

# Performance Analysis of Model Predictive Intersection Control for Autonomous Vehicles <sup>\*</sup>

András Mihály <sup>\*</sup> Zsófia Farkas <sup>\*\*</sup> Bede Zsuzsanna <sup>\*\*</sup>  
Péter Gáspár <sup>\*</sup>

<sup>\*</sup> *Systems and Control Laboratory, Institute for Computer Science and Control, Kende u. 13-17, H-1111, Budapest, Hungary.*

*Email: [mihaly.andras; farkas.zsofia; gaspar.peter]@sztaki.hu*

<sup>\*\*</sup> *Department of Control for Transportation and Vehicle Systems, Budapest University of Technology and Economics, Stoczek u. 2, H-1111 Budapest, Hungary. Email: bede.zsuzsanna@mail.bme.hu*

**Abstract:** The paper focuses on the control challenge of intersections related to the appearance of autonomous vehicles on the roads, which established mixed traffic situations with human-driven vehicles or scenarios with only autonomous vehicles. The goal of the research is to control autonomous vehicles by Model Predictive Control method to guarantee the collision-free passage at the intersection. Generally the outcome of a traffic situation can be varied by human-driven vehicles and fully automated vehicles. Therefore the results of the proposed coordination method used for a given intersection scenario is compared to solution of human-driven vehicles. For the comparison the simulation examples were made in VISSIM and CarSim simulation environments.

Copyright © 2021 The Authors. This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0>)

**Keywords:** intersection control, optimization, model predictive control, autonomous vehicle control.

## 1. INTRODUCTION

The development of automation in the automotive industry raises several questions to be answered before the appearance of autonomous vehicles in traffic. At the initial time of the introduction of highly automated vehicles on the roads, mixed traffic with human-driven and autonomous vehicles will be typical. In case of human-driven cars, the traffic rules are respected by the driver, the solution of traffic situations is depended on the drivers' intention. On the other hand, constraints and performance purposes of the control design are defined in case of autonomous vehicles to operate in traffic without danger and collision. For the development of autonomous vehicles, several research focus on the description of the human driver, its behavior, driving intentions and driving techniques to be built in the control design of automated vehicles. Xinli Geng et al. (2016) analysis human drivers' velocity planning to use the observations for autonomous vehicles. In his research neural network-based models are used to perform the analysis of the velocity profiles. Another paper focuses on the intent of human-driven vehicles to guarantee the collision avoidance in mixed traffic situation consisted of autonomous and human-driven vehicles, see Osipychev et al. (2017). Algorithms for the collision-free driving are developed using multi-stage Gaussian Processes and tested through simulation examples in intersection and high-

way scenarios. Mixed traffic situations with autonomous and human-driven vehicles motivated Liu et al. (2018) to design control for safe intersection crossing of vehicles. Several different constraints are built in the control according to the type of the vehicle (being human-driven or automated). For autonomous vehicles a Model Predictive Control is used while human-driven cars drive according to the traffic lights, signs and rules. Verma and Del Vecchio (2011) also analyse the situation of collision between an autonomous and a human-driven vehicles. The research focuses on human driving behavior to be used to solve safety problems between vehicles in mixed traffic situations.

Related to the development of autonomous vehicles, in 2010 the National Highway Traffic Safety Administration drew the attention of the academic researchers to an important traffic situations and challenges. In the report Administration (2010) it is stated that the 36 percentages of the crashes are related to intersection crossing and 96.1 percent of these intersection-related crashes were caused by the drivers. Therefore the coordination of autonomous vehicles passing through intersections has become important topic. Farkas et al. (2020) propose an iterative control strategy for autonomous vehicles to cross the intersection without collision. The coordination method aims to reach minimum traveling time and efficient energy consumption. Gáspár and Németh (2018) take the performances into consideration in their model demonstrated by an intersection scenario of three vehicles with different characteristics. Hult et al. (2019) proposed a model predictive controller having coordination and vehicle levels to secure

<sup>\*</sup> The research was supported by the Hungarian Government and co-financed by the European Social Fund through the project "Talent management in autonomous vehicle control technologies" (EFOP-3.6.3-VEKOP-16-2017-00001).

collision-free passage of autonomous vehicles at intersections. A distributed Sequential Quadratic Programming method was used for the solution of the optimization problem. Mihály et al. (2020) also present a centralized Model Predictive Controller to guarantee collision avoidance at the intersection. Minimization of traveling time and energy consumption are the targeted performances to be achieved, but a scheduling variable is built in the control to reach the balance between them.

