

Parameter fault estimation in distributed heating/cooling systems

Wijaya Kurniawan*

wjaykurnia@ub.ac.id

Faculty of Computer Science,

University of Brawijaya

Malang, Indonesia

Dept. of Electrical Engineering and

Information Systems, University of

Pannonia

Veszprem, Hungary

Katalin M. Hangos

hangos.katalin@mik.uni-pannon.hu

Dept. of Electrical Engineering and

Information Systems, University of

Pannonia

Veszprem, Hungary

Institute for Computer Science and

Control

Budapest, Hungary

Lorinc Marton

martonl@ms.sapientia.ro

Dept. of Electrical Engineering,

Sapientia Hungarian University of

Transylvania

Targu Mures, Romania

ABSTRACT

In this paper, the problem of fault estimation and localization in the connecting dynamic elements of distributed heating and cooling systems are treated. The fault represents the physical parameter change related to the heat transfer between the system and the external environment. First, based on the bi-linear dynamic state space model of the system in the presence of a fault, structural observability analysis using Signed Directed Graph (SDG) has been performed to investigate the sensor placement problem. Then, a nonlinear observer with a parameter adaptation algorithm was proposed for fault estimation. The simulation results show that it can successfully detect and estimate the fault. Fault localization along the length of the element has also been attempted, but it has been found that the localization cannot be performed using practically changeable input variables. Frequency domain analysis is presented to discuss this phenomenon.

CCS CONCEPTS

• **Applied computing** → **Computer-aided design**; • **General and reference** → **Estimation**.

KEYWORDS

Fault diagnosis, Heating/cooling system, Structural observability, Nonlinear Observer, Parameter estimation

ACM Reference Format:

Wijaya Kurniawan, Katalin M. Hangos, and Lorinc Marton. 2022. Parameter fault estimation in distributed heating/cooling systems. In *Companion to 7th International Conference on Sustainable Information Engineering and Technology (SIET'22)*. ACM, New York, NY, USA, 8 pages. <https://doi.org/10.1145/3568231.3568256>

1 INTRODUCTION

Heating/cooling systems are practically important sub-class of heat exchange systems with source, consumers and connecting (well

isolated) tubes. They commonly are used for heating/cooling systems of a large building or a district of living houses. Here, one can find a heating/cooling source or provider that serves a network of connected consumers, where the heating/cooling medium (e.g. water) circulates in closed and well isolated tubes. In many cases, because of financial reasons, there are only a few sensors in such systems that are placed at the source and consumers.

The units in heating/cooling systems form an important class of operating elements in process systems (see e.g. [Hangos and Cameron 2001]). Their dynamic models can be derived from first engineering principles, in particular from energy balances, and they are in the simplest lumped parameter case linear or bi-linear state space models depending on the chosen/available input variables (see [Hangos et al. 2004]). Thus, the system's states (e.g. temperatures) are spatially distributed along the tube length.

Because of their widespread use, the diagnosis (i.e. fault detection, estimation, and isolation) is of primary importance, and a huge literature exists for this purpose that are focusing on different practically important and meaningful fault types. In the study of [Manservigi et al. 2022], several types of faults in heating networks were identified: leakage, pressure losses, and heat losses. A neural network based leakage detection method was proposed in [Fan et al. 2019] for heating networks with small and constant supply flows. The deterioration detection of the heat transfer surface by ageing was investigated in [Weyer et al. 2000]. A fuzzy observer based fouling detection method for heat exchangers was proposed in [Delrot et al. 2012]. A nonlinear high gain observer based fault diagnosis method was introduced in [Han et al. 2019] for such heat exchangers in which exothermic chemical reactions take place. A review of fault diagnosis strategies for district heating and cooling systems is presented in [Buffa et al. 2021].

The general aim of our research is to develop simple yet powerful diagnosis methods for networked process systems. The dynamic modelling of the heating system unit is based on distributed delay approximation. A general observer design framework for linear systems with distributed delay was presented in [Feng 2022]. However, in this present study, the particularities of the addressed system class will be explored.

Here, we focus on fault estimation in spatially distributed connecting elements (e.g. tubes) in heating/cooling systems. The fault localization (i.e. finding the isolation breakdown position along the tube) is also addressed. The results are built upon our recent work on fault estimation of networked heat exchange systems [Kurniawan et al. 2022].

*Every author contributed equally to this research.

ACM acknowledges that this contribution was authored or co-authored by an employee, contractor or affiliate of a national government. As such, the Government retains a nonexclusive, royalty-free right to publish or reproduce this article, or to allow others to do so, for Government purposes only.

SIET 2022, November 22, 2022, Malang, Indonesia

© 2022 Association for Computing Machinery.

