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Hypercycle Systems of 5-Cycles in Complete 3-Uniform Hypergraphs

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Abstract: In this paper, we consider the problem of constructing hypercycle systems of 5-cycles in complete 3-uniform hypergraphs. A hypercycle system $\mathcal{C}(r, k, v)$ of order v is a collection of r -uniform k -cycles on a v -element vertex set, such that each r -element subset is an edge in precisely one of those k -cycles. We present cyclic hypercycle systems $\mathcal{C}(3, 5, v)$ of orders $v = 25, 26, 31, 35, 37, 41, 46, 47, 55, 56$, a highly symmetric construction for $v = 40$, and cyclic 2-split constructions of orders 32, 40, 50, 52. As a consequence, all orders $v \leq 60$ permitted by the divisibility conditions admit a $\mathcal{C}(3, 5, v)$ system. New recursive constructions are also introduced.

Keywords: hypergraph; hypercycle system; 3-uniform 5-cycle; edge decomposition; Steiner system



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1. Introduction

A hypergraph H of order v is a pair (X, E) , where X is the vertex set with $|X| = v$ and E is a family of subsets of X called edges. If all edges in E have size r , then H is said to be r -uniform. The complete r -uniform hypergraph of order v , denoted by $K_v^{(r)}$, is the hypergraph in which E consists of all the r -element subsets of X .

There are various ways to define cycles in hypergraphs. In this paper, we deal with an object called the tight r -uniform hypercycle of length k ($k > r \geq 3$)—also called cycloid in another context—which consists of k vertices and k edges; namely, it is a cyclic sequence of k vertices of X in which any r consecutive vertices, and only those, form an edge. If r is understood, we simply call it a k -cycle. An r -uniform hypercycle of length k is denoted by $C(r, k)$, and the integers $0, 1, \dots, v-1$ are used for the vertices. For example, the cyclic sequence $(0, 1, 2, 3, 4)$ represents a $C(3, 5)$ cycle for which the edges are the 3-sets $\{0, 1, 2\}$, $\{1, 2, 3\}$, $\{2, 3, 4\}$, $\{3, 4, 0\}$, and $\{4, 0, 1\}$. (Inside a 3-set, the order of vertices does not matter, but the order in a 5-tuple is of essence, except that any cyclic shift and the reversal of the sequence practically mean the same cycle.)

As a relative of Steiner systems, a hypercycle system $\mathcal{C}(r, k, v)$ of order v is a family \mathcal{C} of k -cycles such that each edge of $K_v^{(r)}$ is contained in precisely one k -cycle of \mathcal{C} . A very natural but rather hard problem is to determine those r, k, v for which a $\mathcal{C}(r, k, v)$ exists.

In the case of $k = r + 1$, the cycle is a complete hypergraph at the same time; hence, there is a bijection between $\mathcal{C}(r, r + 1, v)$ systems and Steiner systems $\mathcal{S}(k - 1, k, v)$ of which the study is an important classical issue in design theory. Apart from this, the first paper written on hypercycle systems is [1], where Bailey and Stevens initiated the study of Hamiltonian cycle decompositions, i.e., the case of $k = v$. They showed for $r = 3$ that it is possible to connect any edge of $K_v^{(3)}$ to a triplet of differences, and they developed a numerical algorithm. In this way, they settled the feasible cases for $r = 3$ up to $v \leq 16$ in the affirmative, with two exceptions, and solved the case $v = k = 9$, $r = 4$. Their results on $r = 3$ were extended to the larger range of $v \leq 32$ by Meszka

and Rosa [2], who proposed the study of a general problem with $r < k < v$, too, and introduced further variants. Before turning to 3-uniform 5-cycles, which is the main subject of the present paper, we also mention another track of a remarkably efficient computer search by Jirimutu et al. with the parameters $r = 3$ and $k = 7$, resulting in constructions for $v = 7, 8, 14, 16, 22, 23, 29, 37, 43, 44, 45, 58$; see the series of papers [3–7].

From now on, we deal with $\mathcal{C}(3, 5, v)$ systems and use the shorter term “cycle system” for them. Although such systems gained some attention in the past couple of years, there are still not many previous results regarding this topic. The first constructions were published by Meszka and Rosa in [2]. Certainly, the system of order 5 is trivial as $\mathcal{C}(3, 5)$ is self-complementary. Meszka and Rosa established via a computer search that there are exactly two non-isomorphic cycle systems of order 7 (neither of them is cyclic), and there exist cycle systems of orders 10, 11, and 16. They also described a recursive rule for cases where 3-wise balanced Steiner systems of block size 5 exist and observed further that the spherical geometries $\mathcal{S}(3, 5, 4^n + 1)$ imply the existence of a cycle system for every order v of the form $4^n + 1$. The four base blocks of a cyclic $\mathcal{S}(3, 5, 17)$ system are listed, e.g., on page 319 of [8].

Gionfriddo, Milazzo, and the second author [9] developed further recursions for building larger cycle systems from smaller ones and applied a difference method to design new initial configurations, up to $v = 22$. This work, finally published in [10], was one in an area in which the configurations were constructed by hand rather than by computer search; the paper included several hints concerning the concepts that helped establish results in this way. In the meantime, an independent research was carried out by Li, Lei, and Jirimutu, finding constructions for $v = 20$ [11] and $v = 32$ [12] and for $v = 22$ by Huo, Zhao, Feng, and Yang, as cited in [11]. Beyond explicit constructions, that paper [13] contained the recursion $v \rightarrow 5v$ (which also follows from a more general construction in [10]) and claims a system for $v = 26$. However, the generators for $v = 26$ contain $(0, 4, 21, 9, 16)$ as cycle #12 and $(0, 4, 18, 12, 16)$ as cycle #18, both of them covering the vertex triple $\{0, 4, 16\}$, hence not satisfying the requirements. Later in this paper, we present our solution to $\mathcal{C}(3, 5, 26)$.

1.1. Our Results

In constructions, two types of systems occur frequently. Assuming that the vertices are represented with the integers $0, 1, \dots, v - 1$, a system is cyclic if the mapping $i \mapsto i + 1 \pmod{v}$ is an automorphism. In addition, if a system is of even order $2v$ and contains two vertex-disjoint subsystems of order v , we say with the terminology of [9] that it is a 2-split system.

Every $\mathcal{C}(3, 5)$ contains exactly five edges of $K_v^{(3)}$ and exactly three edges incident with any of its vertices. Hence, $\binom{v}{3}$ has to be a multiple of 5 and $\binom{v-1}{2}$ a multiple of 3. Analogous observations are valid for 2-split systems of order $2v$ and for their subsystems of order v . From these facts, one can conclude that the following conditions are necessary. Natural numbers v belonging to the listed residue classes are referred to as feasible orders.

Lemma 1. (Feasible residue classes for the spectrum)

- (i) If there exist a system $\mathcal{C}(3, 5, v)$ of order v , then $v \equiv 1, 2, 5, 7, 10, 11 \pmod{15}$ [2].
- (ii) If there exists a 2-split system $\mathcal{C}(3, 5, 2v)$ of order $2v$, then $2v \equiv 2, 10, 20, 22 \pmod{30}$ [10].

Divisibility conditions similar to those mentioned before the lemma can be formulated for every Steiner system $\mathcal{S}(t, k, v)$, i.e., for partitioning the edge set of $K_v^{(t)}$ into copies of $K_k^{(t)}$, and more generally for the existence of edge decompositions of $K_v^{(r)}$ into subhypergraphs isomorphic to H for any given r -uniform hypergraph H . Keevash [14] introduced deep methods and proved that all but finitely many orders v satisfying the divisibility conditions admit an $\mathcal{S}(t, k, v)$ system for any parameters t and k . Glock, Kühn, Lo, and Osthus [15] extended this result from $K_v^{(t)}$ -decompositions to H -decompositions for all uniform hypergraphs H . For technical details, we refer to Theorem 1.1 of [15] and Theorem 1.5 of [16]. As a consequence, it follows for every v large enough that the conditions in (i) are not only

necessary but also sufficient for the existence of $\mathcal{C}(3, 5, v)$ systems. Based on our results, we have the impression that they are sufficient in a much stronger sense, namely without any exceptions. This is expressed in the following two-part conjecture.