The comparison of the operation of autonomous vehicles and human-driven vehicles has become the focus of several academic research. Beside the analysis of human drivers' behavior and intentions, it is an important control task to build the results in the design of automated vehicles. Zhou et al. (2020) propose a hierarchical model consisted of a logic model using Bayes' theorem and a plan layer to give the trajectory. The novelty of the paper is the verification of the proposed model demonstrated in simulated intersection scenario, giving better results in case of autonomous vehicles. Meng and Cassandras (2018) also developed control methods for intersection management for autonomous vehicles, focusing on performance requirements as travel time and energy consumption minimization. In this paper the performance of autonomous vehicles is compared to the performance of human-driven vehicles giving better results in case of unmanned automated vehicles.

Although several control solutions are given for autonomous vehicles to avoid collision and satisfy the performances (like minimum traveling time and energy efficiency) in crossing intersection, the contribution of the paper is a Model Predictive Control strategy that is proved to be more efficient than the driving operation of human-driven vehicles. For autonomous vehicles, there are constraints like speed limit, defined accelerations, built in the control design to secure safety at intersections. In this way sudden movement and actions of a human driver can be eliminated to enhance performances. The difference between the operation of human-driven and autonomous vehicles, demonstrated by the simulations, contributes to the change of research directions in the field of autonomous vehicles.

The paper is organized as follows. The motivation of the research is defined in Section 2. Section 3 describes the intersection scenario, defines the conditions for control design as well as traveling time. It shows the steps of control procedure. The comparison of the method for autonomous vehicles and the performance of human-driven vehicles is presented through simulation examples in Section 4. Conclusion remarks are presented in Section 5.

## 2. MOTIVATION

Researchers of several academic fields are encouraged to consider more comprehensive challenges related to the future applications of autonomous vehicles. Before the introduction of highly automated vehicles in traffic system, significant numbers of scenarios, situations and condition must be revised to prepare for the fully intelligent traffic. With the appearance of autonomous vehicles on the roads, a transitional period of mixed traffic will bring people closer to the application of autonomous vehicles. In case of mixed traffic, vehicles with human driver and also

with autonomous functions, pedestrians and other actors of the traffic system have to get the solution for traffic situations. These days, the traffic scenarios are controlled by traffic rules, signs and lights, the drivers additionally define the outcome of these situations by their behavior, intentions and adherence of rules. Therefore the academic sector and the automotive manufacturers must identify the opportunities for autonomous vehicles to get the solution of complex traffic situations, like crossing an intersection. Passing through a non-signalized intersection scenario can be varied by the behavior of human-driven vehicles and also by the controlled automated vehicles. With the introduction of autonomous vehicles in traffic new control methods and strategies are designed to solve traffic situations more efficient. Basically efficiency can be expressed by minimum crossing time of vehicles, collision avoidance, low fuel consumptions, effective design of velocity profiles.

The proposed Model Predictive Control (MPC) method considers design conditions and criteria to secure the safe passage of autonomous vehicles at the intersection. The minimization of traveling time spent at the intersection is in the focus of the coordination strategy. The comparison of the operation of autonomous vehicles with the MPC and human-driven vehicles' performance demonstrates the real advantages of the development and application of autonomous functions in road vehicles.

## 3. MPC CONTROL DESIGN

### 3.1 Intersection scenario

The type of intersection considered in the paper is a double lane four-directional intersection, where vehicles can head straight or turn left/right. Therefore, the risk of collision is present in the scenario, see Shen et al. (2019). The goal of the proposed MPC intersection controller is to ensure collision-free passage for the autonomous vehicles while minimizing the total traveling time spent in the intersection. Hence, congestion developing at the intersections can be avoided, as well as energy consumption of vehicles can be reduced. Moreover, with the automation of intersection crossing, safety of the passengers can also be enhanced. The aim of the design, is to determine the crossing order of the vehicles along with their prescribed accelerations to guarantee collision free passage, while the performance of the control design is to minimize traveling time.

