

Adaptive Iterative Method to Improve the Robustness of PID Regulators

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Abstract: A new approach is proposed to improve the robustness of simple PID controls. This scheme widens the critical medium frequency domain and increases the achievable bandwidth using adaptive-iterative method. Recently not only the optimality but the robustness is also important for the industry.

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1. INTRODUCTION

Consider first a simple closed loop system given in Fig. 1, where R_r, C denote the reference model, the regulator and P the plant transfer function, y_r, e, u, y are the reference, error, input and output signals, respectively.

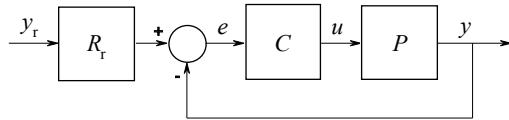


Fig. 1. Block scheme of a simple closed loop system

Let us summarize some basic formulas from the classical control theory. The complementary sensitivity function (CSF) or closed loop output transfer function can be computed by

$$T = \frac{L}{1+L} = \frac{CP}{1+CP} \quad ; \quad L = CP \quad (1)$$

and the sensitivity function (SF) or closed loop error transfer function is given by

$$S = \frac{1}{1+L} = \frac{1}{1+CP} \quad (2)$$

Here $L = CP$ is the open loop transfer function (OLTF) of the closed loop. The most typical case is characterized by the OLTF

$$L = CP \approx \begin{cases} \infty, & \omega \rightarrow 0 \\ 0, & \omega \rightarrow \infty \end{cases} \quad (3)$$

where the CSF and the SF are

$$T = \frac{L}{1+L} = \frac{CP}{1+CP} \approx \begin{cases} 1, & \omega \rightarrow 0 \\ CP = L \rightarrow 0, & \omega \rightarrow \infty \end{cases} \quad (4)$$

$$S = \frac{1}{1+L} = \frac{1}{1+CP} \approx \begin{cases} 0, & \omega \rightarrow 0 \\ 1, & \omega \rightarrow \infty \end{cases} \quad (5)$$

Here ω denotes the natural frequency. The product of the CSF and SF will have an important role in our investigations and can be calculated as

$$\gamma(L) = \frac{L}{(1+L)^2} = \frac{CP}{(1+CP)^2} \approx \begin{cases} 0 \\ CP = L \rightarrow 0 \end{cases} \quad (6)$$

Observe that this product represents a specially computed *medium frequency range (MFR)* weighting. It is easy to see that $\gamma(L)$ has a maximum at $|L|=1$, i.e., at the crossover frequency ω_c .

2. ITERATIVE CONTROL DESIGN APPROACHES

Since the beginning the key paradigm of designing control systems is how to handle uncertainties associated with the plant. An important approach is to implement robustness features at simultaneous identification and control procedures. It is a general observation that

1. The human first learns to control over a limited bandwidth, and learning pushes out the bandwidth over which an accurate model of the plant is known.
2. The human first implements a low gain controller, and learning allows the loop to be tightened.

On the basis of these observations an adaptive robust control philosophy, the windsurfer approach, was proposed by Anderson and Kosut (1991).

It is obvious that control relevant system identification should mostly be formulated as modeling based on a closed loop identification using the criterion

$$\hat{P}^* = \arg \min_{\hat{P} \in \mathcal{M}} \left\| \frac{CP}{1+CP} - \frac{C\hat{P}}{1+C\hat{P}} \right\|_{\infty} \quad (7)$$

for the CSF's. Here \hat{P} denotes the model of the plant and the infinite $\|\cdot\|_{\infty}$ norm provides robust identification.

Rewriting the criterion we get

$$\begin{aligned}\hat{P}^* &= \arg \min_{\hat{P} \in \mathcal{M}} \left\| \left(\frac{C\hat{P}}{1+C\hat{P}} \right) \left(\frac{1}{1+C\hat{P}} \right) \cdots \left(\frac{P-\hat{P}}{\hat{P}} \right) \right\|_{\infty} = \\ &= \arg \min_{\hat{P} \in \mathcal{M}} \left\| \hat{T}S \frac{\Delta}{\hat{P}} \right\|_{\infty}\end{aligned}\quad (8)$$