Conjecture 1. *Let v be a feasible order.*

- (i) *If $v \equiv 1, 2, 5, 7, 10, 11 \pmod{15}$ and $v \neq 7$, then there exists a cyclic $\mathcal{C}(3, 5, v)$ system of order v .*
- (ii) *If $2v \equiv 2, 10, 20, 22 \pmod{30}$, then there exists a cyclic 2-split $\mathcal{C}(3, 5, 2v)$ system of order $2v$.*

Our constructions in Section 5 verify this conjecture for a certain range of orders. We summarize this in the following assertion, indicating the smallest open cases as upper bounds on the orders of cycle systems.

Theorem 1. *Let v be a feasible order.*

- (i) *If $v \equiv 1, 2, 5, 7, 10, 11 \pmod{15}$, and $v \neq 7$, then there exists a cyclic $\mathcal{C}(3, 5, v)$ system of order v for all $v < 61$.*
- (ii) *If $2v \equiv 2, 10, 20, 22 \pmod{30}$, then there exists a cyclic 2-split $\mathcal{C}(3, 5, 2v)$ system of order $2v$ for all $2v < 62$.*

One step in the constructions of Section 5 for odd v is to find a packing of $\binom{v}{2}$ symmetric cycles, a subsystem with a quadratic number of cycles that can be generated by just one cycle and two automorphisms of the subsystem. The feasibility of this step is analyzed in Section 4.

In Section 3, we describe some recursive steps applicable to building larger cycle systems from smaller ones. The prerequisites for them are given in Section 2, where we find decompositions of some auxiliary constructions. We believe that these building blocks will be useful in many further constructions in the future, too.

A remarkable particular consequence of the method is a $\mathcal{C}(3, 5, 40)$ system of order 40, which is generated by as few as seven cycles via two types of mappings (see Corollary 1).

In order to help the reader check the correctness of the constructions, details of the calculation are listed in numerous tables in the Appendix A.

1.2. Difference Triplets

We conclude this introduction with some technical definitions needed in the design and description of cyclic systems.

The vertex set of a cyclic system of order v is assumed to be \mathbb{Z}_v . The distance of two vertices i, j is their shortest distance “along the cycle”, that is $i - j = \min\{i - j, v - i - j\}$. A 5-cycle, say $C = (a, b, c, d, e)$, covers the five 3-element sets (vertex triples) $\{a, b, c\}$, $\{b, c, d\}$, $\{c, d, e\}$, $\{d, e, a\}$, and $\{e, a, b\}$.

Having fixed the value of v , a base cycle is a cycle $C = (a, b, c, d, e)$ for which the orbit under rotation $i \mapsto i + 1 \pmod{v}$ should also be included in the system. Assume that $T = \{p, q, r\}$ is a vertex triple in a base cycle C . As defined in [10], the difference triplet associated with T is a cyclic triplet of integers; it is obtained in the following way. First, find the increasing order of elements in T , i.e., let $0 \leq i < j < k < v$ be such that $\{i, j, k\} = \{p, q, r\}$. Then, either of

$$(i - j, j - k, k - i), \quad (j - k, k - i, i - j), \quad (k - i, i - j, j - k)$$

represents the difference triplet of T . Usually (but not always), we take the lexicographically smallest of these three. A difference triplet of type (i, i, j) is also called a “symmetric difference”, and we use the term “reflected difference” for a pair $\{(i, j, k), (i, k, j)\}$ of difference triplets in which the three distances i, j, k are all distinct.

2. Partial Cycle Systems of Orders 10 and 20

In this section, we present two auxiliary configurations that are useful in building larger cycle systems.

2.1. Decomposing a Doubled 5-Cycle

For the moment, let $X = \{a, b, c, d, e\}$ be a 5-element set on which the 3-uniform 5-cycle $C = (a, b, c, d, e)$ is taken. That is, C has five edges abc, bcd, cde, dea, eab (where the order of vertices in an edge is irrelevant). We double each vertex:

$$X_1 = \{a_1, b_1, c_1, d_1, e_1\}, \quad X_2 = \{a_2, b_2, c_2, d_2, e_2\}.$$

In this way, each edge of C gives rise to eight vertex triples in $X_1 \cup X_2$. For example, bcd yields $b_1c_1d_1, b_1c_1d_2, b_1c_2d_1, b_1c_2d_2, b_2c_1d_1, b_2c_1d_2, b_2c_2d_1$, and $b_2c_2d_2$.

Definition 1. A doubled 5-cycle consists of 10 vertices and 40 edges, as augmented from X to $X_1 \cup X_2$ above.

Lemma 2. The 40 edges of a doubled 5-cycle can be decomposed into eight 5-cycles.

Proof. A decomposition is obtained by taking the following eight cycles:

$$(a_1, b_1, c_1, d_1, e_1), \quad (a_1, b_1, c_2, d_2, e_2), \quad (a_1, b_2, c_1, d_2, e_1), \quad (a_1, b_2, c_2, d_1, e_2), \\ (a_2, b_1, c_1, d_2, e_2), \quad (a_2, b_1, c_2, d_1, e_1), \quad (a_2, b_2, c_1, d_1, e_2), \quad (a_2, b_2, c_2, d_2, e_1).$$

It can be checked that every edge of the doubled cycle occurs in precisely one of the eight cycles above. \square

2.2. Decomposing the Complement of a Blown-Up 4-Cycle

Here, we present a partial cycle system of order 20. The complement of the structure introduced below is the 3-uniform hypergraph, which is constructed by taking four 5-element vertex classes (sets of vertices) in cyclic order and by taking all vertex triples that are either inside one class or in the union of two consecutive classes in the fixed cyclic order.

Definition 2. Let $\mathcal{H}_{(4 \times 5)}$ denote the 3-uniform hypergraph of order 20 for which the vertex set is $X = A \cup B \cup C \cup D$, with $A = B = C = D = 5$, and for which the edge set consists of the vertex triples that

- meet exactly three of the vertex classes A, B, C, D or
- meet both A and C , or both B and D .

Lemma 3. There exists a $\mathcal{C}(3, 5, 20)$ system that contains an embedded decomposition of $\mathcal{H}_{(4 \times 5)}$ as a partial subsystem.

Proof. The structure of the construction is shown in Figures 1 and 2. Inside each of A, B, C, D , we take a $\mathcal{C}(3, 5, 5)$. We also know that there exists a 2-split system of order 10 (see [10]); hence, the families of crossing triples of $A \cup B, B \cup C, C \cup D$, and $D \cup A$ are decomposable into 5-cycles. What remains to be covered are the triples connecting A and C , or B and D or meeting exactly three of A, B, C , and D . Let us call the latter an ABC type (which can mean any of ABC, BCD, CDA , and DAB) and the former two together an AAC type (which can mean AAC, BBD, CCA , and DDA).

We have to cover $4 \cdot 10 \cdot 5 = 20 \cdot 10$ AAC-type triples and $4 \cdot 5^3 = 20 \cdot 25$ ABC-type triples, altogether $20 \cdot 35$ triples, which therefore need $20 \cdot 7$ cycles. This is generated by the seven cycles listed in Table 1a. Figure 2 illustrates ACACB and ACABD cycles of these 7 cycles, highlighted in the table.

One can see that

- each ACACB cycle covers two AAC-type triples and three ABC-type triples,
- each ACABD cycle covers one AAC-type triple and four ABC-type triples.

Hence, the seven cycles cover $3 \cdot 2 + 4 = 10$ AAC-type triples and $3 \cdot 3 + 4 \cdot 4 = 25$ ABC-type triples.

Now, from each of these seven cycles, we generate 20 cycles for $\mathcal{C}(3, 5, 20)$ by defining the following two types of automorphisms:

- rotation of order four among the vertex classes, $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$;
- simultaneous rotation among the indices $a_i \rightarrow a_{i+1}$, $b_i \rightarrow b_{i+1}$, $c_i \rightarrow c_{i+1}$, $d_i \rightarrow d_{i+1}$, for $i = 1, 2, 3, 4, 5$ modulo 5 (where the image of 5 is 1).

The effect of these transformations on the cycles $(a_1 c_1 a_2 c_2 b_1)$ and $(a_2 c_5 a_1 b_1 d_4)$ is detailed in Table 1b,c, respectively.