The MPC controller design is based on some preconditions. First of all, the considered intersection is separated into different zones, as shown in Figure 1.

Before entering the control zone, autonomous vehicles are guided independently. After reaching the control zone, based on their initial states and turning intentions their velocity trajectories will be calculated by the proposed MPC method and controlled by the centralized controller of the intersection. For sending and receiving position and velocity data among autonomous vehicles and the centralized controller, V2I communication techniques are required. Note, that although the presented iterative calculation considers four vehicles at the same time, the method can consider newly entering vehicles by reconfiguring the initial conditions.

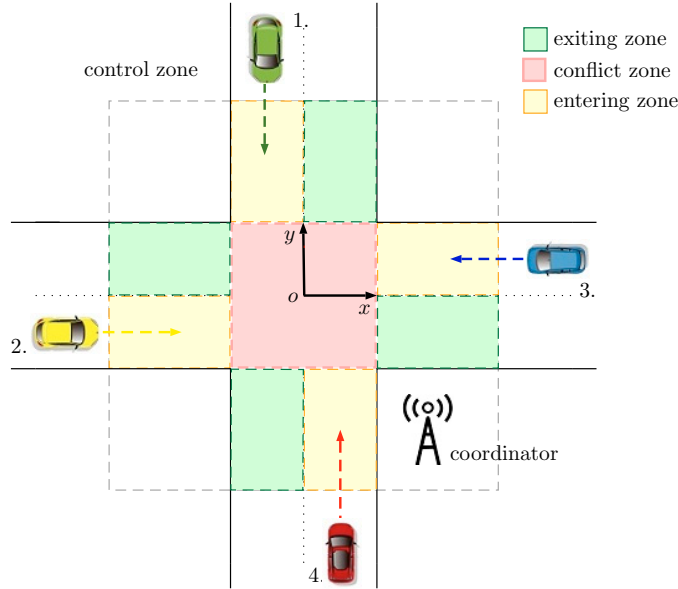


Fig. 1. Intersection scenario with control zones

### 3.2 Limitations for the MPC design

Due to the different possible trajectories followed by the autonomous vehicles in the intersection, in order to avoid instability or skidding of the controlled vehicles due to loss of adhesion in the cornering maneuver, it is necessary to define safe velocities connected to the turning intentions of each vehicles. Hence, using simplified vehicle dynamic equations, the velocity limit of turning left is defined as  $v_l = \sqrt{R_l g \mu}$ , while the right turn as  $v_r \sqrt{R_r g \mu}$ . Here,  $R_l$  and  $R_r$  are the turning radius of left and right turn trajectories in the intersection calculated based on the intersection geometry. The gravitational constant is noted with  $g = 9.81 \text{ m/s}^2$ ,  $\mu$  is the tire-road adhesion coefficient which can be estimated, see e.g. Gustafsson (1997); Li et al. (2007); Alvarez et al. (2005). Note, that in case the autonomous vehicle heads straight at the intersection,  $v_s = v_{lim}$  is set as a constraint, which is the speed limit at the given intersection. For instant, at a four-directional intersection having a lane width of 5 meters and assuming a road adhesion of  $\mu = 0.8$ ,  $v_l \approx 28 \text{ km/h}$  and  $v_r \approx 16 \text{ km/h}$  are calculated for the safety velocities, while the speed limit in connection to the straight heading is typically  $v_{lim} = 50 \text{ km/h}$ .

Another constraint is introduced in the control design in connection with the comfort of the passengers. In order to avoid abrupt changes in the velocity of autonomous vehicles, minimal and maximal acceleration threshold values are given as  $a_{min} = -5 \text{ m/s}^2$  and  $a_{max} = 5 \text{ m/s}^2$  based on Bichiou and Rakha (2019).

More importantly, for avoiding possible collisions in the intersection, another constraint is built in the MPC design. Namely, it is expected that in the conflict zone of the intersection only one autonomous vehicle can stay at a given time. Hence, independently of the designed trajectory, oncoming vehicles can only enter the conflict zone at the time instant when the previous vehicle has already left the conflict zone of the intersection.

