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ABSTRACT

The aggregation of evaluators' preferences is a key problem in group decision making. We examine the recently proposed distance-based techniques and compare their efficiency to the traditional aggregation of individual preferences (AIP) methods in simulated Analytic Hierarchy Process (AHP) cases. We use the Kendall W statistic to measure the rank correlation among the individual priority vectors of the group and the common priority vector for the different aggregation approaches. Extensive simulations (altogether 88000 cases) show that both the Euclidean Distance-Based Aggregation Method (EDBAM) and the Aitchison Distance-Based Aggregation Method significantly outperform the traditional techniques in case of smaller and mid-sized priority vectors (at most six items to be compared). However, EDBAM outperform the AIP methods for all dimensions that is conventionally used in AHP, and its computation time is also low.

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Group decision making; AHP; distance-based; preference aggregation

1. Introduction

Group decision making is one of the most frequently used and most important applications of multicriteria decision making (MCDM) methods, such as the popular Analytic Hierarchy Process (AHP; Saaty, 1977); hence, it is in the focus of a wide range of recent research as well, both from a theoretical (Amenta et al., 2020, 2021) and a practical (Ishizaka & Labib, 2011; Marcarelli & Squillante, 2020) point of view. Today most of real-world decision making problems belong to the large-scale group decision making (Yang et al., 2022); thus, it also worth to examine larger number of decision makers.

A crucial step of group AHP (GAHP) is to aggregate individual preferences and determine the common priority vector that shows the opinion of the decision makers in the best way. The primarily used aggregation techniques are the Aggregation of Individual Judgements (AIJ; Aczél & Alsina, 1986) and the Aggregation of Individual Preferences (AIP; Basak & Saaty, 1993), both of which use some sense of mean (arithmetic or geometric) to create the group

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priority vector. However, AIJ determines a common pairwise comparison matrix (PCM) of the group and compute the vector from that, while AIP calculates all the individual preference vectors and draws the common priority vector from those.

Different aggregation techniques have been compared based on several metrics (Grošelj et al., 2015), as well as theoretical properties (Ossadnik et al., 2016) in the literature. A large number of studies deal with the fuzzy extension of GAHP as well (Grošelj & Stirn, 2018).

The currently used aggregation techniques can be sensitive to extreme opinions. We propose another family of aggregation methods, where we get the common priority vector by minimising the (some sense of) distance from the individual preferences in the decision space. In our current research, we concentrate on the Euclidean distance and the Aitchison distance based on some preliminary comparisons with other cases. These methods were first discussed in Duleba and Szádoczki (2022); however, some of the results were inconclusive, and many parameter combinations have not been examined. Besides the new cases that we reveal, we also change the approach of the comparison of aggregation techniques, as we not only focus on the method that is performing the best in a case but the average performance on all instances as well.

We carry out a wide range of simulations (88,000 altogether) that examine from 2 to 9 objects to be evaluated and 5, 10, 20, …, 100 decision makers in order to compare the new aggregation methods with the AIP, which is the most comprehensive aggregation technique according to many researchers (Brunelli, 2019; Munim et al., 2020). For all dimensions and number of decision makers, as well as both for the best and average comparing approaches the results show that the distance-based aggregation techniques tend to outperform the traditional ones.

The rest of the paper is structured as follows. Section 2 presents the different aggregation techniques and the measure of comparison in detail. The results are included in Section 3, while the discussion of the results take place in Section 4. Finally, we conclude and propose further research questions in Section 5.

2. Materials and methods

GAHP is based on PCM, that is an $n \times n$ matrix A, and its general element a_{ij} shows how many times item i is larger (better, more important) than item j.

There are many different techniques to calculate a priority vector from a PCM, however, we assume that the individual priority vectors are known in our case, and we only focus on the aggregation itself. Based on that, we use two types of the AIP method as benchmarks and two types of distance-based aggregation techniques as well.