Here $\Delta = P - \hat{P}$ is the additive, Δ/\hat{P} is the relative model uncertainty, \hat{T} means that the *CSF* is calculated by the model \hat{P} and \mathcal{M} denotes the model class used. One can derive that the modeling error at this control relevant system identification (*CLCR*) is most important in the medium frequency range (around ω_c) given (or weighted) by

$$\hat{\gamma}(L, \hat{L}) = \frac{C\hat{P}}{(1+C\hat{P})(1+C\hat{P})} = \frac{\hat{L}}{(1+L)(1+\hat{L})} \quad (9)$$

where $\hat{L} = C\hat{P}$. If the model is exact this corresponds to the *MFR* weighting (see equation (6))

$$\gamma(L, \hat{L}) \Big|_{\hat{P} \rightarrow P} = \frac{L}{(1+L)^2} = \gamma(L) \quad (10)$$

The above background (Eq. (7)) is an implicit relationship for M^* implies an iterative regulator design method

$$\begin{aligned}\hat{P}_{i+1} &= \arg \min_{\hat{P}_i \in \mathcal{M}} \left\| \left(\frac{C_i \hat{P}_i}{1+C_i \hat{P}_i} \right) \left(\frac{1}{1+C_i \hat{P}_i} \right) \cdots \left(\frac{P-\hat{P}_i}{\hat{P}_i} \right) \right\|_{\infty} \\ &= \arg \min_{\hat{P}_i \in \mathcal{M}} \left\| \hat{T}_i S_i \frac{\Delta_i}{\hat{P}_i} \right\|_{\infty}\end{aligned}\quad (11)$$

and

$$C_{i+1} = \mathfrak{R}(\hat{P}_{i+1}) \quad (12)$$

Here $\mathfrak{R}(\dots)$ means supposedly a robust controller design. Once \hat{P}_{i+1} and C_{i+1} are found we can continue to increase the closed loop bandwidth repeating the procedure. The iterative process is continued until the ultimate control objective is achieved or is terminated by reaching some vital constraints.

One should know that there exist robust design procedures among the classical regulator design procedures, too. For industrial experts even the above *windsurfer approach* is not robust enough. They prefer the very stable catamarans of aborigines instead of the fancy fashion windsurfs.

3. A CATAMARAN APPROACH TO ADAPTIVE CONTROL

Since the closed loop identification generally involves a sophisticated parameterization, we will investigate how these combined criteria can be provided through open loop identification.

In the classical design approach the -40dB/d slope section of the open loop BODE-diagram is the *MFR*.

Assuming monotonous frequency characteristics a typical open loop amplitude frequency curve is shown on Fig. 2.

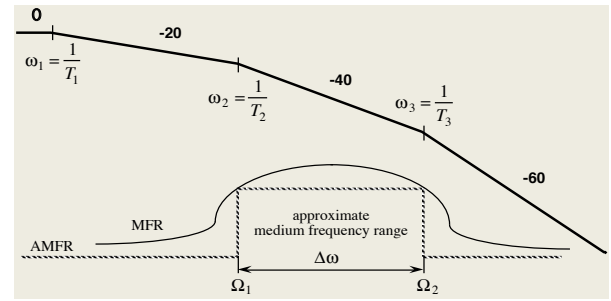


Fig. 2. Typical open loop amplitude-frequency curve

This means that if we use an open loop identification method based on a frequency weighting of the thin line (corresponding to *MFR*), then we substitute the general *CLCR* identification criterion. The *MFR* can be well approximated by an Approximate Medium Frequency Range (*AMFR*) weighting provided by the $\Delta\omega = \{\Omega_1 \leftrightarrow \Omega_2\}$ window (dashed line and in this case $\Omega_1 = \omega_2$ and $\Omega_2 = \omega_3$), or by any other proper approximate weighting filter. This approximate *CLCR* identification requires the a-priori knowledge of Ω_1 and Ω_2 , so only an iterative algorithm can be performed. The two legs of the catamaran are the Ω_1 and Ω_2 corner frequencies and a *CLCR* robust identification should provide the most accurate model properties in this *AMFR*.