ACM ISBN 978-1-4503-9711-7/22/11...\$15.00

<https://doi.org/10.1145/3568231.3568256>

2 MODELLING OF DYNAMIC HEATING/COOLING SYSTEMS AND THEIR FAULTS

Distributed heating/cooling systems transfer energy from one place to another using a heat transfer media (usually a fluid) transported in tubes. Unlike industrial heat exchanger networks, the energy transfer is taking place between the fluid in the tube and its environment in the source and the consumer equipment, and one tries to perfectly isolate the connecting tubes between this equipment. This section deals with the simplest way of modelling the units or elements of distributed heating/cooling systems for fault estimation.

2.1 System elements: sources, consumers and dynamic connections

As it was discussed above, distributed heating/cooling systems as process systems have three types of elements as operating units:

- a *source* element, that is a spatially distributed unit where there is an energy transfer between the heating/cooling source to the transfer fluid through the tube wall,
- *consumer* elements, that are also spatially distributed elements for heat transfer through the tube wall,
- *connecting* elements that are spatially distributed elements with good/perfect isolation, i.e. no intended energy transfer to/from the environment.

The *common faults* are different among the above elements. The source and consumer elements suffer from the deterioration of the heat transfer area that reduces the efficiency of the heat transfer, while the *connecting elements may have faults in their isolation that causes unwanted loss to/from the environment*.

Dynamic models of the elements. In the simplest case, one can construct a linear time invariant (LTI) state space model of all of the above three system elements by spatially lumping the distributed engineering model derived from dynamic energy balance [Hangos and Cameron 2001]. This way the state variables will be the fluid temperatures along the tube length (in the lumps), and there will be two input variables: the inlet temperature and the environmental temperature (see details in [Kurniawan et al. 2022]).

It is important to note that all of the system elements cause a time delay in the dynamics of the overall system that is approximated by the above mentioned lumped LTI state space model. Based on the idea of “linear chain trick” [Krasznai et al. 2010], the recent results indicate that a stable linear time-invariant single input single output (SISO) system can equivalently represents a linear connection with distributed delay [Lipták et al. 2019].

2.2 The 2-input single output (2ISO) dynamic connection model and its parameter change as fault

The simplest dynamic model of the elements in distributed heating/cooling systems is obtained if the following *assumptions* are made [Hangos and Cameron 2001].

- A tube is considered as the balance volume, where there is perfect mixing in the cross-section and it is lumped into n lumps (sections) along its length.
- Incompressible fluid with constant physical properties (e.g. density, specific heat) is present in the tube.
- We have a constant heat transfer coefficient k_E in each section.
- The state variables $x_i, i = 1, \dots, n$ are the temperatures in the sections, the input variables u are the inlet temperature T_I and the environmental temperature T_{EXT} .

Further assumptions are needed for the fault detection, estimation, and localization that are as follows:

- The fault in the isolation of a connecting element is considered to act only on a certain position along the length of the tube, i.e. it directly affects only one state variable.
- There is only a single fault affecting a connecting element at a time.
- The measurable output variable is the temperature at the end of the tube, i.e. $y = x_n$

State space model with fault. We can make a simple model of the above fault by noticing that k_E represents a *physical parameter* that may change in a certain location (say, in the j th section of the tube) if the isolation of the tube goes wrong so that its value becomes k_{Ef} . By assuming that there are no simultaneous changes, this k_{Ef} can be incorporated into the state space model as follows:

$$\begin{aligned} \dot{x} &= A_{f,j}x + B_{f,j}u, \quad y = Cx \\ A_{f,j} &= \begin{bmatrix} -(v + k_E) & 0 & 0 & \dots & 0 \\ v & -(v + k_E) & 0 & \dots & 0 \\ 0 & v & -(v + k_E) & \dots & 0 \\ 0 & 0 & \dots & \dots & \dots \\ 0 & \dots & v & \dots & -(v + k_{Ef}) \\ 0 & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots & \dots \\ \dots & 0 & \dots & \dots & \dots \\ -(v + k_E) & 0 & \dots & \dots & \dots \\ v & -(v + k_E) & \dots & \dots & \dots \end{bmatrix} \\ B_{f,j} &= \begin{bmatrix} v & 0 & \dots & 0 & \dots & 0 \\ k_E & k_E & \dots & k_{Ef} & \dots & k_E \end{bmatrix}^T, \quad C = [0 \quad 0 \quad 0 \quad \dots \quad 1] \end{aligned} \quad (1)$$

where $x = [x_1 \ x_2 \ \dots \ x_n]^T$ is the system's states, $u = [T_I \ T_{EXT}]^T$ is the inputs, y is the measured outputs, and $v > 0$ is the velocity constant of the fluid. $A_{f,j} \in \mathbb{R}^{n \times n}$ and $B_{f,j} \in \mathbb{R}^{n \times 2}$ are the state and input matrices in the presence of fault in the j th section. $C \in \mathbb{R}^{1 \times n}$ is the output matrix.

This can be seen in Fig 1.