Let us denote the number of decision makers in our problem by m, let $w^{(k)} = (w_1^{(k)}, w_2^{(k)}, \ldots, w_n^{(k)})^T$ be the individual preference vector for evaluator k ($w_i^{(k)} > 0$ for $\quad i = 1, 2, \ldots, n$ and $\sum_{i=1}^{n} w_i^{(k)} = 1$ for $\quad k = 1, 2, \ldots, m$). In case of the AIP Weighted Arithmetic

Mean (WAMM) technique the common preference vector $w^{(A)}$ is obtained as the weighted arithmetic mean of the individual priorities, while for AIP Weighted Geometric Mean (WGMM), we use the geometric mean in a similar way to get the group priority vector $w^{(G)}$.

As for the distance-based aggregation methods, the group preference vector w is the solution of Equation (1) normalised to one.

$$argmin \sum_{\mathrm{k}=1}^{m} d(w^{(\mathrm{k})}, x) \tag{1}$$

where $x \in \mathbb{R}^n$ and $d(w^{(\mathrm{k})}, x)$ is the given distance. Equations (2) and (3) show the Euclidean Distance-Based Aggregation Method (EDBAM) and the Aitchison Distance-Based Aggregation Method (ADBAM) cases, respectively.

$$d\left(w^{(k)}, x\right) = \sqrt{\sum_{i=1}^{n} \left(w_i^{(k)} - x_i\right)^2} \tag{2}$$

$$d\left(w^{(k)}, x\right) = \sqrt{\sum_{i=1}^{n} \left[\log\left(\frac{w_i^{(k)}}{g(w^{(k)})}\right) - \log\left(\frac{x_i}{g(x)}\right)\right]^2} \tag{3}$$

where $g(w^{(\mathrm{k})})$ and $g(x)$ denote the geometric mean of the given vectors. The main idea is to find the closest vector to the individual priorities according to the respective distance. To solve the optimisation problems, we use the method of Nelder and Mead (1965), which is a robust technique that only uses function values.

To measure the performance of the different techniques, we apply the tie-corrected Kendall coefficient of concordance (Kendall W) that is a non-parametric statistic, its value is in the common $[0, 1]$ range, and it measures the overall agreement of different vectors in ranking (Kendall, 1938). In our case, we have to supplement the rankings determined by the individual decision makers with the common preference vector calculated with one of the above-mentioned aggregation techniques, and measure the strength of concordance. The aggregation method that provides the highest Kendall W is the best for that particular example.

3. Results

We carried out numerical simulations to compare the performance of the different aggregation methods. We examined preference vectors from two to nine dimensions (n), while the number of decision makers (m) was 5, 10, 20, ... and 100. The simulation for a given (n, m) pair consists of the following steps.

(1) We generate m random n-dimensional preference vectors (normalised to one, namely $\sum_{i=1}^{n} w_i = 1$) based on continuous uniform distribution separately in each coordinate.

(2) The tie-corrected Kendall coefficient of concordance is calculated for AIP WAMM, AIP WGMM, EDBAM and ADBAM and the methods that provide the highest Kendall W are saved.
(3) Steps 1 and 2 are repeated for 1000 times.
(4) The average Kendall W of the four different methods are also calculated based on the 1000 iterations.

This way we can compare the results of the different techniques. It is important to note that ties might occur between the methods according to the Kendall W measure (they provide the same ranking). Figures 1 and 2 show the detailed results of the simulation. Namely, Figure 1 illustrates how many times a given aggregation technique provided the highest Kendall W, while Figure 2 presents the average Kendall W values of the different methods for different parameter combinations.

4. Discussion

As we can see, in Figure 1, EDBAM provides the best results in case of all dimensions and for all examined number of decision makers. For smaller and mid-sized dimensions (up until $n = 6$), ADBAM also outperforms the AIP techniques, but its performance is decreasing in n. This is probably due to the higher unknowns in the optimisation problem, but it is important to mention that large dimensional cases are rare in GAHP.

It is also notable that there are more than 1000 first places in almost every case, because of the ties. However, for larger priority vectors, the number of ties is much lower.