### 3.3 Time-optimal MPC design

One of the possible control goal in the design of the centralized controller is to minimize total traveling time  $T_{total}$  of autonomous vehicles. If this performance is prioritized, the possibility of congestion forming at the intersection can be reduced significantly. To achieve this goal, autonomous vehicles are ordered and accelerated to reach the highest allowed velocity in the intersection. Hence, the proposed algorithm firstly defines constant accelerations  $a_i$  ( $i \in [1..n]$ ) for vehicles reaching the entering zone, by which the maximal velocities  $v_{i,max}$  for the given trajectories can be achieved reaching the conflict zone.

Thus, as a initial step for the MPC method, the following equation is applied for all vehicles entering the intersection:

$$a_i = \frac{v_{i,max}^2 - v_{i,0}^2}{2s_{i,ent}} \quad (1)$$

where  $v_{i,0}$  are initial speeds, whereas  $s_{i,ent}$  are initial distances from the origin of the intersection.

Note, that  $a_i = \{a_{max}; a_{min}\}$  are substituted for the accelerations given by (1) if the previously defined limits are exceeded.

The time-optimal control method is based on the comparison of autonomous vehicle traveling times. Conflict situation among vehicles are analyzed and accelerations are modified to avoid any possible accidents. The core of the coordination method is founded on detecting the overlaps in traveling times of autonomous vehicles when they cross the conflict zone. In case of a detected conflict situation, priority is given to the vehicle having the smallest traveling time, i.e. leaving the conflict zone first. In order to give priority, other conflicting vehicles reduce their accelerations iteratively until the time overlap with the first vehicle disappears. The calculation is then followed by analyzing the conflict situations among the remaining vehicles, and handled in a similar manner. In case of multiple vehicle conflict, it might be necessary to calculate a waiting time for some vehicles.

First, based on (1) entrance and exit times at the intersection conflict zone is calculated. For defining the entrance times, a second order equation must be solved:

$$\frac{1}{2}a_i t_{i,ent}^2 + v_{i,0} t_{i,ent} - s_{i,ent} = 0 \quad (2)$$

where  $t_{i,ent} \geq 0$   $i \in [1..n]$  is the entrance time for autonomous vehicles. Assuming constant accelerations for the entering zone, (2) can be rearranged as follows:

$$t_{i,ent} = \frac{s_{i,ent}}{(v_{i,max} + v_{i,0})/2} \quad (3)$$

Next, assuming vehicles to maintain their speeds constant in the conflict zone, time spent inside is calculated as follows:

$$t_{i,con} = s_{i,int}/v_{i,max}, \quad (4)$$

where  $s_{i,int}$  is the trajectory length in the conflict zone given in coherence with the vehicle turning intention.

Moreover, if the situation arises when a vehicle has to stop to give priority, a waiting time  $t_{i,wait}$  is calculated based on the difference between the final time of the previous vehicle

and the entering time of the subject vehicle. For restarting the vehicle, the autonomous vehicle selects the predefined maximal acceleration  $a_{max}$ , hence the time spent in the conflict zone is

$$t_{i,con} = \sqrt{2s_{i,int}/a_{max}}. \quad (5)$$

The traveling time of each autonomous vehicle is then given as the sum of the entrance time, waiting time and the time spent in the intersection conflict zone:

$$t_{i,fin} = t_{i,ent} + t_{i,wait} + t_{i,con} \quad (6)$$

Hence, total traveling time is given as  $T_{total} = \max(t_{i,fin}) \forall i \in [1..n]$ .

The analytical calculation is evaluated in an iterative manner with the following steps:

- Safe maximal velocities  $v_{i,max}$   $i \in [1..n]$  of autonomous vehicles are calculated based on the geometry of the intersection.
- A constant acceleration  $a_i$   $i \in [1..n]$  is given for vehicles using (1). In case threshold values are reached, safe maximal velocities must be altered by substituting  $a_i = (a_{max}; a_{min})$ .
- For all vehicles participating the algorithm, entrance and exit times  $t_{i,ent}$  and  $t_{i,fin} \in [1..n]$  are calculated. By analyzing overlaps in the time domain, conflict situations are detected in this step of the procedure.
- In case no conflict is found between autonomous vehicles, every vehicle applies above calculated acceleration values.
- In case of detected conflict situation, vehicle having the smaller exit time  $t_{i,fin}$   $i \in [1..n]$  gets priority. The other vehicle or vehicles must reduce their acceleration until their entering time  $t_{i,ent}$  is larger than the exit time of the priority vehicle. This iterative calculation is done for every conflicting situation detected. Note, that in case a vehicle must stop before the conflict zone to give priority, a waiting time  $t_{i,wait}$  is calculated as well. Hence, if the waiting time  $t_{i,wait}$  is greater than zero, the autonomous vehicle stops for the calculated time at the beginning of the conflict then accelerates with  $a_i = a_{max}$ .
- Finally, the resulting accelerations  $a_i$   $i \in [1..n]$  are prescribed for each vehicle at the intersection through the vehicle control model detailed in Section 3.5.