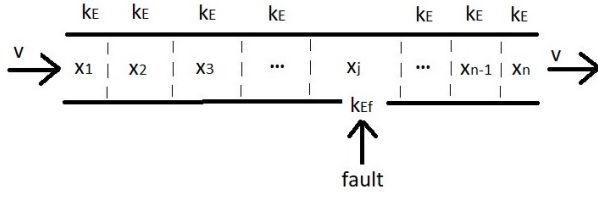


Figure 1: k_{Ef} fault position in the j th section of the tube

We treat the change in the k_E parameter as the fault f and try to identify it. The fault is modelled as a multiplicative parameter uncertainty in the system as follows:

$$k_{Ef} = (1 - f_j)k_E \quad (2)$$

where $f_j \in [0, 1)$ is a piece-wise continuous fault signal with sparse changes and j represents the j th section in the tubes where the fault happened.

By substituting Eq (2) into Eq (1) and derive it more, we get:

$$\dot{x} = Ax + Bu + F_j, \quad y = Cx$$

$$A = \begin{bmatrix} -(v + k_E) & 0 & 0 & \dots \\ v & -(v + k_E) & 0 & \dots \\ 0 & v & -(v + k_E) & \dots \\ 0 & \dots & \dots & v \\ 0 & \dots & \dots & \dots \end{bmatrix}$$

$$B = \begin{bmatrix} v & 0 & \dots & 0 \\ k_E & k_E & \dots & k_E \end{bmatrix}^T, \quad F_j = [0 \quad \dots \quad f_j h(x_j) \quad \dots \quad 0]^T \quad (3)$$

where $h(x_j) = k_E(x_j - T_{EXT})$ and only the j th entry of $F_j \in \mathbb{R}^{n \times 1}$ has a non-zero value. $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times 2}$ are the state and input matrices in the non-faulty case.

It should also be noted that *the presence of fault yields bi-linear term into the model.*

3 FAULT DETECTION AND ESTIMATION

This section describes the methods applied for parameter fault diagnosis. First, structural observability analysis is performed. Then an observer is provided to estimate the magnitude of the fault. Thereafter, we analyse the possibilities of fault localization along the tube length.

3.1 Structural observability analysis for fault diagnosis using Signed Directed Graph

Here, using observability analysis, we investigated the sensor placement problem to estimate the parameter fault in dynamic heating/cooling systems. Apart from the observability matrix based approach, it can also be analysed using Signed Directed Graph (SDG) which is a graph theoretic approach. In SDG, the vertices represent the inputs, outputs, and states, while the edges represent the relations between them. An edge (p_i, p_j) from p_i vertex to

p_j vertex exists if and only if the entry p_{ji} of the matrix P is not zero. After that, the observability analysis is done by checking the fulfilment of the following two conditions:

- (1) There is at least one path from every state vertices to at least one of the outputs vertices.
- (2) There is at least one cycle family which touches every state vertices.

A "cycle family" means a set of vertices with disjoint cycles. The first condition is also called output reachability. It implies that each change in the states can be detected at least in one of the output vertices. Meanwhile, the second condition is related to the structural rank (s-rank) of the observable pair of (A, C) to ensure that it has a full rank (see [Reinschke 1988] for details). If both of those conditions are satisfied, then the whole states in the system are called "structurally observable" or "s-observable".

From Eq. (1), it is seen that each state affects its consecutive state. To make it simpler, but without loss of generalization, assume that the system consists of 5 states ($n = 5$) and the third state is affected by a parameter fault. Hence, we can draw the SDG as shown in Fig 2. The output vertices y_1, y_2, y_3, y_4 , and y_5 represent the possibility of sensor placement in the distributed heating/cooling system. Meanwhile, the f vertex represents the parameter fault.

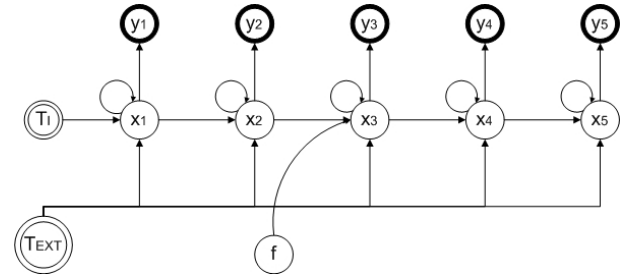


Figure 2: SDG for distributed heating/cooling system

If minimizing the number of sensors is preferred, it is seen that y_5 is the best position to place the sensor. From that position, every state vertices have a path to the output vertex and there is one cycle family which touches all of the state vertices ($y_5, x_1, x_2, x_3, x_4, x_5$). Thus, both of the s-observability conditions are fulfilled. This shows that using just one sensor at the end of the connection to measure the last state is enough to estimate all of the system's states. Moreover, it is also seen that there exists a path from f to y_5 which implies that we can use that same sensor to estimate the parameter fault in the distributed heating/cooling system. This will always hold even if the fault affects other states.

3.2 Observer design for fault estimation

In [Jiang and Chowdhury 2005], a fault estimation method was developed using parameter adaptation. By taking the basic idea from this previous research, we modified it to suit our case.

