]	Waveform (fiducial points) + RR	LD	86.1%
Llamedo et al. 2011 [15]	Waveform (VCG, wavelet) + RR	LD	93%
Ye et al. 2012 [16]	Wavelet + ICA (PCA) + RR	SVM	86%
Dózsa et al. 2019 [10]	LC (rational) + RR + PRD	SVM	82.1%
	LC (Hermite) + RR + PRD	SVM	90.9%
	LC (Hermite) + NLC + RR PRD	SVM	93.6%
Bognár et al. 2020 [9]	LC (rational) + NLC + RR	SVM	94.5%
Rational VP Encoder	F_0 : LC (rational)	SVM	89.4%
		Random Forest	91.3%
	F_1 : LC (rational) + DIST + RR	SVM	**94.6%**
		Random Forest	92.7%

Table 1: Comparison of the proposed method with earlier projection based classifiers and other state-of-the-art methods.

For the extracted features the following acronyms were used: **LC**: Linear coefficient from applying rational VP encoding on the data; **NLC**: Nonlinear coefficients of VP encoding, when a different VP projection was trained for each patient; **RR**: The RR interval length before and after the beats [1, 16]; **ICA**: Independent romponent Analysis; **DIST**: Distance from the approximating subspace (absolute error of the approximation); **PRD**: Percent root mean square difference (relative error of the approximation).

feature map can be written as:

$$F_2(x) = \big(F_1 \oplus \Delta \oplus RR\big)(x). \qquad (2)$$

The classification scheme is built upon the above projection encoding operators and is performed via the following steps:
1. Based on the recommendation by [1] for subject-oriented (inter-patient) classification, the heartbeats were separated into distinct training (DS1) and test (DS2) sets.
2. For each class in $\{N, S, V, F, Q\}$ a Rational VP Autoencoder was trained.
3. The encoding map F_i $(i = 1, 2)$ was calculated from the Rational VP Autoencoders as defined in (1) and (2).
4. Both training and test datasets were transformed by F_i.
5. Classification models (SVM and RF) was trained on the encoded data.

4. Dataset

We evaluated the proposed method on the MIT-BIH Arrhythmia Database [17] from PhysioNet [18]. In accordance to the AAMI recommendations, we excluded the four paced records, and regrouped the annotations into 5 classes: normal (N), supraventricular ectopic (S), ventricular ectopic (V), fusion (F), and unknown (Q) heartbeats.

The ECG signals are preprocessed and segmented following [9]: a wavelet-based baseline wandering removal, and a lowpass filter at 35 Hz is applied, then fixed windows of $M = 300$ samples are selected for each heartbeat (100 samples before and 200 samples after R peak annotations of the database).

We employed the subject-oriented (inter-patient) evaluation scheme, and divided the database into DS1 and DS2 for training and testing, as proposed by de Chazal [1], each containing around 50,000 heartbeats. This scheme provides a realistic and comparable evaluation, since the separation of the records in DS1 and DS2 prevents intra-patient data leakage and unrealistic overfitting of the patients data.

5. Results

The conducted experimental results are summarized in Table 1. The first block of the table contains the leading state-of-art research results in ECG heartbeat classification based on the experiment setting of [1]. The middle part shows the results from research based on training a VP operator for each heartbeat separately, while the last block presents the results arising from our methodology described in Section 3. The first column references the compared research, the second lists the extracted features used for the classification, the third names the applied classification algorithm, while the last presents the achieved classification accuracy. As can be seen from Tab. 1, the presented scheme outperforms both the state-of-art methods [1, 15] and the other variable projection based classification schemes [9, 10]. It is also worth mentioning that in [9, 10] the variable projection operators was optimized separately for every patient in both the training and and the test dataset, in contrast to the recommended scheme presented in this article, where only five nonlinear parameter optimization had to be done for the considered arrhythmia classes. This makes the presented encoder structure computationally inexpensive for both the test set and the application to new data compared to the previous methods.

We also note that only objectively comparable state-of-the-art results are considered, i.e. where the authors followed the AAMI recommendations for 5 classes, employed the subject-oriented (inter-patient) evaluation scheme, and reported overall accuracy.

6. Conclusions

In summary the presented shallow autoencoder structure extracts ECG features using the direct sum of parametrized orthogonal transformations. By training a rational VP autoencoder globally and using the linear coefficients as features, we achieved over 94% accuracy on the 5-class AAMI heartbeat classification problem. This method matches state-of-the-art performance while being computationally efficient, offering a lightweight alternative to deep learning in medical data analysis.

Acknowledgements

The research was supported by the European Union within the framework of the National Laboratory for Autonomous Systems. (RRF-2.3.1-21-2022-00002). Supported by the ÚNKP-23-3 New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund. This project was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences. Project no. K146721 and TKP2021-NVA-29 have been implemented with the support provided by the Ministry of Culture and Innovation of Hungary from the National Research, Development and Innovation Fund, financed under the K_23 "OTKA" and the TKP2021-NVA funding schemes.

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