To implement $\mathfrak{R}(\dots)$ investigate the robustness of a classical regulator design procedure, e.g., a *PID* regulator design based on dominant poles first. This class of design methods is based on a simple procedure using pole cancellation and providing a prescribed phase advance $\Delta\phi$. The classical design procedure providing prescribed $\Delta\phi$ is shown on Fig. 3, where $a(\omega)$ and $\phi(\omega)$ are the amplitude and phase frequency characteristics of $L(j\omega) = a(\omega) e^{j\phi(\omega)}$

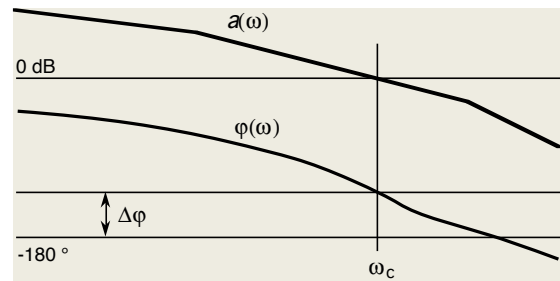


Fig. 3. Classical design procedure providing prescribed phase advance $\Delta\phi$

The robustness of such a design depends on the gain sensitivity of the phase-frequency characteristics at the crossover frequency ω_c

$$\delta\phi = \left(\frac{d\phi}{da} \right) \delta K = \frac{d\phi/d\omega}{da/d\omega} \delta K \quad (13)$$

Here $da/d\omega$ is limited by -40dB/d , so $d\phi/d\omega$ should be minimal. This means that the slope of the $\phi(\omega)$ curve should be minimized at ω_c or the NYQUIST curve should cross the unit circle perpendicularly close (see Fig. 4). So this robust regulator C^* is obtained by

$$C^* = \arg \min_{C \in \mathcal{R}} \left| \frac{d\phi}{d\omega} \right|_{\omega_c} \quad (14)$$

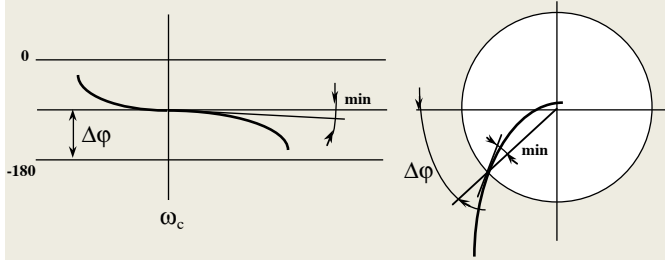


Fig. 4. The robustness of a classical design procedure providing prescribed phase advance $\Delta\phi$

The above condition can be heuristically provided, if the -40dB/d section is *as wide as possible*. Anyway this criterion is well known from the classical references, providing the least maximum of the magnitude of the SF in the frequency domain (second theorem of BODE).

Let us assume a monotonous $a(\omega)$ frequency curve given by

$$L(j\omega) = \frac{r_1}{1+j\omega T_1} + \frac{r_2}{1+j\omega T_2} \dots = \sum r_i f(\omega, T_i) \quad (15)$$

$$T_1 > T_2 > T_3 > T_4 > \dots$$

Let us assume the PID regulator in the most generally used form

$$C(s) = \frac{(1+sT_1)(1+sT_D)}{T_I s(1+s\alpha T_D)} = \frac{K(1+sT_1)(1+sT_D)}{s(1+\alpha sT_D)} \quad (16)$$

where α is for the non-ideal derivative action or providing a finite control action at step disturbances ($0.1 < \alpha < 0.5$) and $T_1 > T_D$.

The classical design procedure based on dominant pole cancellation is given by

$$T_1 = T_I = 1/\omega_1 \quad \text{and} \quad T_D = T_2 = 1/\omega_2 \quad (17)$$

Instead we propose a theoretical robust pole cancellation procedure first given by

$$T_1 = T_2 = 1/\omega_2 \quad \text{and} \quad T_D = T_3 = 1/\omega_3 \quad (18)$$

which provides the widest -40dB/d section and the highest bandwidth at the same time. This procedure seems to ensure the most robust PID regulator design based on the above conditions. Fig. 5 shows the increase of both the bandwidth (ω_b) and the AMFR ($\Delta\omega$) with the new design.

Thus we can summarize the special catamaran approach to adaptive control:

1. A cautious human designing a robust control system identifies an approximate plant model fitting best on the *AMFR* first.
2. Then a robust regulator based on the *AMFR* is designed.