The dynamics of the connection with fault is as shown in the Eq (3). Define the estimation errors as:

$$\begin{aligned} e_x &= x - \hat{x} \\ e_y &= y - \hat{y} = C e_x \\ e_f &= f_j - \hat{f}_j \end{aligned} \quad (4)$$

where \hat{f}_j is the estimated fault.

Then, a bank of observers is constructed by which each of them is specifically designed to detect a fault in a section of the tube. For example, observer 1 is designed to estimate a fault that happened in the 1st section of the tube, observer 2 is designed to estimate a fault that happened in the 2nd section of the tube, and so on.

To make $\lim_{t \rightarrow \infty} e_f = 0$ for the j th section ($j = 1 \dots n$), the following fault estimator observer can be constructed (see details in [Kurniawan et al. 2022]):

$$FE_j := \begin{cases} \dot{\hat{x}} &= A\hat{x} + Bu + \hat{F}_j + K_x e_y \\ y &= C\hat{x} \\ \hat{F}_j &= \begin{bmatrix} 0 & \dots & \hat{f}_j h(\hat{x}_j) & \dots & 0 \end{bmatrix}^T \end{cases} \quad (5)$$

with the following adaptation equations:

$$\dot{\hat{f}}_j := \begin{cases} K_f h(\hat{x}_j) e_y, & \text{if } \|f\| \leq \sigma \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

Here, FE_j is an observer specifically designed to estimate a fault that happened in the j th section in the tube. $K_x, K_f^T \in \mathbb{R}^n$ are the observer and adaptation gain vectors. σ is a design constant to guarantee the stability and robustness of the adaptation law so that e_f is always bounded (see [Levine 2010]).

The diagram of this bank of observers is shown in Fig 3.

3.3 Fault localization analysis

Using a bank of observers described by Eq (5) and Eq (6), it is expected that the fault can also be isolated/localized in addition to be estimated. To investigate it, by assuming zero initial conditions, we took the Laplace transform of Eq (1) so that we get:

$$\begin{aligned} Y(s) = X_n(s) &= \frac{v^n}{[s + (v + k_E)]^{n-1} [s + (v + k_{Ef})]} T_I(s) \\ &+ \sum_{k=1}^{n-j} \frac{v^{k-1}}{[s + (v + k_E)]^k} k_E T_{EXT}(s) \\ &+ \frac{v^{n-j}}{[s + (v + k_E)]^{n-j} [s + (v + k_{Ef})]} k_{Ef} T_{EXT}(s) \\ &+ \sum_{k=1}^{j-1} \frac{v^{n-j+k}}{[s + (v + k_E)]^{n-j+k} [s + (v + k_{Ef})]} k_E T_{EXT}(s) \end{aligned} \quad (7)$$

From that Eq (7), it is seen that the transfer function from $T_I(s)$ to $Y(s)$ does not contain the j variable which represents the j th section in the tube where the fault occurred. Thus, even if we manipulate or tweak the input T_I with a signal which contains a wide spectrum of frequency (e.g. inserting a white noise signal, a sawtooth signal, or a Pseudo Random Binary Signal (PRBS)), the information about the fault position can not be obtained.

On the other hand, the transfer function from T_{EXT} to $Y(s)$ contains the j variable that looks promising to use so that the fault

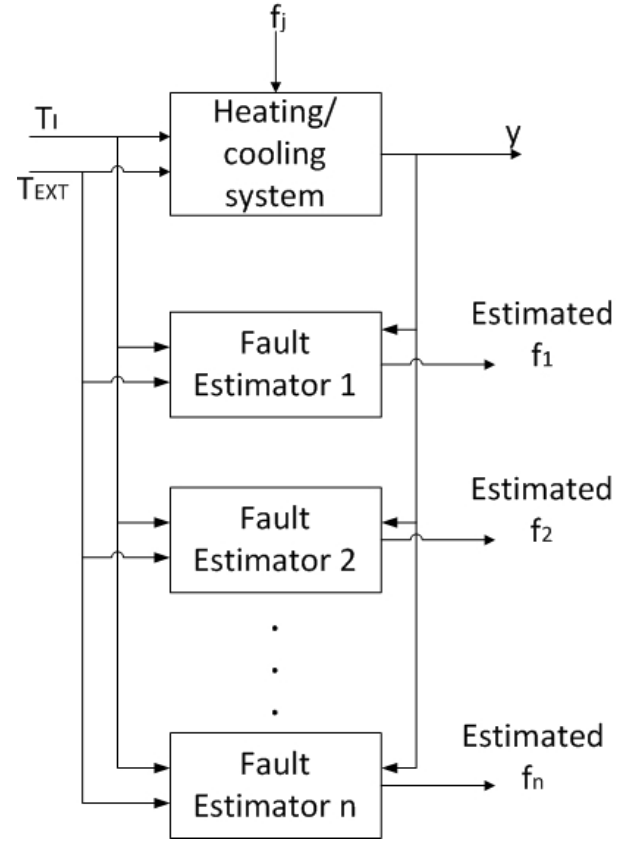


Figure 3: A Bank of Observers for Fault Estimation in the Dynamic Connections

position can be obtained. However, it is impractical to manipulate or tweak the input T_{EXT} because it represents the external temperature which comes from the outside environment.