In this way, $20 \cdot 7$ cycles are defined as needed, and one can check that each vertex triple is covered exactly once. \square

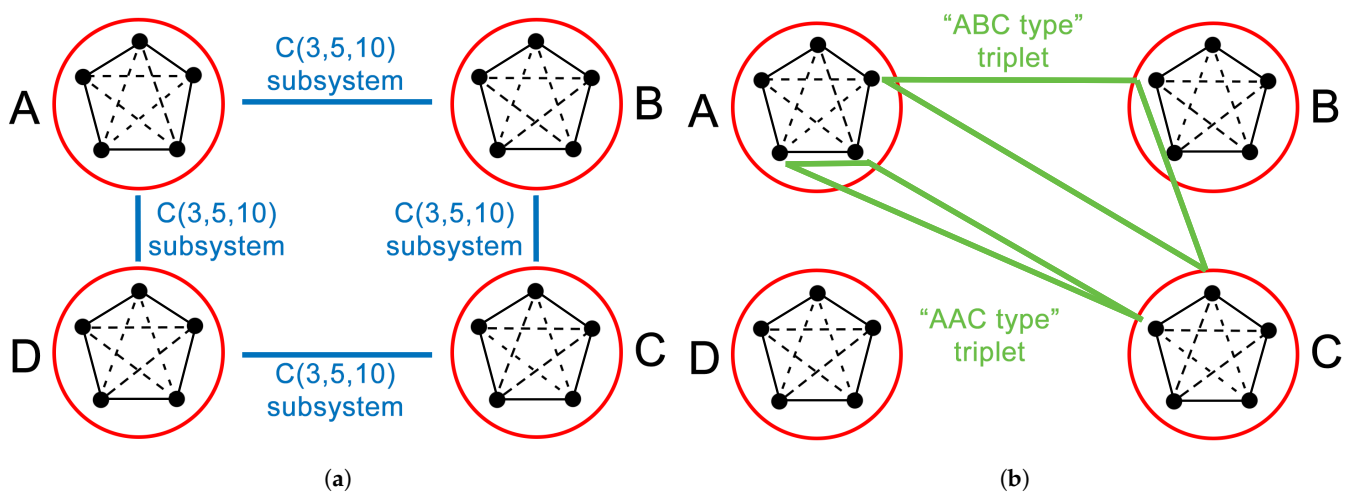


Figure 1. $\mathcal{C}(3, 5, 20)$ construction step 1. (a) Inside each of A, B, C , and D , we take $\mathcal{C}(3, 5, 5)$. The families of crossing triples of $A \cup B, B \cup C, C \cup D$, and $D \cup A$ are decomposable into 5-cycles since we know there exists a 2-split system of order 10 (see [10]). (b) The remaining triples either connect A and C , or B and D or meet exactly three of A, B, C , and D . The latter is called an ABC type (which can mean any of ABC, BCD, CDA , and DAB) and the former two together are an AAC type (which can mean AAC, BBD, CCA , and DDB).

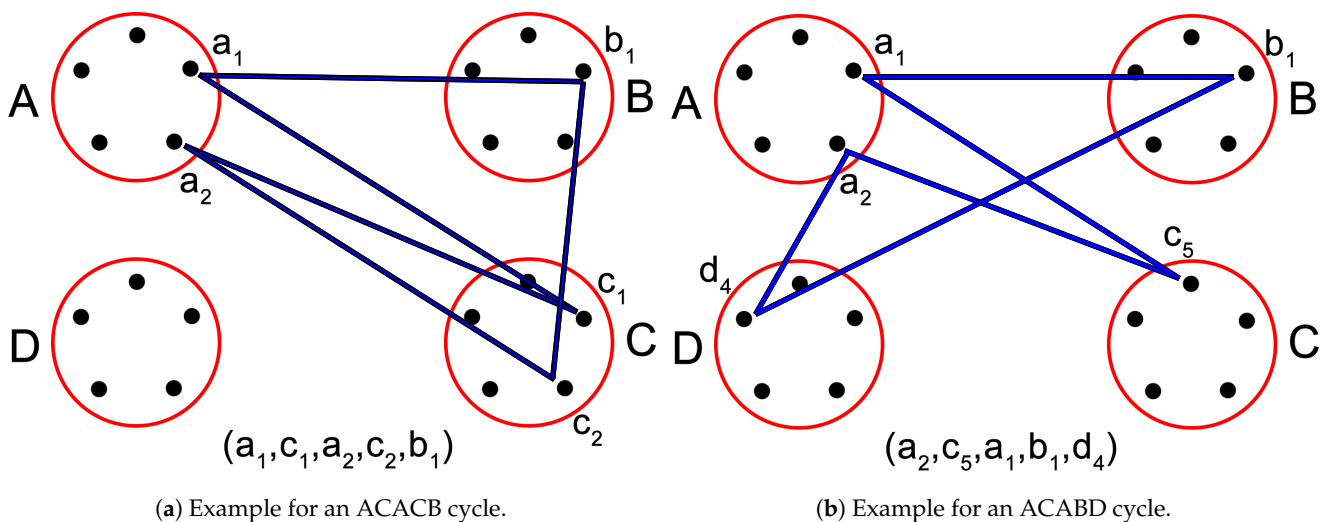


Figure 2. $\mathcal{C}(3, 5, 20)$ construction step 2. Two of the seven base cycles of Table 1a are illustrated.

Table 1. Subsystem $\mathcal{H}_{\langle 4 \times 5 \rangle}$ inside $\mathcal{C}(3, 5, 20)$.

(a) Seven base cycles: three ACACB cycles and four ACABD cycles.																								
Base Cycles (5 Vertices)																								
v1					v2					v3					v4					v5				
a1					c1					a2					c2					b1				
a1					c1					a3					c5					b3				
a1					c2					a3					c1					b4				
a2					c5					a1					b1					d4				
a3					c5					a1					b5					d4				
a4					c1					a1					b2					d4				
a4					c2					a1					b2					d3				
(b) Generating 20 cycles from the green base cycle of type ACACB.																								
ACACB cycles					BDBDC cycles					CACAD cycles					DBDBA cycles									
v1	v2	v3	v4	v5	v1	v2	v3	v4	v5	v1	v2	v3	v4	v5	v1	v2	v3	v4	v5					
a1	c1	a2	c2	b1	b1	d1	b2	d2	c1	c1	a1	c2	a2	d1	d1	b1	d2	b2	a1					
a2	c2	a3	c3	b2	b2	d2	b3	d3	c2	c2	a2	c3	a3	d2	d2	b2	d3	b3	a2					
a3	c3	a4	c4	b3	b3	d3	b4	d4	c3	c3	a3	c4	a4	d3	d3	b3	d4	b4	a3					
a4	c4	a5	c5	b4	b4	d4	b5	d5	c4	c4	a4	c5	a5	d4	d4	b4	d5	b5	a4					
a5	c5	a1	c1	b5	b5	d5	b1	d1	c5	c5	a5	c1	a1	d5	d5	b5	d1	b1	a5					
(c) Generating 20 cycles from the purple base cycle of type ACABD.																								
ACABD cycles					BDBCA cycles					CACDB cycles					DBDAC cycles									
v1	v2	v3	v4	v5	v1	v2	v3	v4	v5	v1	v2	v3	v4	v5	v1	v2	v3	v4	v5					
a2	c5	a1	b1	d4	b2	d5	b1	c1	a4	c2	a5	c1	d1	b4	d2	b5	d1	a1	c4					
a3	c1	a2	b2	d5	b3	d1	b2	c2	a5	c3	a1	c2	d2	b5	d3	b1	d2	a2	c5					
a4	c2	a3	b3	d1	b4	d2	b3	c3	a1	c4	a2	c3	d3	b1	d4	b2	d3	a3	c1					
a5	c3	a4	b4	d2	b5	d3	b4	c4	a2	c5	a3	c4	d4	b2	d5	b3	d4	a4	c2					
a1	c4	a5	b5	d3	b1	d4	b5	c5	a3	c1	a4	c5	d5	b3	d1	b4	d5	a5	c3					

Remark 1. In the system $\mathcal{C}(3, 5, 20)$ constructed above, each of $A \cup B$, $B \cup C$, $C \cup D$, and $D \cup A$ induces a 2-split $\mathcal{C}(3, 5, 10)$ system.

3. Recursive Constructions

In this section, we build some constructions that may be applied recursively. The method of the first subsection works for every order v for which there is a known 5-cycle system. The construct of the second subsection is applicable when an auxiliary partial system is available too. This requirement is fulfilled by $\mathcal{H}_{\langle 4 \times 5 \rangle}$, with the help of which a highly symmetric $\mathcal{C}(3, 5, 40)$ system can be designed.