### 3.4 Operation of the MPC Controller

The scheme of the proposed time-optimal MPC control is depicted in Figure 2. Note, that the control method works with a discrete time step  $k$ , with a corresponding sampling time  $T_s$ . For the operation of the centralized controller, the intersection coordinator receives position and velocity data  $s_{i,ent}(k)$ ,  $v_{i,ent}(k)$   $i \in [1..n]$  of all autonomous vehicles approaching the intersection entering zone, along with their turning intention  $d_i$ . Note, that in case of a vehicle entering the intersection, firstly a decision has to be made based on the preceding vehicle (being in the conflict zone) position. In case the preceding vehicle left the intersection conflict zone, the newly entered vehicle is added to the optimization method, otherwise the coordinator switches to an adaptive cruise control mode.

Next, at every time step  $k$  the solution of the time-optimal analytical calculation detailed in Section 3.3 is given for a time horizon  $T = \max(t_{i,fin})$   $i \in [1..n]$ , resulting in the ordering of vehicles and their prescribed accelerations. Note, that although a constant sampling time  $T_s = 0.1s$  is applied, the time horizon for the calculation is a function of the intersection geometry and initial conditions of the autonomous vehicles. Hence, in every time step a control input  $a_i(k+1)$   $i \in [1..n]$  is given by the optimization method, which is followed by the controlled vehicles until the subsequent time step, when the analytical optimization is repeated with a shifted time horizon.

Since the calculation is very sensitive to the data of vehicle states, it is necessary to handle uncertainties related to communication links (noises, packet drops, delays), see Chohan (2019); Khayatian et al. (2018).

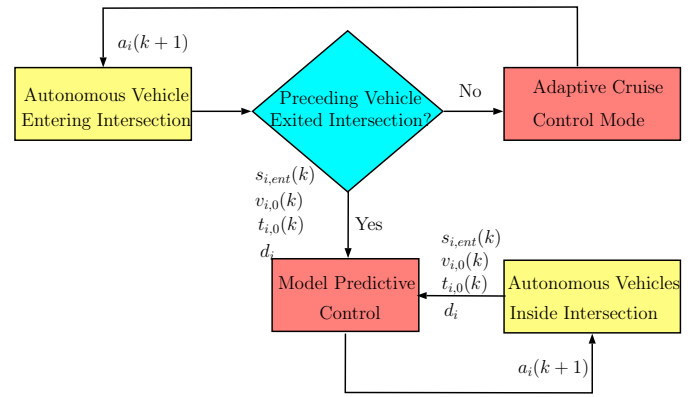


Fig. 2. Intersection control process

### 3.5 Vehicle control model

The control of the autonomous vehicles with the proposed MPC method is realized by defining the necessary longitudinal force coherent with the calculated acceleration  $a_i$   $i \in [1..n]$ . The sufficient forces for the vehicles are given as  $F_{i,l} = m_i a_i + F_{i,d}$ , where  $m_i$   $i \in [1..n]$  is the mass of the vehicle,  $F_{i,d}$  is the disturbance force affecting the longitudinal dynamics. Note, that disturbance force  $F_{i,d}$  consists of the aerodynamic drag, the rolling resistance and the effect of road slope on the vehicle calculated as detailed in Rajamani (2005). In the CarSim simulation listed in the next section, the given longitudinal force  $F_{i,l}$   $i \in [1..n]$  is realized by applying longitudinal wheel forces equally for all four wheels.