Besides the transfer function as in Eq (7), we can also use the steady state value of the plant's output with respect to the position variation of the fault.

In Laplace transform, there is a property about Final Value Theorem (FVT) as follows:

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) \quad (8)$$

where $F(s)$ is the Laplace transform of $f(t)$.

By assuming that $T_I(t)$ and $T_{EXT}(t)$ are step function from 0 to T_I and from 0 to T_{EXT} respectively so that $T_I(s) = \frac{T_I}{s}$ and $T_{EXT}(s) = \frac{T_{EXT}}{s}$, we can use FVT to get a steady state value of $y(t) = x_n(t)$ in Eq (7) as follows:

$$\begin{aligned} y_{ss} &= \frac{v^n}{(v + k_E)^{n-1} (v + k_{Ef})} T_I + \sum_{k=1}^{n-j} \frac{v^{k-1}}{(v + k_E)^k} k_E T_{EXT} \\ &+ \frac{v^{n-j}}{(v + k_E)^{n-j} (v + k_{Ef})} k_{Ef} T_{EXT} \\ &+ \sum_{k=1}^{j-1} \frac{v^{n-j+k}}{(v + k_E)^{n-j+k} (v + k_{Ef})} k_E T_{EXT} \end{aligned} \quad (9)$$

where y_{ss} is the steady state value of $y(t)$ given that the input $T_I(t)$ and $T_{EXT}(t)$ are step function.

Now, consider 2 faults occurred in 2 different positions j_1 and j_2 where $j_2 > j_1$. Let the steady state values in the presence of these faults are y_{ss1} and y_{ss2} respectively. If we compute the difference between these 2 steady state values, we get:

$$y_{ss2} - y_{ss1} = - \sum_{k=n-j_2+1}^{n-j_1} \frac{v^{k-1}}{(v+k_E)^k} k_E T_{EXT} + T_{EXT} \left[\left(\frac{(v+k_E)^{j_2}}{v^{j_2}} - \frac{(v+k_E)^{j_1}}{v^{j_1}} \right) \frac{v^n}{(v+k_E)^n} \right] \quad (10)$$

$$\left(\frac{k_{Ef}}{(v+k_{Ef})} + \frac{k_E}{(v+k_{Ef})} \sum_{k=j_1}^{j_2-1} \frac{v^k}{(v+k_E)^k} \right)$$

Note that $\frac{v}{v+k_E} \in (0, 1)$. Eq (10) shows that if $K_E > 0$ is sufficiently large compared to v and n is also large, then the effect of the fault can hardly be localized.

4 CASE STUDY

4.1 Fault detection and estimation results

To verify and validate the proposed fault estimation, a simulation of a distributed dynamic heating system is built using MATLAB Simulink. In this simulation, it is assumed that the dynamics can efficiently be represented using 5 states/sections ($n = 5$). The other parameters are $v = 3$ and $k_E = 2$. The inputs are $T_I = 600K$ and $T_{EXT} = 500K$. All of those numbers are chosen for the sake of convenience because, in the real world, they have many various values based on the units and equipment/medium type.

For the observers, the fault detector gains K_x is chosen using the pole placement method while the fault estimator gain K_f is chosen to achieve a fast but sufficiently damped observer response. Meanwhile, the chosen design constant is $\sigma = 1.5$ to allow some small overshoot in the observer's estimation process when the fault is near its maximum value.

There are 3 scenarios of fault occurrence in different sections that are simulated. For every scenario, a fault is introduced into the system at the 20th second. Fig 4 shows the results when a fault with an amplitude of 0.4 ($f = 0.4$) happened in the 1st section of the tube ($j = 1$). Fig 5 shows the results when a fault with an amplitude of 0.2 ($f = 0.2$) happened in the 3rd section of the tube ($j = 3$). Lastly, Fig 6 shows the results when a fault with an amplitude of 0.6 ($f = 0.6$) happened in the 5th section of the tube ($j = 5$). In all of those figures, the real fault is plotted using dotted lines while the estimated fault from each observer is plotted using dashed lines.

In the figures, it can be seen that all the faults in every scenario are correctly estimated by the observers. However, it is also shown that every observer in each scenario produced the same fault estimation results. Thus, even though the fault estimation is successful, it can not detect in which section the fault has occurred in the system.

4.2 Fault localization results

In the previous section (see subsection 3.3), we have shown that the fault position can only be extracted by tweaking the input T_{EXT} which is impractical to do. Moreover, in this subsection, we will

also show that it is still difficult to get this information even if we can tweak it.