3.1. General Step $v \rightarrow 10v$

The next construction builds 2-split systems; hence, it generates systems that are suitable for further recursions.

Theorem 2. If there exists a $\mathcal{C}(3, 5, v)$, then a 2-split $\mathcal{C}(3, 5, 10v)$ also exists.

Proof. Applying the $(\mathcal{C}(3, 5, v) \wedge \mathcal{C}(3, 5, w)) \rightarrow \mathcal{C}(3, 5, vw)$ recursion of [10], [Theorem 4.3], for the particular case of $w = 5$ starting from a $\mathcal{C}(3, 5, v)$ system, we obtain a $\mathcal{C}(3, 5, 5v)$ that has the following structure. The vertex set is partitioned into 5-element classes, say $X = X_1 \cup \dots \cup X_v$; each class X_i induces a $\mathcal{C}(3, 5, 5)$; the union of any two classes induces a

2-split $\mathcal{C}(3, 5, 10)$; and every vertex triple meeting three of the classes is covered by a cycle, all of the vertices of which are in mutually distinct classes.

The construction for $\mathcal{C}(3, 5, 10v)$ is illustrated with Figure 3. We create two vertex-disjoint copies of a $\mathcal{C}(3, 5, 5v)$ with the structure described above, say on the vertex sets $X_1 = X_{1,1} \cup \dots \cup X_{1,v}$ and $X_2 = X_{2,1} \cup \dots \cup X_{2,v}$. For $i = 1, \dots, v$, we keep the $\mathcal{C}(3, 5, 5)$ subsystems inside both $X_{1,i}$ and $X_{2,i}$ and insert the crossing cycles of a 2-split $\mathcal{C}(3, 5, 10)$ between $X_{1,i}$ and $X_{2,i}$.

For all $1 \leq i < j \leq v$, we also keep the crossing cycles of a 2-split $\mathcal{C}(3, 5, 10)$ inside $X_{1,i} \cup X_{1,j}$ as well as inside $X_{2,i} \cup X_{2,j}$. The remaining triples inside $X_{1,i} \cup X_{1,j} \cup X_{2,i} \cup X_{2,j}$ together form a hypergraph isomorphic to $\mathcal{H}_{(4 \times 5)}$, which we decompose into 5-cycles by Lemma 3.

Finally, in order to cover the remaining vertex triples, we perform the following. Any cycle C that meets more than two classes of $\mathcal{C}(3, 5, 5v)$ specifies a 5-cycle C_1 in X_1 and a 5-cycle C_2 in X_2 . We replace $C_1 \cup C_2$ with the eight 5-cycles constructed in Lemma 2. This completes the construction of $\mathcal{C}(3, 5, 10v)$.

As it can be seen, in the system constructed in this way, every vertex triple is contained in one and only one 5-cycle. Hence, a $\mathcal{C}(3, 5, 10v)$ system is obtained. \square

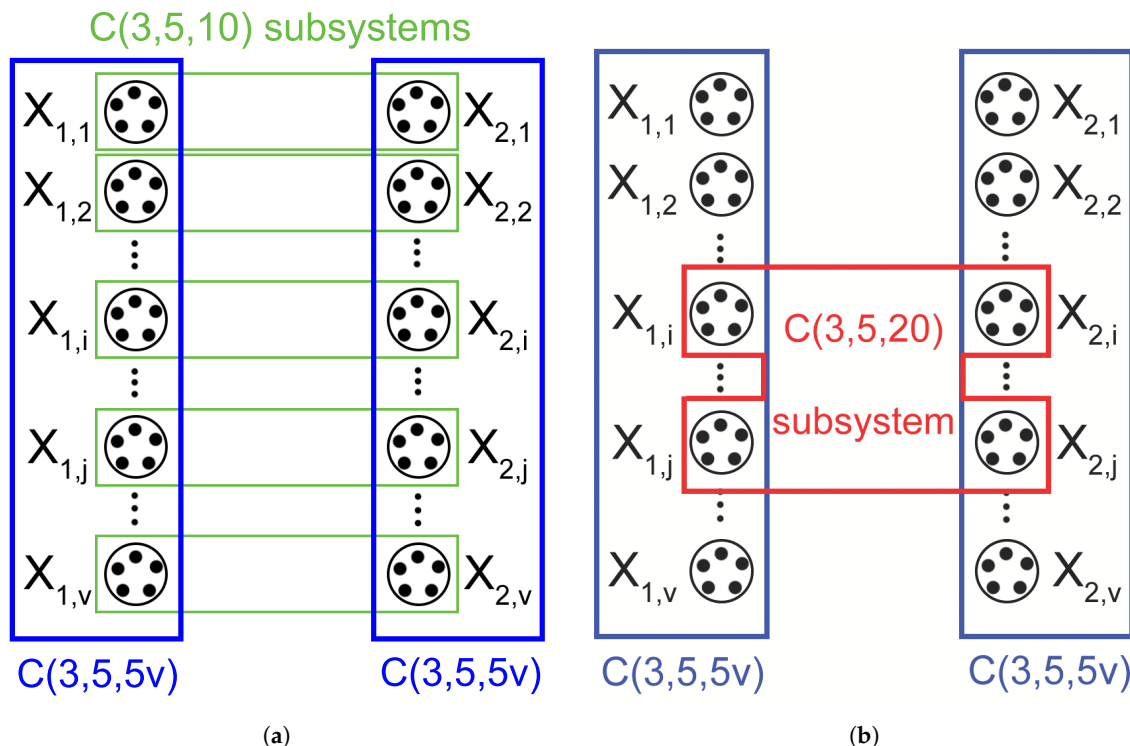


Figure 3. The $v \rightarrow 10v$ construction. (a) Two vertex-disjoint copies of a $\mathcal{C}(3, 5, 5v)$ are created. The details are explained in the proof of Theorem 2. Within the green rectangles, the two $\mathcal{C}(3, 5, 5)$ subsystems are kept with crossing cycles of a 2-split $\mathcal{C}(3, 5, 10)$ between them. (b) For all $1 \leq i < j \leq v$, we also keep the crossing cycles of a 2-split $\mathcal{C}(3, 5, 10)$ inside $X_{1,i} \cup X_{1,j}$ as well as inside $X_{2,i} \cup X_{2,j}$. The remaining triples inside $X_{1,i} \cup X_{1,j} \cup X_{2,i} \cup X_{2,j}$ together form a hypergraph isomorphic to $\mathcal{H}_{(4 \times 5)}$.

Remark 2. Applying this result for known values and for the newly constructed systems, the existence problem is now solved for some further $v = 70, 160, 200, 260, 310, 320, 350, 370, 460, 470, 520, 550, 560, 700, 800, 820, 1000, \dots$. Moreover, there are further values for which the existence of systems was already known, but systems with the 2-split property are new: 100, 110, 400, 410, and 500.

3.2. Conditional Step $v \rightarrow 8v$

First, we define a parametrized hypergraph that generalizes the structure of $\mathcal{H}_{\langle 4 \times 5 \rangle}$. For the sake of applicability, the parameter k is assumed to be feasible for the existence of a $\mathcal{C}(3, 5, k)$ system.

Definition 3. Let $\mathcal{H}_{\langle 4 \times k \rangle}$ denote the 3-uniform hypergraph of order $4k$ for which the vertex set is $X = A \cup B \cup C \cup D$, with $|A| = |B| = |C| = |D| = k$ and for which the edge set consists of those vertex triples that

- meet exactly three of the vertex classes A, B, C , or D or
- meet both A and C , or both B and D .

Theorem 3. If $\mathcal{H}_{\langle 4 \times v \rangle}$ admits a decomposition into 5-cycles and there exists a 2-split $\mathcal{C}(3, 5, 2v)$ system, then a system $\mathcal{C}(3, 5, 8v)$ exists.

Proof. The basic scheme of the construction is as follows.

1. Start with a Steiner Quadruple System of order 8.
2. Take a $2K_2$ -decomposition of the complete graph K_8 embedded into $\mathcal{SQS}(8)$.
3. Substitute a v -element set into each point of $\mathcal{SQS}(8)$ and a $\mathcal{C}(3, 5, v)$ system into each of those v -sets.
4. Insert 14 copies of the 5-cycle decomposition of $\mathcal{H}_{\langle 4 \times v \rangle}$ in each blown-up block of $\mathcal{SQS}(8)$.