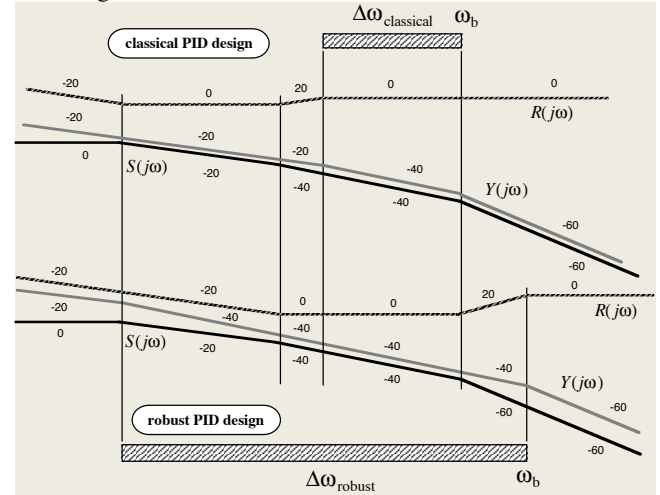


Fig. 5. The new design principle for robust PID regulators based on identification over the *AMFR*

The iterative process is continued until the control objectives (or certain constraints) are reached.

The approximate *CLCR* identification using the *AMFR* weighting defined by the window $\Delta\omega = \{\Omega_1 \leftrightarrow \Omega_2\}$ can be performed by

- an input design procedure emphasizing the *AMFR*
- an (open or closed loop) identification scheme using the *MFR* frequency weighting is computed by (9).

4. A PRACTICAL ROBUST PID TUNING PROCEDURE

In the previous section it was shown how the T_1 and T_D parameters of a PID regulator should be chosen to provide the least gain sensitivity. The value of K can be obtained from the design goal providing prescribed phase advance $\Delta\phi$ as shown in Fig. 3. This calculation can be simply done on a computer (e.g., using MATLAB) having the *OLTF* of the closed loop system. Note that there is a special relationship between $\Delta\phi$ and the overshoot of the step response of the closed loop. The selection $\Delta\phi = 60^\circ$ (or $\Delta\phi \cong 1$ radian) results in an overshoot value of about 5 percent for monotonous open loop frequency characteristics (Banyasz and Keviczky (1982)).

Unfortunately this procedure is only a nice *theoretical* approach so far to explain the necessary frequency characteristic behavior of the *OLTF* and it has only little importance in the practice. The reason is that the prescribed $\Delta\phi$ is usually greater or equal to 45° , however, $\phi(\omega)$ is already less than -135° at $\omega_2 = 1/T_2$, so the tuning principle (18) widening the -40dB/d section

does not help so much around any $\omega_c \leq \omega_2$. Another *practical* procedure is necessary to utilize the above robustness principle.

The approximate and exact BODE diagram of the *PID* regulator (16) is shown in Fig. 6. One can observe that the slope of $\varphi_R(\omega)$ is highest at the points $1/T_1$ and $1/T_D$ ($T_1 > T_D$), therefore if a tuning procedure located one of them to the crossover frequency a *PID* regulator could maximally reduce the sensitivity formulated by (14). It is obvious that from these points only $1/T_1$ could be an approximate cross-over frequency $\tilde{\omega}_c = \omega_R^0$. At this loop-shaping procedure T_D can be again selected according to the *theoretical* robust strategy, i.e.,

$$T_D = T_3 = 1/\omega_3 \quad (19)$$

however, T_1 must be different from (18). Thus, first determine an approximate crossover frequency

$$\tilde{\omega}_c = \omega_S^0 = \omega_R^0 = \arg(\varphi_S(\omega) = 180^\circ - \Delta\varphi) \quad (20)$$

which is obtained from the process phase-frequency curve $\varphi_S(\omega)$ and not from the open loop $\varphi(\omega)$. The approximate value of \tilde{K} can be calculated by the above mentioned numerical procedure. (The simplest method is to increase \tilde{K} step by step from a proper low value, while the intersection given by (20) is reached.) The exact K can be obtained anyway if

$$\omega_c = \omega_R^0 = \arg(\varphi(\omega) = 180^\circ - \Delta\varphi) \quad (21)$$

is used instead of (20). It is easy to see from the shape of $\varphi_R(\omega)$ in Fig. 6, that the optimal contribution of the *PID* regulator to minimize the slope of $\varphi_S(\omega)$ at $\tilde{\omega}_c$ (and this way the slope of $\varphi(\omega)$ at ω_c) if the integrating time constant is calculated as