Using Eq (9), the steady state value y_{ss} of a system with 5 states ($n = 5$) is calculated for each value of $j = 1 \dots 5$ representing the position of the fault for some fixed parameters value of T_I , T_{EXT} , v , k_E , and k_{Ef} . The results are shown in Table 1. It is seen that the steady state value of the plant's output is always the same wherever the fault is happening.

Fig 7 shows the Bode plot from input 1 ($T_I(s)$) and input 2 ($T_{EXT}(s)$) to $Y(s)$ for $j = 1, 2, 3, 4, 5$ (f_1, f_2, f_3, f_4, f_5) which looks almost identical to each other for each value of j . This reinforces the previous results that the position of the fault can hardly be isolated using only a measurement at the end of the connection (last state variable x_n) even if we excite the system in the entire reasonable frequency range.

5 CONCLUSIONS

In this paper, a fault diagnosis method for connecting tubes in a distributed heating/cooling system is proposed. The applied model is a 2ISO LTI system which can also represent the existing distributed delay phenomenon in a linear connection. The concerned fault is a change of a physical parameter in a section in the tube. This physical parameter is related to the heat transfer coefficient between the tube and the external environment. The presence of this fault yields a bi-linear term in the model.

An SDG based analysis is done to check the structural observability necessary for the parameter fault diagnosis. This solves and also strengthens the argument that using only one sensor at the end of connecting tube is enough to estimate the states when there is no fault and the fault when there is a single fault. After that, based on the previous research, the design of a nonlinear observer with parameter adaptation is proposed for fault detection and estimation. However, it is also found that the fault localisation/isolation is impractical and can hardly be performed. Frequency domain analysis is presented to discuss this phenomenon.

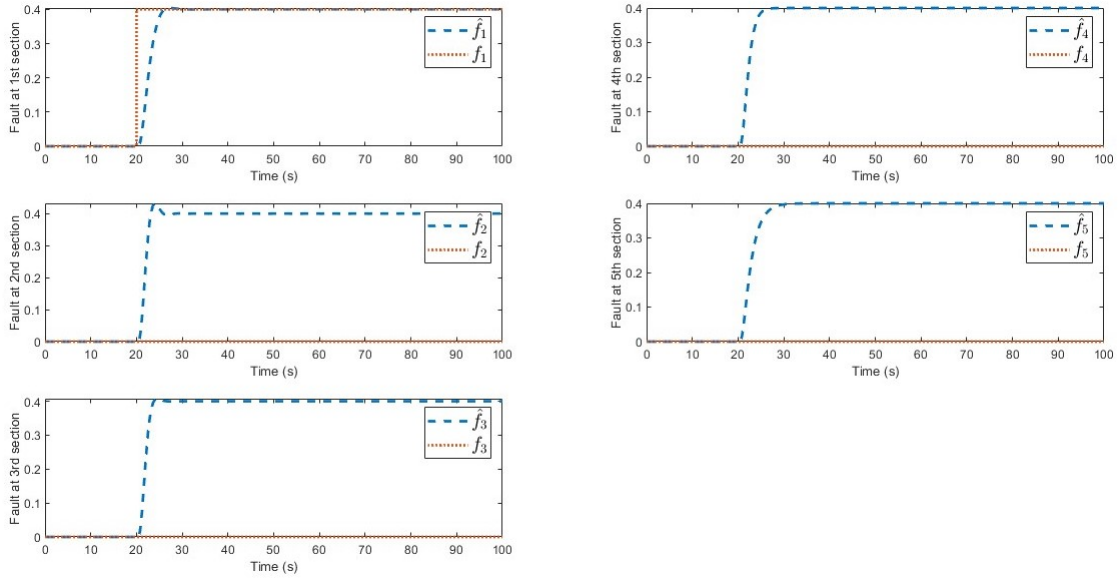
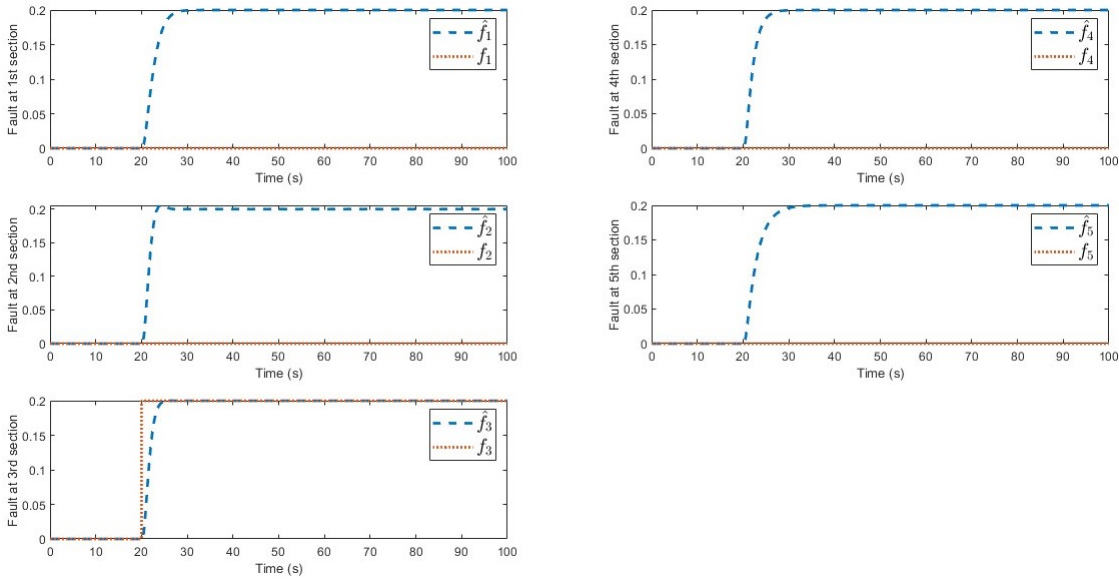
To verify and validate the proposed method, a simulation is done using MATLAB and Simulink. The results show that the proposed method can successfully detect and estimate the fault. Besides that, a steady state value calculation and frequency response using a Bode plot is also presented to show the difficulties of localising the fault.