Based on Figure 2, the results are similar if we focus on the average performance of the methods compared to the best cases. The distance-based methods provide a higher average Kendall W compared to the traditional techniques. Their dominance is especially clear in the smaller and mid-dimensional cases (for at most six alternatives). The EDBAM technique outperforms the AIP methods for all dimensions, however, the ADBAM tends to provide similar averages to the AIP WAMM method in higher dimensions. It is also true that the difference of the averages is much smaller in the larger cases, however, there is a gap between the AIP WGMM method that performs the worst, and the other techniques. One should also note that the exact values of the averages mainly depend on the number of decision makers, because the Kendall coefficient of concordance measures an overall agreement in the preference vectors; however, the main point is the performance of the techniques compared to each other.

Based on these results, the distance-based methods outperform the commonly used AIP techniques, and for larger cases the EDBAM method is the preferred one. It is important to mention that the computational process of EDBAM is low (around one second for large cases).

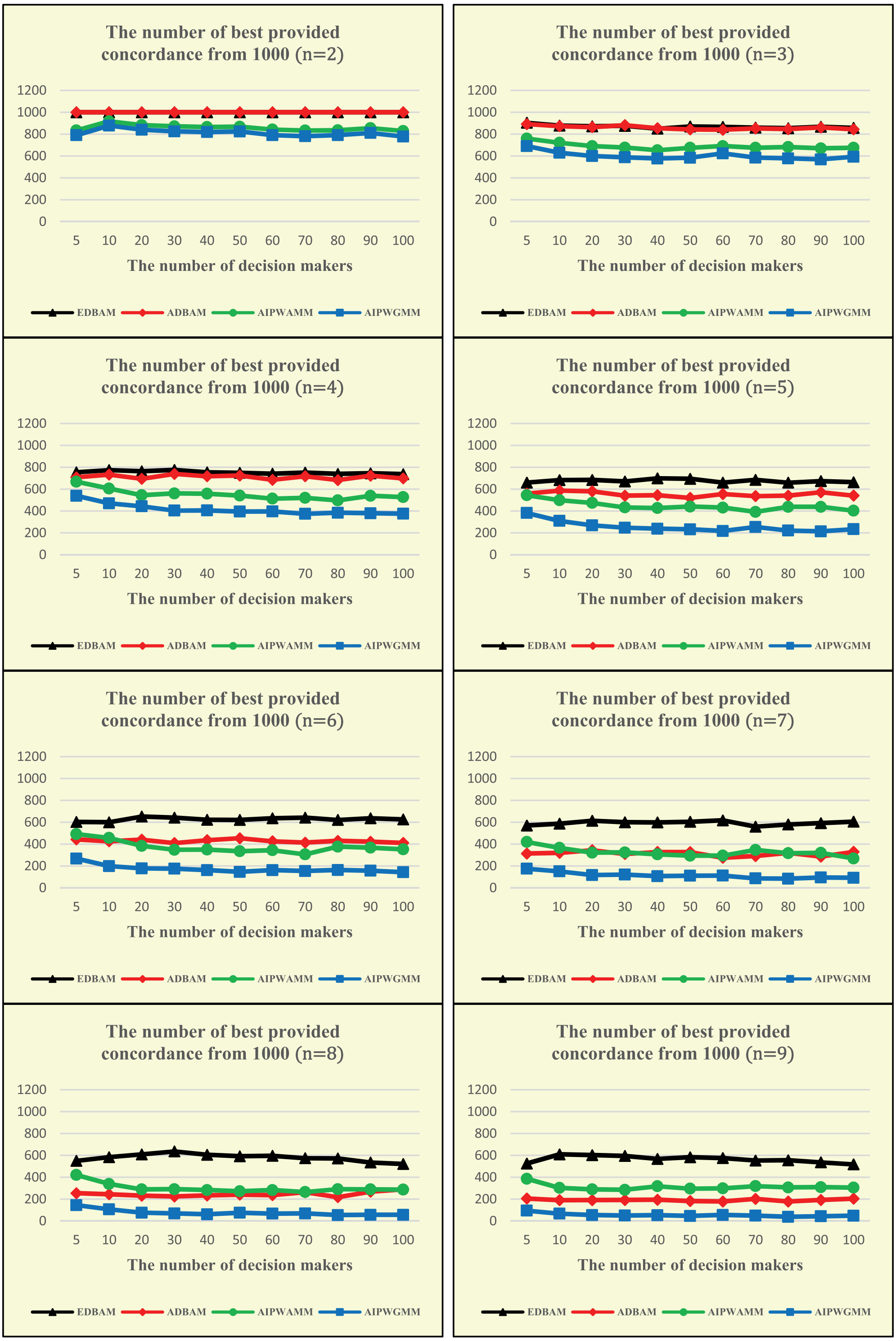