These steps are performed in the following way, illustrated in Figure 4.

1° For a simple notation, let us assume that the eight points of $\mathcal{SQS}(8)$ are $1, 2, \dots, 8$. We consider the following 14 blocks:

$$\begin{aligned} A &= \{1, 2, 5, 6\}, & B &= \{1, 2, 7, 8\}, & C &= \{3, 4, 5, 6\}, & D &= \{3, 4, 7, 8\}, \\ E &= \{1, 3, 5, 7\}, & F &= \{1, 3, 6, 8\}, & G &= \{2, 4, 5, 7\}, & H &= \{2, 4, 6, 8\}, \\ J &= \{1, 4, 5, 8\}, & K &= \{1, 4, 6, 7\}, & L &= \{2, 3, 5, 8\}, & M &= \{2, 3, 6, 7\}, \\ & & P &= \{1, 2, 3, 4\}, & Q &= \{5, 6, 7, 8\}. \end{aligned}$$

2° The complete graph K_8 has 28 edges. Assume that its vertex set is the same $\{1, 2, \dots, 8\}$ as the set of points of $\mathcal{SQS}(8)$. We decompose the edge set into 14 subgraphs isomorphic to $2K_2$ in such a way that the union of the two edges in each copy of $2K_2$ is a distinct block of $\mathcal{SQS}(8)$. One solution is as follows:

$$\begin{aligned} A &- 1, 2 : 5, 6; & B &- 1, 7 : 2, 8; & C &- 3, 5 : 4, 6; & D &- 3, 4 : 7, 8; \\ E &- 1, 3 : 5, 7; & F &- 1, 8 : 3, 6; & G &- 2, 7 : 4, 5; & H &- 2, 4 : 6, 8; \\ J &- 1, 5 : 4, 8; & K &- 1, 6 : 4, 7; & L &- 2, 5 : 3, 8; & M &- 2, 6 : 3, 7; \\ & & P &- 1, 4 : 2, 3; & Q &- 5, 8 : 6, 7. \end{aligned}$$

3° For each point i ($i = 1, 2, \dots, 8$) of $\mathcal{SQS}(8)$, we take a v -element set X_i , which will be the vertex set of a $\mathcal{C}(3, 5, v)$, hence forming eight vertex-disjoint subsystems of the $\mathcal{C}(3, 5, 4v)$ under construction.

4° For each block—say, $i_1, i_2 : i_3, i_4$ as partitioned in 2°—embed the 5-cycle decomposition of $\mathcal{H}_{\langle 4 \times v \rangle}$, applying the rule

$$A \rightarrow X_{i_1}, \quad C \rightarrow X_{i_2}, \quad B \rightarrow X_{i_3}, \quad D \rightarrow X_{i_4}.$$

Hence, the two edges of $2K_2$ inside each block specify the positions of AAC triples in the blow-up of the block in question.

It can be checked that each vertex triple appears in exactly one 5-cycle:

- If a triple is a subset of one X_i , then it is in a cycle in the $\mathcal{C}(3, 5, v)$ system embedded in X_i .

- If a triple meets exactly two parts, say X_{i_1} and X_{i_2} , then the edge $i_1 i_2$ determines a $2K_2$ in the decomposition of K_8 ; this $2K_2$ specifies a block in $\mathcal{SQS}(8)$, and then in the copy of $\mathcal{H}_{\langle 4 \times v \rangle}$ embedded in the blow-up of that block, the vertex triple in question is a triple of AAC type; hence, it is contained in a 5-cycle of the system.
- If a triple meets exactly three parts, say X_{i_1} , X_{i_2} , and X_{i_3} , then by the definition of Steiner quadruple systems, the three points i_1, i_2, i_3 determine a block, and then in the copy of $\mathcal{H}_{\langle 4 \times v \rangle}$ embedded in the blow-up of that block, the vertex triple in question is a triple of ABC type; hence, it is contained in a unique cycle.

This verifies the claimed properties and completes the proof. \square

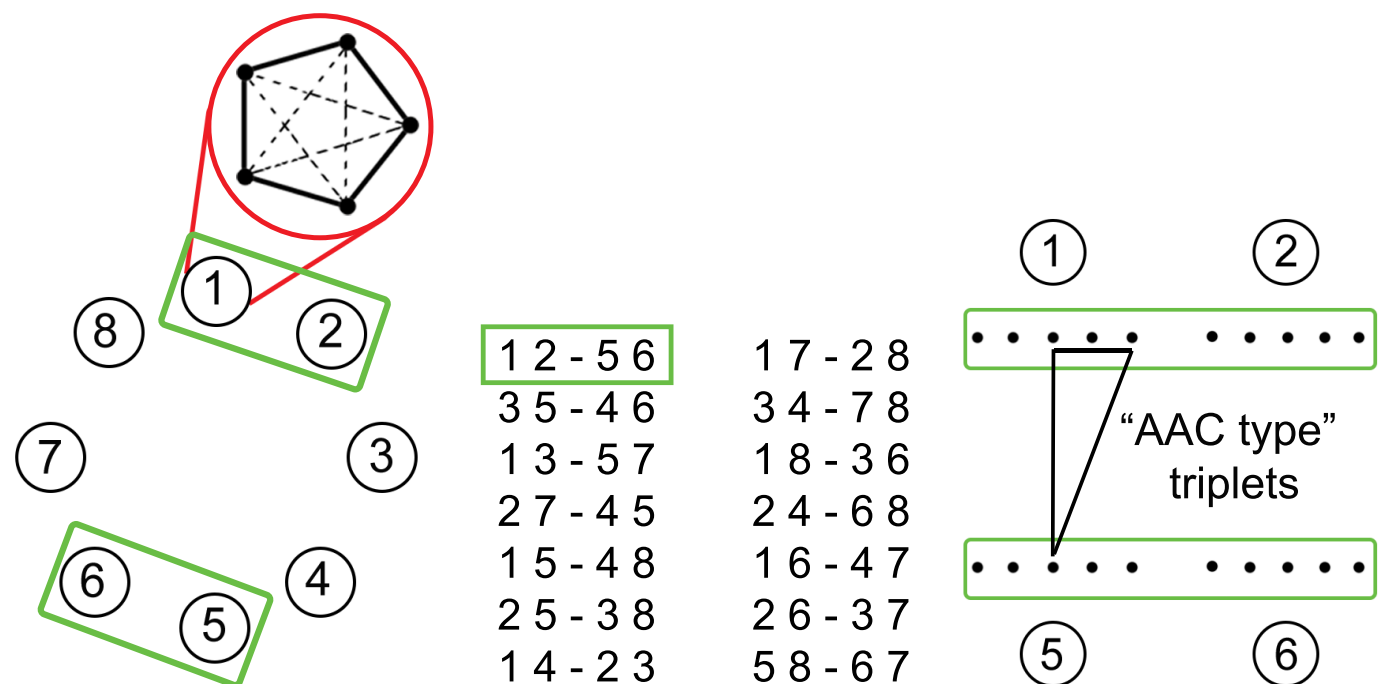


Figure 4. $\mathcal{C}(3, 5, 40)$ construction as described in the proof of Theorem 3.

As a first application of this method, we derive that a highly symmetric $\mathcal{C}(3, 5, 40)$ system exists.

Corollary 1. *There exists a $\mathcal{C}(3, 5, 40)$ system generated by seven 5-cycles.*

Proof. This follows from Theorem 3, applying Lemma 3, which ensures that $\mathcal{H}_{\langle 4 \times 5 \rangle}$ admits a 5-cycle decomposition generated by seven cycles and two automorphisms. \square

4. General Method for Cyclic Packing of Symmetric Cycles with $\gcd(v, 10010) = 1$

Cyclic systems are generated by a certain number of base cycles. In this way, instead of $O(v^3)$ cycles, the task is to find $O(v^2)$ base cycles for which the orbits cover all vertex triples. In this section, we present a method that reduces the number of base cycles to be found. Although the gain is only linear in v , it is significant when v is not too large. It is applicable whenever v satisfies a simple arithmetic condition.

More explicitly, the construction works for all odd v not divisible by 5, 7, 11, and 13. We first explain these exceptions and then prove that the method is applicable for all the

other values of v . This analysis also tells us which ones are the few symmetric differences to be handled separately when v is a multiple of 5, 7, 11, or 13.