ACKNOWLEDGMENTS

This study was supported by the National Research, Development and Innovation Fund of Hungary, financed under the K_19 funding scheme, project no. 131501.

REFERENCES

- S. Buffa, M. H. Fouladfar, G. Franchini, I. L. Gabarre, and M. A. Chicote. 2021. Advanced control and fault detection strategies for district heating and cooling systems—A review. *Applied Sciences* 11, 1 (2021), 455.
- S. Delrot, M. G. Thierry, M. Dambrine, and F. Delmotte. 2012. Fouling detection in a heat exchanger by observer of Takagi–Sugeno type for systems with unknown polynomial inputs. *Engineering Applications of Artificial Intelligence* 25, 8 (2012), 1558–1566.
- Q. Fan, Y. Guo, S. Wu, and X. Liu. 2019. Two-Level Diagnosis of Heating Pipe Network Leakage Based on Deep Belief Network. *IEEE Access* 7 (2019), 182983–182992.
- Q. Feng. 2022. Dissipative Control and Observation of Linear Time-Delay Systems: Full State Feedback.

Figure 4: f and \hat{f} for $f = 0.4$ in the 1st section of the connectionFigure 5: f and \hat{f} for $f = 0.2$ in the 3rd section of the connectionTable 1: Steady State Value y_{ss} for each fault's position for some parameter values

| parameters value/ j | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ | $j = 5$ |
|--|----------|-----------|----------|----------|----------|
| $T_I = 600, T_{EXT} = 500, v = 2, k_E = 1, k_{Ef} = 0.5$ | 515.8025 | 515.8025 | 515.8025 | 515.8025 | 515.8025 |
| $T_I = 600, T_{EXT} = 500, v = 3, k_E = 2, k_{Ef} = 1.5$ | 508.6400 | 508.6400 | 508.6400 | 508.6400 | 508.6400 |
| $T_I = 600, T_{EXT} = 500, v = 8, k_E = 3, k_{Ef} = 0.75$ | 525.5783 | 525.57839 | 525.5783 | 525.5783 | 525.5783 |
| $T_I = 600, T_{EXT} = 500, v = 10, k_E = 4, k_{Ef} = 2$ | 521.6924 | 521.6924 | 521.6924 | 521.6924 | 521.6924 |
| $T_I = 600, T_{EXT} = 500, v = 15, k_E = 5, k_{Ef} = 3.75$ | 525.3125 | 525.3125 | 525.3125 | 525.3125 | 525.3125 |