$$T_1 = 1/\omega_S^0 = 1/\tilde{\omega}_c \quad (22)$$

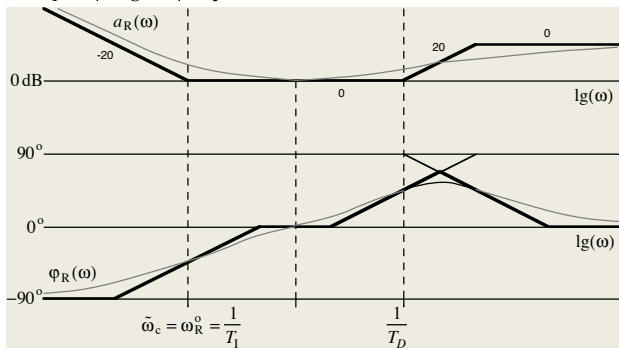


Fig. 6. The approximate and exact BODE diagram of a *PID* regulator

If the approximate design is not accurate enough, the procedure can be continued by solving the exact relationship (21). Fig. 7 shows the comparison of the *classical*, the *theoretical* and *practical* robust design of a *PID* regulator by indicating the $\Delta\omega_{\text{classical}}$, $\Delta\omega_{\text{robust}}$ and

$\Delta\tilde{\omega}_{\text{robust}}$ intervals, respectively. The zero slope sections of the regulator magnitude curves, playing also important role in the minimization of the slope of the phase curves are denoted by double thickness.

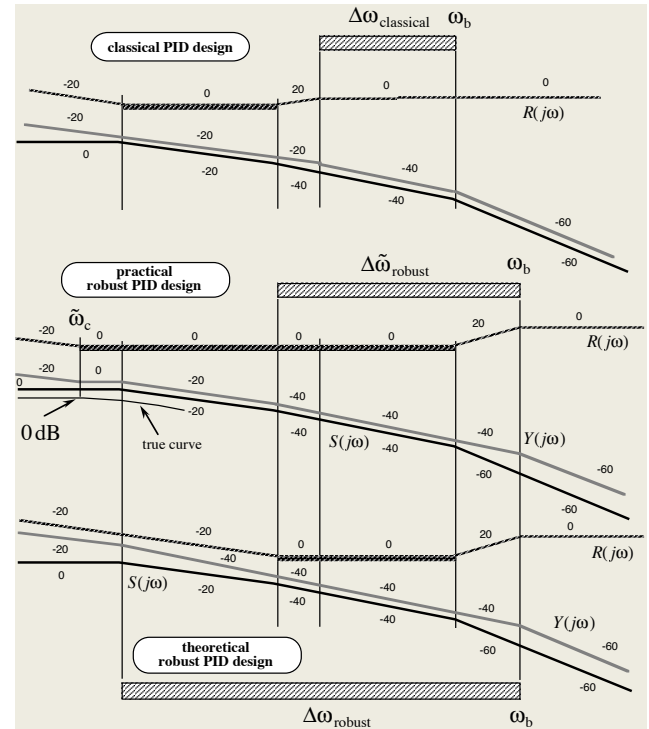


Fig. 7. Comparison of the simple design principles of a *PID* regulator

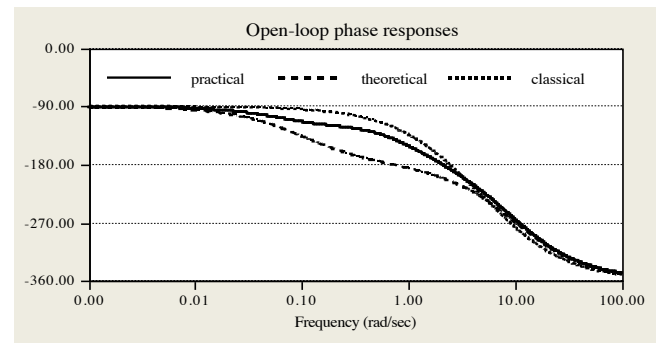


Fig. 8. Comparison of $\varphi(\omega)$ curves obtained by the three different design procedures

Example 4.1 Let the process be given by

$$P = \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)(1+sT_4)} \quad (23)$$

where $T_1 = 10$ sec; $T_2 = 1$ sec; $T_3 = 0.5$ sec; $T_4 = 0.1$ sec. The first step in the design procedure provided the approximate $\tilde{\omega}_c = \omega_S^0 = \omega_R^0 = 0.4788$ (see (20)) and $\tilde{K} = 5.583$. After the suggested iterative procedure $\omega_c = 0.1883$, $T_1 = 5.3115$, $T_D = 0.5$ and $K = 5.583$ were obtained. The tuning procedure was performed for the other two methods, too. Fig. 8 shows the comparison of the open loop $\varphi(\omega)$ phase responses for the *classical*

(dotted), the *theoretical* (dashed) and *practical* (continuous) robust design procedures. It can be well seen that the continuous line curve is the closest to our desired goal.