Throughout this section, we assume that v is odd and consider the base cycles

$$(0, i, v - 3i, 3i, v - i), \quad i = 1, 2, \dots, (v - 1)/2.$$

If v is a multiple of 5 or 7 or 11 or 13, then the difference triplets (see the definition in Section 1.2) listed in Table 2 are covered more than once. Marking with S means that a symmetric difference belonging to a value other than i also occurs in the 5-cycle $(0, i, v - 3i, 3i, v - i)$. Marking with M means that a reflected difference occurs for more than one value of i .

Table 2. Disqualified triplet types: S = symmetric triplet, and M = multiply covered triplet.

Divisor of v	Difference Triplet	Reason of Conflict	i of Cycle that Covers Triplet
5	$(v/5, v/5, 2v/5)$	S	$v/5$
5	$(v/5, 2v/5, 2v/5)$	S	$2v/5$
7	$(v/7, v/7, 2v/7)$	S	$2v/7$
7	$(2v/7, 2v/7, 3v/7)$	S	$3v/7$
7	$(3v/7, 3v/7, v/7)$	S	$v/7$
7	$(v/7, 2v/7, 3v/7)$	M	$v/7, 2v/7, 3v/7$
11	$(v/11, 2v/11, 3v/11)$	M	$3v/11, 5v/11$
11	$(v/11, 3v/11, 4v/11)$	M	$v/11, 2v/11$
11	$(v/11, 4v/11, 5v/11)$	M	$3v/11, 4v/11$
11	$(2v/11, 3v/11, 5v/11)$	M	$2v/11, 4v/11$
11	$(2v/11, 4v/11, 5v/11)$	M	$v/11, 5v/11$
13	$(v/13, 3v/13, 4v/13)$	M	$v/13, 3v/13, 4v/13$
13	$(2v/13, 5v/13, 6v/13)$	M	$2v/13, 5v/13, 6v/13$

The case of $v = 7k$ is special, as it has both types of anomalies: all symmetric triplets and the (unique) reflected triplet are covered three times. To make the reason transparent, we exhibit the situation in Table 3 for $v = 7$.

Table 3. Covered vertex triples and their difference triplets of $(0, i, v - 3i, 3i, v - i)$ for $v = 7$.

cycle for $i = 1$			$(0, 1, 4, 3, 6)$		
covered vertex triples	$\{0, 1, 4\}$	$\{1, 3, 4\}$	$\{3, 4, 6\}$	$\{0, 3, 6\}$	$\{0, 1, 6\}$
difference triplets	$(1, 3, 3)$	$(2, 1, 3)$	$(1, 2, 3)$	$(3, 3, 1)$	$(1, 1, 2)$
cycle for $i = 2$			$(0, 2, 1, 6, 5)$		
covered vertex triples	$\{0, 1, 2\}$	$\{1, 2, 6\}$	$\{1, 5, 6\}$	$\{0, 5, 6\}$	$\{0, 2, 5\}$
difference triplets	$(1, 1, 2)$	$(2, 1, 3)$	$(1, 2, 3)$	$(1, 1, 2)$	$(2, 2, 3)$
cycle for $i = 3$			$(0, 3, 5, 2, 4)$		
covered vertex triples	$\{0, 3, 5\}$	$\{2, 3, 5\}$	$\{2, 4, 5\}$	$\{0, 2, 4\}$	$\{0, 3, 4\}$
difference triplets	$(2, 2, 3)$	$(1, 2, 3)$	$(2, 1, 3)$	$(2, 2, 3)$	$(3, 3, 1)$

Theorem 4. (Symmetric Packing Lemma) If v is an odd feasible order, then for the symmetric base cycles

$$(0, i, v - 3i, 3i, v - i), \quad i = 1, 2, \dots, (v - 1)/2,$$

the necessary and sufficient condition of covering each difference triplet at most once and covering each symmetric difference exactly once is that v is not divisible by any of 5, 7, 11, and 13. Moreover, if such a forbidden divisibility holds, then the multiple covers of difference triplets can be eliminated by the removal of symmetric cycles belonging to those i that are listed in the last column of Table 2.

Proof. The discussion preceding the theorem showed that divisibility by 5, 7, 11, and 13 must be excluded if we want to avoid multiple covers of vertex triples. Below, we see that, if v is a multiple of one or more of these numbers, then exactly the multiple covers listed in Table 2 occur. Moreover, in any other case, each cycle $(0, i, v - 3i, 3i, v - i)$ contains

precisely one symmetric difference—namely, only the one occurring for its 3-element subset $\{0, i, v - i\}$ —and that no reflected difference is generated by more than one value of i .

To do this, we consider the vertex triples $\{0, i, v - 3i\}$ and $\{i, v - 3i, 3i\}$, compute the three differences determined by the vertex pairs in each triple, and put them in increasing order. Although we consider only two vertex triples here, they also represent the other two triples $\{v - 3i, 3i, v - i\}$ and $\{3i, v - i, 0\}$ because the latter two yield the same increasing sequences when the three distances are put in order.

The formula for a difference, as a function of i and v , depends on the ratio i/v . For example, the difference between 0 and $3i$ is equal to $3i$ if $0 \leq i/v \leq 1/6$, $v - 3i$ if $1/6 \leq i/v \leq 1/3$, and $3i - v$ if $1/3 \leq i/v \leq 1/2$. (We may write strict inequalities here because a feasible v cannot be divisible by 2 and 3.) The same ratio i/v determines which of the three differences is smallest and which of them is largest.

A further characteristic that we take into account is what we call “position”. We say that a triplet has a normal position if its largest difference equals the sum of its other two (smallest and middle) differences, and it has extra position if its three differences sum up to v . Table 4 exhibits all these pieces of information for all i , where $[a, b]$ means the range in the closed interval determined by the condition $a \leq i/v \leq b$. Completely identical (small, middle, and large) triplets are not repeated; for instance, $(i, 3i, 4i)$ is valid in the entire interval $[0, 1/8]$.

Table 4. Characteristics of difference triplets depending on i/v .

Interval [a, b]	Small	(0, i, v - 3i) Middle	Large	Cycle Position	Small	(i, v - 3i, 3i) Middle	Large	Cycle Position
[0, 1/12]	i	$3i$	$4i$	N	$2i$	$4i$	$6i$	N
[1/12, 1/10]				N	$2i$	$4i$	$v - 6i$	E
[1/10, 1/8]				N	$2i$	$v - 6i$	$4i$	E
[1/8, 1/7]	i	$3i$	$v - 4i$	E	$v - 6i$	$2i$	$v - 4i$	N
[1/7, 1/6]	i	$v - 4i$	$3i$	E				N
[1/6, 1/5]	i	$v - 4i$	$v - 3i$	N	$6i - v$	$v - 4i$	$2i$	N
[1/5, 1/4]	$v - 4i$	i	$v - 3i$	N	$v - 4i$	$6i - v$	$2i$	N
[1/4, 2/7]	$4i - v$	$v - 3i$	i	N	$4i - v$	$2v - 6i$	$v - 2i$	N
[2/7, 3/10]	$v - 3i$	$4i - v$	i	N				N
[3/10, 1/3]				N	$2v - 6i$	$4i - v$	$v - 2i$	N
[1/3, 3/8]	$3i - v$	i	$4i - v$	N	$6i - 2v$	$v - 2i$	$4i - v$	N
[3/8, 2/5]	$3i - v$	i	$2v - 4i$	E	$v - 2i$	$6i - 2v$	$2v - 4i$	E
[2/5, 5/12]	$3i - v$	$2v - 4i$	i	E	$v - 2i$	$2v - 4i$	$6i - 2v$	E
[5/12, 3/7]				E	$v - 2i$	$2v - 4i$	$3v - 6i$	N
[3/7, 1/2]	$2v - 4i$	$3i - v$	i	E				N