Figure 1. Simulation results for the different aggregation techniques, which show how many times a given technique provided the highest Kendall W.

Figure 2. Simulation results for the different aggregation techniques, which show the average Kendall W for the given methods.

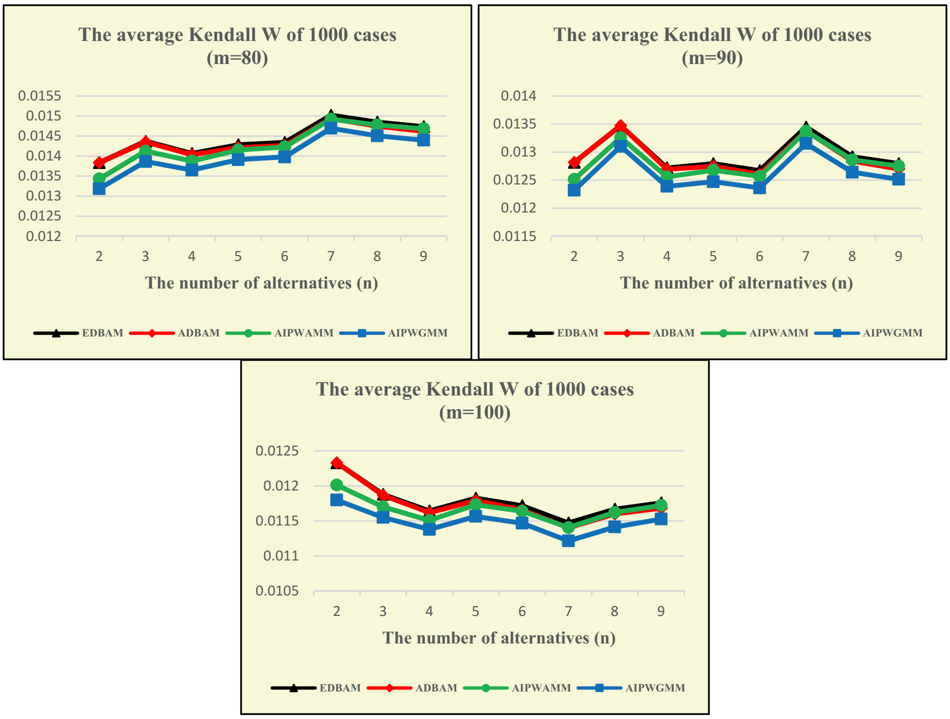

Figure 2. (Continued).

5. Conclusion

We proposed the EDBAM and ADBAM and with the help of 88,000 simulation cases on randomly generated priority vectors we demonstrated that these dominate the popular and commonly used AIP WAMM and AIP WGMM techniques, especially for small and mid-sized cases. However, EDBAM outperforms the AIP methods for large dimensional vectors as well, and its computation time is low. We discussed many parameter combinations that have not been examined in the literature yet, and also focused on the average performance of the methods, not just the best cases.

In the future, besides the study of other comparison measurements instead of the Kendall W, not only other aggregation techniques but also other MCDM methodologies can be investigated to compare them with the new distance-based techniques. Later, it would be also nice to provide a well-established theoretical explanation of the dominance of EDBAM or the decreasing performance of ADBAM in the number of dimensions.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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