Fig. 9 shows the comparison of the *OLTF*'s on the NYQUIST plot for the three cases. It is also easy to see that the so-called *practical* (continuous line) robust design procedure provides the NYQUIST curve, crossing the unit circle perpendicularly close. One should note that the increase of robustness is mostly against the requirement to increase the bandwidth or speed of the closed loop. Fig. 10 shows how much we paid in this process presenting the unit step closed loop system responses for the three cases comparing to the open loop process transient response.

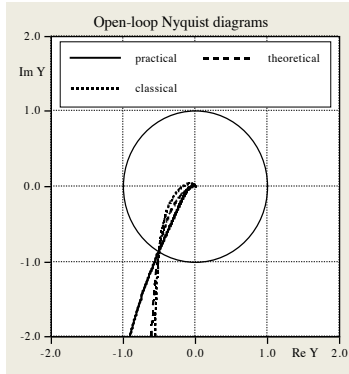


Fig. 9. Comparison of the three different design procedures on the NYQUIST plot

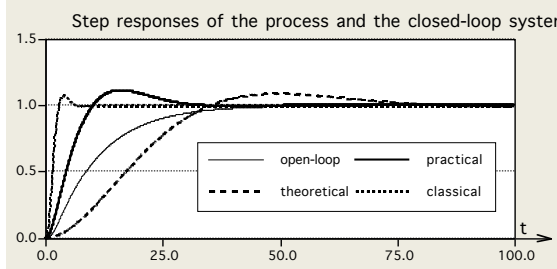


Fig. 10. Comparison of the unit step closed loop system responses

The experience of other examples gives that the robustness cannot be increased in a similar extent if the time constants are much closer than in the above example.

5. A RECURSIVE OPEN LOOP INPUT DESIGN FOR CLCR IDENTIFICATION

Consider the frequency characteristics of the plant given by

$$P(j\omega) = \frac{r_1}{1+j\omega T_1} + \frac{r_2}{1+j\omega T_2} + \frac{r_3}{1+j\omega T_3} + U = \sum r_i f(\omega, T_i) \quad (24)$$

in *Elementary Sub-Systems (ESS)* form (Bokor and Keviczky (1984)) with un-modeled uncertainty U . For the open loop *CLCR* identification the $\|\cdot\|_\infty$ norm is important

only in the window $\Delta\omega = \{\Omega_1 \leftrightarrow \Omega_2\}$ which can be reached by using the *AMFR* weighting with

$$W(\omega) = \begin{cases} 0, & \omega < \Omega_1 \\ 1, & \Omega_1 \leq \omega \leq \Omega_2 \\ 0, & \omega > \Omega_2 \end{cases} \quad (25)$$

Here $W(\omega)$ is the "catamaran" (infinite order) approximation of the ideal weighting $\gamma(\omega)$. Remember that the window $\Delta\omega$ can be determined according to several design procedures, too, see the previous section:

$\Omega_1 = \omega_1$ and $\Omega_2 = \omega_2$ for the *classical* design procedure
 $\Omega_1 = \omega_2$ and $\Omega_2 = \omega_3$ for the *theoretical* robust design procedure

$\Omega_1 = \omega_c$ and $\Omega_2 = \omega_3$ for the *practical* robust design procedure

One can obtain a much simpler $\tilde{\gamma}(\omega)$ which is the first order approximation of $\gamma(\omega)$ for the robust *theoretical* design procedure, see Fig. 11, if

$$\begin{aligned} \tilde{\gamma}(\omega) &= |\tilde{\gamma}(j\omega)| = \left| \frac{sT_2}{(1+sT_2)(1+sT_3)} \right|_{s=j\omega} = \\ &= \left| \frac{s\omega_3}{(\omega_2+s)(\omega_3+s)} \right|_{s=j\omega} \end{aligned} \quad (26)$$