Hence, there are 23 possible cases altogether, 14 of which are normal and 9 of which are extra. More explicitly, the 14 normal triplets are

$$\begin{aligned}
 A &= (i, 3i, 4i); & B &= (i, v - 4i, v - 3i); & C &= (2i, 4i, 6i); \\
 D &= (3i - v, i, 4i - v); & E &= (4i - v, v - 3i, i); & F &= (4i - v, 2v - 6i, v - 2i); \\
 G &= (6i - 2v, v - 2i, 4i - v); & H &= (6i - v, v - 4i, 2i); & J &= (v - 6i, 2i, v - 4i); \\
 K &= (v - 4i, i, v - 3i); & L &= (v - 4i, 6i - v, 2i); & M &= (v - 3i, 4i - v, i); \\
 N &= (v - 2i, 2v - 4i, 3v - 6i); & P &= (2v - 6i, 4i - v, v - 2i),
 \end{aligned}$$

and the 9 extra triplets are

$$\begin{aligned}
 Q &= (i, 3i, v - 4i); & R &= (i, v - 4i, 3i); & S &= (2i, 4i, v - 6i); \\
 T &= (2i, v - 6i, 4i); & U &= (3i - v, i, 2v - 4i); & V &= (3i - v, 2v - 4i, i); \\
 X &= (v - 2i, 6i - 2v, 2v - 4i); & Y &= (v - 2i, 2v - 4i, 6i - 2v); & Z &= (2v - 4i, 3i - v, i).
 \end{aligned}$$

One has to show that no coincidences other than the ones listed in Table 2 can occur. This can be done through a case-by-case analysis, as demonstrated in Tables 5 and 6. Note that normal triplets—as well as extra triplets—may coincide among each other, but a normal triplet can never be equal to an extra triplet. Hence, the inspections branch into two directions, one for normal triplets and one for extra triplets. In the simplified scheme, as exhibited in the tables, we use the following abbreviations:

Table 5. Infeasibility of pairs of normal triplets.

	P	N	M	L	K	J	H	G	F	E	D	C	B
A	2 v	2 v	⟨13⟩	3 v	⟨13⟩	2 v	⟨11⟩	2 v	3 v	3 v	2 v	contr	interv
B	2 v	2 v	2 v	2 v	⟨5⟩	2 v	⟨5⟩	2 v	2 v	interv	interv	2 v	
C	2 v	2 v	2 v	2 v	2 v	2 v	2 v	2 v	2 v	2 v	2 v		
D	3 v	interv	3 v	2 v	interv	2 v	2 v	3 v	⟨11⟩	interv			
E	2 v	⟨11⟩	⟨7⟩	2 v	2 v	2 v	2 v	2 v	3 v				
F	2 v	⟨7⟩	2 v	2 v	2 v	⟨7⟩	2 v	2 v					
G	3 v	2 v	3 v	2 v	2 v	2 v	2 v						
H	2 v	2 v	2 v	2 v	⟨5⟩	2 v							
J	2 v	⟨7⟩	2 v	2 v	contr								
K	2 v	3 v	⟨13⟩	⟨5⟩									
L	2 v	2 v	contr										
M	3 v	2 v											
N	2 v												

Table 6. Infeasibility of pairs of extra triplets.

	Z	Y	X	V	U	T	S	R
Q	3 v	2 v	2 v	interv	2 v	2 v	contr	⟨7⟩
R	⟨13⟩	2 v	2 v	2 v	⟨13⟩	2 v	2 v	
S	⟨11⟩	2 v	2 v	2 v	2 v	2 v		
T	3 v	2 v	2 v	2 v	2 v			
U	⟨13⟩	⟨5⟩	⟨5⟩	⟨5⟩				
V	⟨7⟩	⟨5⟩	⟨5⟩					
X	2 v	⟨5⟩						
Y	2 v							

contr : no two values i_1, i_2 can make the two triplets identical;

interv : the possible values i_1, i_2 for identical triplets are not in the intervals where the formulas apply;

2 | v : the possible values i_1, i_2 for identical triplets would imply that v is even;

3 | v : the possible values i_1, i_2 for identical triplets would imply that v is a multiple of 3;

⟨5⟩ : a disqualified triplet of Table 2 occurs if v is a multiple of 5;

⟨7⟩ : a disqualified triplet of Table 2 occurs if v is a multiple of 7;

⟨11⟩ : a disqualified triplet of Table 2 occurs if v is a multiple of 11;

⟨13⟩ : a disqualified triplet of Table 2 occurs if v is a multiple of 13.

The cases are many but not hard. Here are some examples:

A vs. B : the first terms of $(i_1, 3i_1, 4i_1) = (i_2, v - 4i_2, v - 3i_2)$ would require $i_1 = i_2 = i$ for some i and then $i = v/7$ by either of the second and third terms, but the formula $(i_1, 3i_1, 4i_1)$ is valid only for $i_1/v \in [0, 1/8]$ (and $(i_2, v - 4i_2, v - 3i_2)$ is valid only for $i_2 \in [1/6, 1/5]$).

E vs. H : a comparison of the corresponding terms in $(4i_1 - v, v - 3i_1, i_1) = (6i_2 - v, v - 4i_2, 2i_2)$ would imply $4i_1 = 6i_2$, $3i_1 = 4i_2$, and $i_1 = 2i_2$, but any two of these lead to the contradiction $i_1 = i_2 = 0$.

V vs. X : in $(3i_1 - v, 2v - 4i_1, i_1) = (v - 2i_2, 6i_2 - 2v, 2v - 4i_2)$, the equalities between the first terms and between the third terms together imply $3i_1 + 2i_2 = 2v = i_1 + 4i_2$; hence, $i_1 = i_2 = i$ for some i and then $v = 5i/2$. That is, $i_1 = i_2 = 2v/5$. For the validity of the formulas, we need $i_1 \in [2/5, 3/7]$ and $i_2 \in [3/8, 2/5]$. Both conditions are satisfied, yielding the disqualified triplet $(v/5, 2v/5, 2v/5)$. This means that the 5-cycle defined for $i = 2v/5$ covers this symmetric triplet three times.

The same conclusion is obtained for the combinations (U, V), (U, X), (U, Y), (V, Y), and (X, Y), too, i.e., all six pairs from (U, V, X, Y) have the same implication.

U vs. Z : starting from $(3i_1 - v, i_1, 2v - 4i_1) = (2v - 4i_2, 3i_2 - v, i_2)$, we multiply the second terms by 2 and compare the third terms. Then, $6i_2 - 2i_1 = 2v = 4i_1 + i_2$; hence, $6i_1 = 5i_2$. Writing $i_1 = 5k$ and $i_2 = 6k$, we obtain $v = 3i_2 - i_1 = 13k$; then, $i_1 = 5v/13$

and $i_2 = 6v/13$. These solutions are feasible because, for validity of the formulas, we need $i_1/v \in [3/8, 2/5]$ and $i_2/v \in [3/7, 1/2]$. These conditions are satisfied by $5/13$ and $6/13$, yielding the disqualified triplet $(2v/13, 5v/13, 6v/13)$.

The validity of all entries in Tables 5 and 6 can be checked in a similar way. This proves the assertion since v is assumed to be odd, and there does not exist any 5-cycle system for which the order is divisible by 3. \square

5. Complete Spectrum of $\mathcal{C}(3, 5, v)$ Systems for $v \leq 60$

Recall that the feasible residue classes are $v \equiv 1, 2, 5, 7, 10, 11 \pmod{15}$. The results presented in this section, together with earlier publications, imply that every feasible order $v \leq 60$ admits a 3-uniform 5-cycle system. The previously known system $\mathcal{C}(3, 5, 55)$ (which is not cyclic) was derived from $\mathcal{C}(3, 5, 11)$ by the recursions of [10,12], and before the time of writing this paper, no construction was known for $v = 56$.

All our constructions are cyclic here, and the systems for $v = 32$, $v = 40$, $v = 50$, and $v = 52$ are not only cyclic but also 2-split. These three are presented in the second subsection, separately from the systems of the first subsection that are cyclic but not 2-split. (The even orders 26 and 46 do not admit 2-split constructions.)

5.1. Cyclic Systems of Orders 25, 26, 31, 35, 37, 41, 46, 47, 55, and 56

Proposition 1. *There exists a cyclic $\mathcal{C}(3, 5, 25)$ system.*

Proof. We have $\frac{25 \cdot 24 \cdot 23}{6} = 25 \cdot 92$ vertex triples and 12 symmetric differences.

- At the beginning, we take five subsystems isomorphic to the cyclic $\mathcal{C}(3, 5, 5)$ system, hence with the cycles

$$(0, 5, 10, 15, 20), \quad (0, 10, 20, 5, 15)$$

in their five possible positions:

$$(i, 5 + i, 10 + i, 15 + i, 20 + i), \quad (i, 10 + i, 20 + i, 5 + i, 15 + i) \quad i = 0, 1, 2, 3, 4.$$

These cover $5 \cdot 10 = 25 \cdot 2$ vertex triples and two symmetric differences. There remain $25 \cdot 90$ triples to cover for which we need $90/5 = 18$ base cycles.