## 4. SIMULATION RESULTS

In order to demonstrate the effectiveness of the proposed MPC method for autonomous vehicles, as an example two different simulations have been evaluated and compared. First, human driven vehicles approaching an intersection have been simulated in VISSIM environment. Next, autonomous vehicles using the proposed MPC intersection control have been simulated in CarSim environment with similar intersection setup and vehicle initial conditions.

For the sake of comparison, a four-directional double lane intersection has been chosen with four approaching vehicles having initial conditions and turning intentions

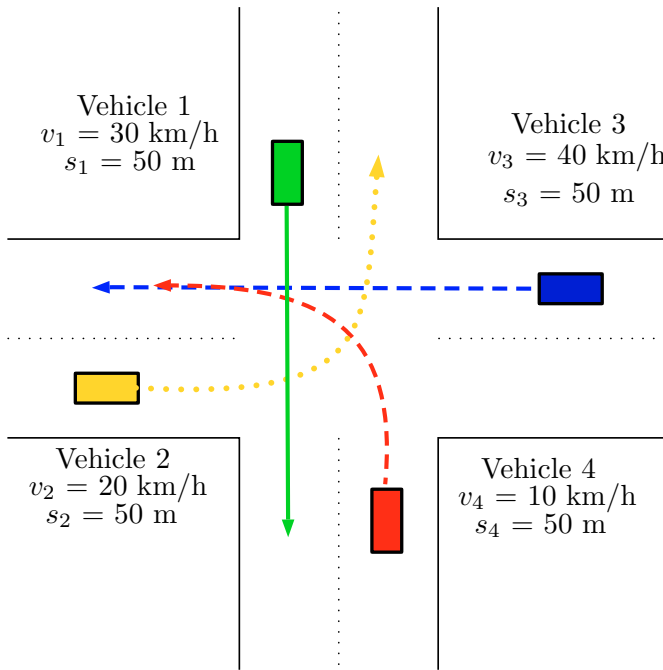


Fig. 3. Simulation setup with four vehicles

shown in Figure 3. As it can be seen, Vehicle 1 and Vehicle 3 are intending to head straight at the intersection, while Vehicle 2 and Vehicle 4 are intending to turn left, with all vehicles approaching the intersection with different velocities.

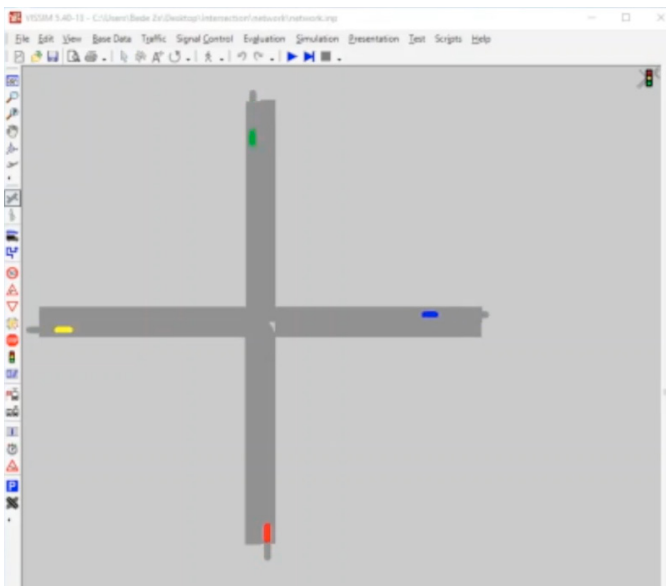


Fig. 4. VISSIM simulation setup

Firstly, VISSIM simulation has been built and ran with the above detailed scenario, as depicted in Figure 4. Note, that VISSIM uses a built-in car-following driver model, which is founded on a psychophysical model. The Wiedemann model defines four type of modes for the human drivers: free driving, approaching, following and braking mode. These modes are applied based on the distance and speed differences among vehicles, see Olstam and Tapani (2004).

Next, the introduced MPC intersection controller has been implemented in CarSim environment for the autonomous vehicles as illustrated in Figure 5. Note, that in contrast to the previous VISSIM simulation with built-in driver models, here all of the vehicles are controlled with the proposed centralized controller.