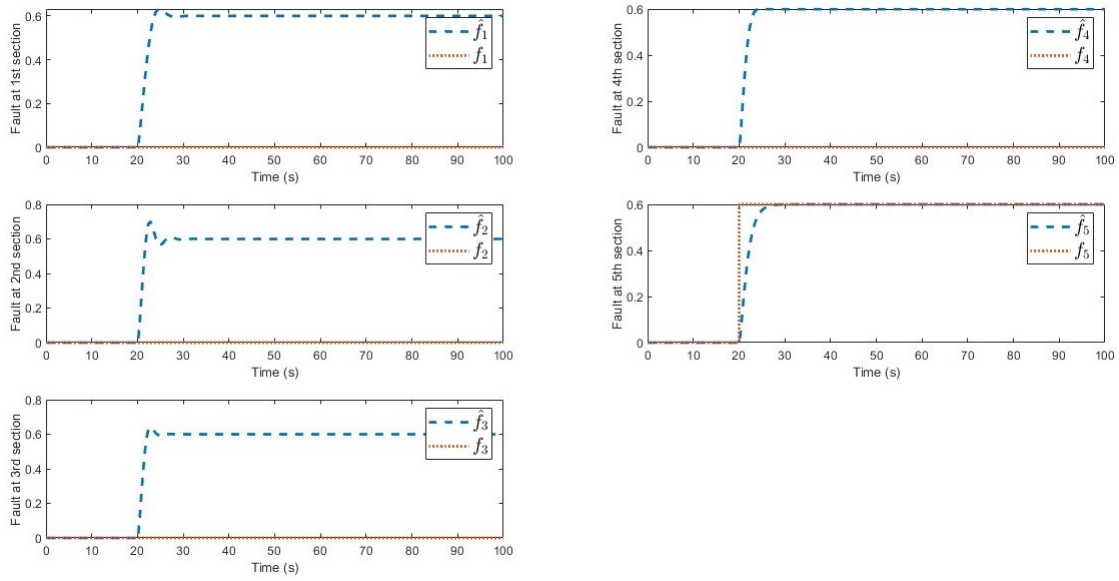


Figure 6: f and \hat{f} for $f = 0.6$ in the 5th section of the connection

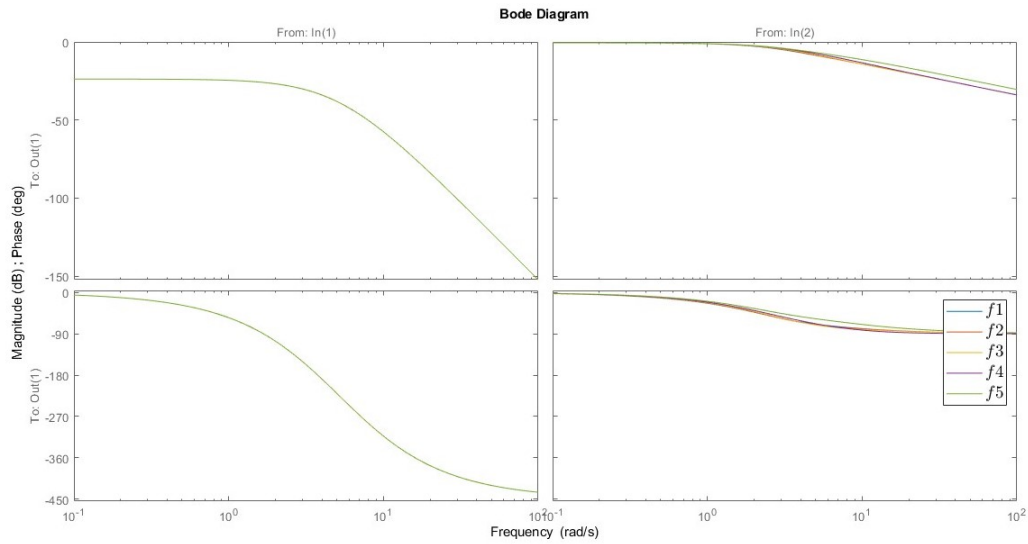


Figure 7: Bode Diagram for fault in each section of the connection

- X. Han, Z. Li, B. Dahhou, M. Cabassud, and M. He. 2019. Nonlinear Observer Based Fault Diagnosis for an Innovative Intensified Heat-Exchanger/Reactor. In *Lecture notes in electrical engineering*. 423–432.
- K. M. Hangos, J. Bokor, and G. Szederkényi. 2004. *Analysis and control of nonlinear process systems*. Springer.
- K. M. Hangos and I. T. Cameron. 2001. *Process Modelling and Model Analysis*. Academic Press, London.
- B. Jiang and F. N. Chowdhury. 2005. Parameter fault detection and estimation of a class of nonlinear systems using observers. *Journal of the Franklin Institute* 342, 7 (2005), 725–736.

- B Krasznai, I Györi, and M. Pituk. 2010. The modified chain method for a class of delay differential equations arising in neural networks. *Mathematical and computer modelling* 51, 5-6 (2010), 452–460.
- W. Kurniawan, K.M. Hangos, and L. Márton. 2022. Parameter fault diagnosis in heat exchange networks with distributed time delay. *IFAC-PapersOnLine* 55, 18 (2022), 39–44.
- W. Levine. 2010. The Control Handbook: Control System Fundamentals, Control System Applications, Control System Advanced Methods. *Electrical Engineering Handbook Series*. Taylor & Francis Group (2010).
- Gy. Lipták, M. Pituk, and K.M. Hangos. 2019. Modelling and stability analysis of complex balanced kinetic systems with distributed time delays. *Journal of Process*

- Control* 84 (2019), 13–23.
- L. Manservigi, H. Bahlawan, E. Losi, M. Morini, P. R. Spina, and M. Venturini. 2022. A diagnostic approach for fault detection and identification in district heating networks. *Energy* 251 (2022), 123988.
- K. J. Reinschke. 1988. *Multivariable control: a graph-theoretic approach*. Springer.
- E. Weyer, G. Szederkényi, and K. M. Hangos. 2000. Grey box fault detection of heat exchangers. *Control Engineering Practice* 8 (2000), 121–131.