The step response equivalent (*SRE*) discrete transformation of the partial fractional form

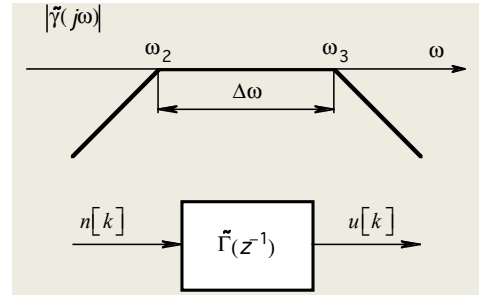


Fig. 11. The amplitude frequency characteristics of $\tilde{\gamma}(j\omega)$ and the discrete-time filter $\tilde{\Gamma}(z^{-1})$

$$\tilde{\gamma}(s) = \frac{\omega_3}{\Delta\omega} \left(\frac{1}{1+sT_3} - \frac{1}{1+sT_2} \right) = \frac{s(T_2 - T_3)\omega_3}{\Delta\omega(1+sT_3)(1+sT_2)} \quad (27)$$

of $\tilde{\gamma}(\omega)$ can be easily obtained by straightforward computations (Keviczky (1979)) and given by

$$\begin{aligned} \tilde{\Gamma}(z^{-1}) &= \frac{\omega_3}{\Delta\omega} \left(\frac{b_3 z^{-1}}{1+a_3 z^{-1}} - \frac{b_2 z^{-1}}{1+a_2 z^{-1}} \right) = \\ &= \frac{\omega_3}{\Delta\omega} \frac{(b_3 - b_2)z^{-1} + (a_2 b_3 - a_3 b_2)z^{-2}}{1 + (a_2 + a_3)z^{-1} + a_2 a_3 z^{-2}} \end{aligned} \quad (28)$$

Here

$$\begin{aligned} b_2 &= 1 - e^{-x_2} = 1 + a_2 ; a_2 = -e^{-h/T_2} = -e^{-h\omega_2} \\ b_3 &= 1 - e^{-x_3} = 1 + a_3 ; a_3 = -e^{-h/T_3} = -e^{-h\omega_3} \end{aligned} \quad (29)$$

h is the sampling time and $h\omega_3 < 0.1$ is required to a good continuous-discrete transformation. If we apply a white noise sequence $n[k]$ at the input of $\tilde{\Gamma}(z^{-1})$ as Fig. 11 shows, then the output $u[k]$ of this filter can serve as an approximately optimal input excitation in the *CLCR* identification.

If the $\tilde{\Gamma}(z^{-1})$ discrete-time filter is given there is another simple procedure (Keviczky and Banyasz (2015)) to generate an amplitude constrained input signal, having approximately the desired frequency spectrum. This algorithm provides a "maximum-variance" exciting signal under the mentioned constraint. The simple formulation of the problem is how to generate a signal $|n[k]| \leq 1$, if it is required to provide maximum-variance for the output of the filter $\tilde{\Gamma}(z^{-1})$. Thus the algorithm solves the following local optimality problem

$$\max_{|n(k)| \leq 1} \left\{ \text{var} \left[\frac{G}{D} n(k) \right] \right\} = \max_{|n(k)| \leq 1} \left\{ \text{var} [u(k)] \right\} \quad (30)$$

where $G(z^{-1})$ and $D(z^{-1})$ are the numerator and denominator of $\tilde{\Gamma}(z^{-1})$. Here let

$$\tilde{\Gamma}(z^{-1}) = \frac{G(z^{-1})}{D(z^{-1})} = \frac{g_0 + \tilde{G}(z^{-1})}{1 + \tilde{D}(z^{-1})} z^{-d} \quad (31)$$

then the generating algorithm is given by

$$\begin{aligned} n[k+d] &= \text{sign}(g_0) \times \\ &\times \text{sign} \left[\tilde{G}(z^{-1})n[k-1] - \tilde{D}(z^{-1})u[k] \right] \end{aligned} \quad (32)$$

(see Keviczky and Banyasz (2015) for the details). The last equation is a simple recursive input design procedure for *CLCR* identification in open loop if $\tilde{\Gamma}(z^{-1})$ is given.