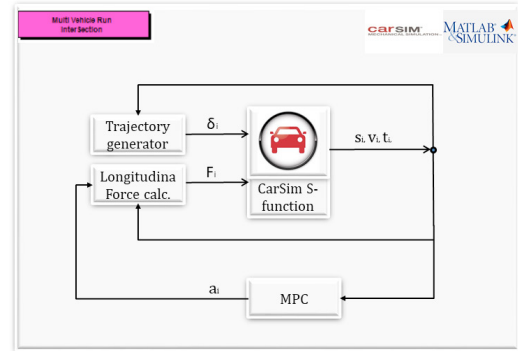


Fig. 5. MPC intersection control in CarSim

The results of the VISSIM and CarSim simulations with human drivers and autonomous vehicles are depicted in Figure 6. It is well demonstrated, that both the ordering of the vehicles and their velocity profiles are significantly different. With human drivers following the right hand rule at the intersection as depicted in Figure 6 (a), Vehicle 3 crosses firstly, followed by Vehicle 4 and Vehicle 2, while Vehicle 1 finally leaves the intersection at 11.2 seconds. It is well demonstrated, that following the right hand rule requires the driver of Vehicle 1 and Vehicle 2 to decrease their velocities heavily, with the driver of Vehicle 1 coming to a complete halt and restarting after giving way to Vehicle 2.

In contrast, with the proposed MPC centralized controller, autonomous vehicles select a completely different crossing strategy, as it is shown in Figure 6 (b). As in the previous case, Vehicle 3 with the largest initial velocity leaves the intersection first, increasing its speed similarly to the human driver. Next, Vehicle 1 and Vehicle 2 cross the intersection, while Vehicle 4 finally exits the intersection at 10.45 seconds. Thus, compared to the simulation results of the human driven vehicles, the proposed MPC intersection controller guides the autonomous vehicles through the intersection almost one second faster. Even more importantly, speed profiles of the autonomous vehicles are smooth without deceleration, whereas human drivers brake and accelerate abruptly due to the imperfect perception of human drivers and the obligatory right hand rule followed by them. Note, that these uneven velocity profiles significantly increase energy consumption of the vehicles, while passenger comfort is also harmfully effected. Hence, application of the proposed MPC intersection controller can decrease traveling times while increase passenger comfort and fuel economy of the vehicles.

## 5. CONCLUSION

The paper proposed a time-optimal MPC intersection controller for autonomous vehicles and analyzed its performance by multiple simulations evaluated in VISSIM traffic



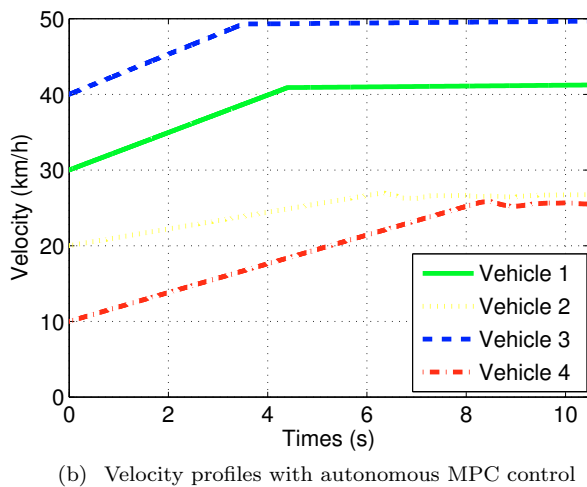
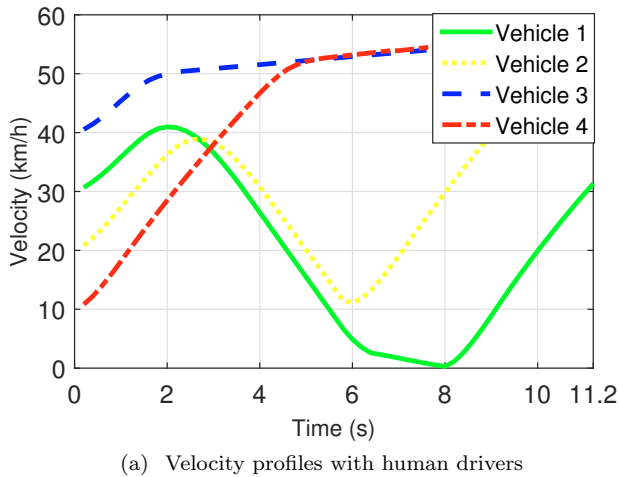


Fig. 6. Velocities of human driven and autonomous vehicles and CarSim vehicle simulator environment. The goal of the analysis was to compare the velocity trajectories of vehicles entering the intersection with two different strategy: in a conventional manner with human drivers via with the proposed MPC centralized control assuming autonomous vehicles. Although the simulation results validated the efficiency of the proposed method, future work must focus on analyzing the performance of the MPC intersection controller with larger traffic densities, showing a more detailed comparison with the conventional intersection management.

## REFERENCES

Administration, N.H.T.S. (2010). Crash factors in intersection-related crashes: An on-scene perspective. <https://crashstats.nhtsa.dot.gov>.

Alvarez, L., Yi, J., Horowitz, R., and Olmos, L. (2005). Dynamic friction model-based tire-road friction estimation and emergency braking control. *J. Dynamic Systems Measurements and Control*, 127(1), 22–32.

Bichiou, Y. and Rakha, H.A. (2019). Real-time optimal intersection control system for automated/cooperative vehicles. *Int. J. Transp. Sci. Techn.*, 8(1), 1–12.

Chohan, N. (2019). Robust trajectory planning of autonomous vehicles at intersections with communication impairments. *MS Thesis*.

Farkas, Z., Mihály, A., and Gáspár, P. (2020). Control methods for the coordination of autonomous vehicles at intersections. In *2020 European Control Conference (ECC)*, 668–673.

Gáspár, P. and Németh, B. (2018). *Predictive Cruise Control for Road Vehicles Using Road and Traffic Information*. Springer International Publishing.

Gustafsson, F. (1997). Slip-based tire-road friction estimation. *Automatica*, 33(6), 1087–1099.

Hult, R., Zanon, M., Gros, S., and Falcone, P. (2019). Optimal coordination of automated vehicles at intersections: Theory and experiments. *IEEE Transactions on Control Systems Technology*, 27(6), 2510–2525.

Khayatian, M., Mehrabian, M., and Shrivastava, A. (2018). Rim: Robust intersection management for connected autonomous vehicles. doi: 10.1109/RTSS.2018.00014.

Li, K., Misener, J.A., and Hedrick, K. (2007). On-board road condition monitoring system using slip-based tyre-road friction estimation and wheel speed signal analysis. *Automatica*, 221(1), 129–146.

Liu, X., Hsieh, P., and Kumar, P.R. (2018). Safe intersection management for mixed transportation systems with human-driven and autonomous vehicles. In *2018 56th Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, 834–841.

Meng, X. and Cassandras, C.G. (2018). Optimal control of autonomous vehicles for non-stop signalized intersection crossing. In *2018 IEEE Conference on Decision and Control (CDC)*, 6988–6993.

Mihály, A., Farkas, Z., and Gáspár, P. (2020). Multi-criteria autonomous vehicle control at non-signalized intersections. *Applied Sciences*, 10(20).

Olstam, J.J. and Tapani, A. (2004). Comparison of car-following models. *VTI Meddelande 990A, Swedish National Road Transport Research Institute. Project Code 40503 and 40485*.

Osipychov, D., Duy Tran, Weihua Sheng, and Chowdhary, G. (2017). Human intention-based collision avoidance for autonomous cars. In *2017 American Control Conference (ACC)*, 2974–2979.

Rajamani, R. (2005). *Vehicle Dynamics and Control*. Springer.

Shen, Z., Mahmood, A., Wang, Y., and Wang, L. (2019). Coordination of connected autonomous and human-operated vehicles at the intersection. *Int. Conf. Adv. Intell. Mech.*

Verma, R. and Del Vecchio, D. (2011). Development and experimental validation of a semi-autonomous cooperative active safety system. In *2011 50th IEEE Conference on Decision and Control and European Control Conference*, 4849–4854.

Xinli Geng, Huawei Liang, Hao Xu, Biao Yu, and Maofei Zhu (2016). Human-driver speed profile modeling for autonomous vehicle's velocity strategy on curvy paths. In *2016 IEEE Intelligent Vehicles Symposium (IV)*, 755–760.

Zhou, D., Ma, Z., and Sun, J. (2020). Autonomous vehicles turning motion planning for conflict areas at mixed-flow intersections. volume 5, 204–216.