Because $\tilde{\Gamma}(z^{-1})$ depends on the identified model and the control requirements, it is obvious that an iterative refinement algorithm can only be the final solution.

Example 5.1 Investigate the above simple recursive optimal input design for the process model used in Example 4.1. Selecting a sampling interval $h = 0.05$ the $\tilde{\Gamma}(z^{-1})$ discrete-time filter is

$$\tilde{\Gamma}(z^{-1}) = \frac{0.0928z^{-1} - 0.0928z^{-2}}{1 - 1.8561z^{-1} + 0.8607z^{-2}} \quad (33)$$

The BODE-diagrams of the continuous (continuous-line) and discrete (dashed-line) input generating frequency characteristics are shown in Fig. 12. The frequency

spectrum of the optimal generated input signal obtained for 500 samples is shown in Fig. 13.

It is easy to see that this simple recursive method approximates our design goal quite well, i.e., concentrates the exciting input spectrum according to the required "catamaran" frequency gate around the *CLCR* domain required for optimal identification.

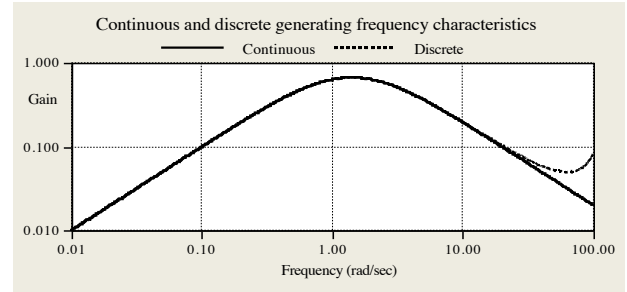


Fig. 12. The amplitude Bode-diagram of $\tilde{\gamma}(j\omega)$ and $\tilde{\Gamma}(j\omega)$

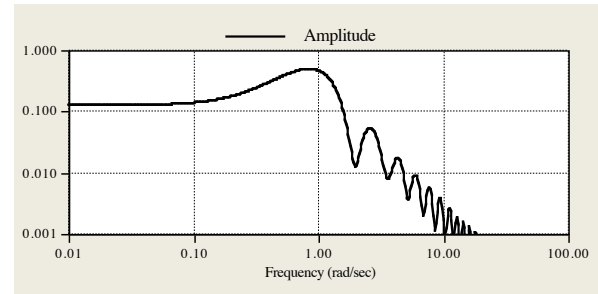


Fig. 13. Frequency spectrum of the generated optimal input signal

Note that the above procedures are different, simple, more or less heuristic approaches to obtain an input signal exciting the process in the relevant medium frequency range around ω_c and there are many other possibility to obtain similar results. The exact solution of this input design task applying either a classical (sometimes called *B-optimal*) input, or to use a robust \mathcal{H}_∞ problem formulation to minimize the un-modeled uncertainty U (Keviczky and Banyasz (2015)), is a very complex and sophisticated task.

6. CONCLUSIONS

In our paper a new approach is proposed to improve the robustness of simple *PID* controls. This scheme widens the critical medium frequency domain and increases the achievable bandwidth. It is shown how this robust design method differs from the classical approaches. A simple procedure based on dominant pole cancellation is used for illustrating the proposed method.

A simple heuristic and a recursive input design method are also given to ease the generation of the input excitation for a *CLCR* identification.

REFERENCES

Anderson, B.D.O. and R.L. Kosut (1991) Adaptive robust control: on-line

- learning, 30th IEEE Conf. Decision and Control, Brighton, England
- Bányász, Cs. and L. Keviczky (1982) Direct methods for self-tuning PID regulators, 6th IFAC Symposium on Identification and System Parameter Estimation, Washington, USA, 1249-1254.
- Bokor, J. and L. Keviczky (1984). Structure and parameter estimation of *MIMO* systems using elementary subsystem representation, *Int. J. of Control*, Vol. 39, 5, pp. 965-986.
- Diaz-Rodriguez, I.D., S. Han and S.P. Bhattacharyya (2019). Analytical Design of PID Controllers. Springer.
- Keviczky, L. (1979). On the transfer functions of sampled continuous systems, Technical Report, University of Minnesota, Department of Electrical Engineering, Minneapolis (USA).
- Keviczky, L. and Cs. Banyász (2015). *Two-Degree-of-Freedom Control Systems (The Youla Parameterization Approach)*, Elsevier, Academic Press.