- The 10 symmetric base cycles can be defined by the rule

$$(0, i, 25 - 3i, 3i, 25 - i), \quad i = 1, 2, 3, 4, 6, 7, 8, 9, 11, 12$$

according to Theorem 4.

- We just need 8 reflected base cycles that can be arranged in four pairs:

$$(0, 1, 5, 11, 10), \quad (0, 2, 5, 12, 10), \quad (0, 3, 13, 20, 8), \quad (0, 4, 14, 20, 9), \\ (0, 24, 5, 9, 10), \quad (0, 23, 5, 8, 10), \quad (0, 13, 20, 5, 8), \quad (0, 14, 20, 5, 9).$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A2 and A3. \square

Proposition 2. *There exists a cyclic $\mathcal{C}(3, 5, 26)$ system.*

Proof. We have $\frac{26 \cdot 25 \cdot 24}{6}$ vertex triples; therefore, we need $26 \cdot 20$ cycles that should be generated by 20 base cycles.

- The 12 symmetric base cycles are

$$(0, 1, 24, 2, 25), \quad (0, 3, 20, 6, 23), \quad (0, 5, 16, 10, 21), \quad (0, 7, 12, 14, 19), \\ (0, 9, 8, 18, 17), \quad (0, 11, 4, 22, 15), \quad (0, 2, 22, 4, 24), \quad (0, 4, 18, 8, 22), \\ (0, 6, 22, 4, 20), \quad (0, 8, 21, 5, 18), \quad (0, 10, 25, 1, 16), \quad (0, 12, 23, 3, 14).$$

Here, for odd i , the cycles are taken according to the pattern $(0, i, 26 - 2i, 2i, 26 - i)$, but for even i , we need to make several modifications.

- The 8 reflected base cycles are in four pairs:

$$(0, 1, 8, 17, 13), (0, 1, 12, 25, 6), (0, 12, 20, 23, 2), (0, 3, 7, 2, 19), \\ (0, 22, 5, 12, 13), (0, 7, 20, 5, 6), (0, 5, 8, 16, 2), (0, 17, 12, 16, 19).$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once as it can be verified from the data in Tables A4 and A5. \square

Proposition 3. *There exists a cyclic $\mathcal{C}(3, 5, 31)$ system.*

Proof. We have $\frac{31 \cdot 30 \cdot 29}{6}$ vertex triples; therefore, we need $31 \cdot 29$ cycles, which should be generated by 29 base cycles.

- The 15 symmetric base cycles can be taken as

$$(0, i, 31 - 3i, 3i, 31 - i), \quad i = 1, \dots, 15$$

as guaranteed by Theorem 4.

- The 14 reflected base cycles are in seven pairs:

$$(0, 1, 5, 8, 20) - (0, 12, 15, 19, 20); (0, 1, 7, 17, 9) - (0, 23, 2, 8, 9); \\ (0, 1, 13, 15, 18) - (0, 3, 5, 17, 18); (0, 2, 9, 22, 7) - (0, 16, 29, 5, 7); \\ (0, 13, 27, 23, 3) - (0, 11, 7, 21, 3); (0, 15, 6, 25, 4) - (0, 10, 29, 20, 4); \\ (0, 5, 11, 6, 14) - (0, 8, 3, 9, 14).$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A6 and A7. \square

Proposition 4. *There exists a cyclic $\mathcal{C}(3, 5, 35)$ system.*

Proof. We have $\frac{35 \cdot 34 \cdot 33}{6} = 35 \cdot 187$ vertex triples and 17 symmetric differences. From Theorem 4, we know that multiples of 5 and 7 have to be handled separately from the other symmetric differences.

- At the beginning, we take seven subsystems isomorphic to the cyclic $\mathcal{C}(3, 5, 5)$ system, hence with the cycles

$$(i, 7 + i, 14 + i, 21 + i, 28 + i), (i, 14 + i, 28 + i, 7 + i, 21 + i) \quad i = 0, 1, \dots, 6.$$

These cover $7 \cdot 10 = 35 \cdot 2$ vertex triples and two symmetric differences. There remain $35 \cdot 185$ triples to cover, for which we need $185/5 = 37$ base cycles.

- Now, 12 of the remaining 15 symmetric base cycles can be defined by the rule

$$(0, i, 35 - 3i, 3i, 35 - i), \quad i = 1, 2, 3, 4, 6, 8, 9, 11, 12, 13, 16, 17$$

according to Theorem 4. For the other three, i.e., the multiples of 5, the almost general rule would not work. Therefore, we make a little modification, taking the base cycles

$$(0, 5, 33, 2, 30), (0, 10, 31, 4, 25), (0, 15, 29, 6, 20).$$

- Furthermore, we need 22 reflected base cycles that can be arranged in 11 pairs. As the first half, we take

$$(0, 1, 5, 10, 7), (0, 1, 6, 9, 20), (0, 1, 8, 10, 22), (0, 1, 10, 16, 21), \\ (0, 2, 10, 22, 17), (0, 3, 15, 31, 17), (0, 4, 11, 24, 19), (0, 4, 24, 34, 10), \\ (0, 5, 14, 33, 23), (0, 6, 16, 33, 13), (0, 7, 15, 28, 18);$$

the reflected pairs of them are

$$(0, 32, 2, 6, 7), (0, 11, 14, 19, 20), (0, 12, 14, 21, 22), (0, 5, 11, 20, 21), \\ (0, 30, 7, 15, 17), (0, 21, 2, 14, 17), (0, 30, 8, 15, 19), (0, 11, 21, 6, 10), \\ (0, 25, 9, 18, 23), (0, 15, 32, 7, 13), (0, 25, 3, 11, 18).$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A10 and A11. \square

Proposition 5. *There exists a cyclic $\mathcal{C}(3, 5, 37)$ system.*

Proof. We have $\frac{37 \cdot 36 \cdot 35}{6}$ vertex triples; therefore, we need $37 \cdot 42$ cycles, which should be generated by 42 base cycles.

- The 18 symmetric base cycles can be taken as

$$(0, i, 37 - 3i, 3i, 37 - i), \quad i = 1, \dots, 18$$

as guaranteed by Theorem 4.

- For the first 12 reflected base cycles, we take

$$(0, 1, 5, 8, 27), (0, 1, 6, 3, 14), (0, 1, 7, 11, 29), (0, 1, 8, 3, 15), \\ (0, 17, 25, 22, 1), (0, 2, 9, 15, 25), (0, 2, 17, 30, 12), (0, 18, 14, 28, 2), \\ (0, 4, 13, 29, 9), (0, 5, 11, 26, 17), (0, 6, 14, 4, 17), (0, 7, 20, 34, 18).$$

- Their reflected pairs are

$$(0, 19, 22, 26, 27), (0, 11, 8, 13, 14), (0, 18, 22, 28, 29), (0, 12, 7, 14, 15), \\ (0, 16, 13, 21, 1), (0, 10, 16, 23, 25), (0, 19, 32, 10, 12), (0, 11, 25, 21, 2), \\ (0, 17, 33, 5, 9), (0, 28, 6, 12, 17), (0, 13, 3, 11, 17), (0, 21, 35, 11, 18).$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A12 and A13. \square

Proposition 6. *There exists a cyclic $\mathcal{C}(3, 5, 41)$ system.*

Proof. We have $\frac{41 \cdot 40 \cdot 39}{6}$ vertex triples; therefore, we need $41 \cdot 4 \cdot 13$ cycles, which should be generated by 52 base cycles.

- The 20 symmetric base cycles can be taken as

$$(0, i, 41 - 3i, 3i, 41 - i), \quad i = 1, \dots, 20$$

as guaranteed by Theorem 4.

- For the first 16 reflected base cycles, we take

$$(0, 1, 5, 8, 10), (0, 1, 6, 8, 18), (0, 1, 7, 11, 16), (0, 1, 8, 10, 19), \\ (0, 1, 9, 4, 17), (0, 1, 12, 7, 29), (0, 2, 14, 23, 27), (0, 2, 17, 5, 25), \\ (0, 3, 16, 33, 27), (0, 3, 30, 14, 10), (0, 6, 13, 2, 20), (0, 6, 19, 38, 21), \\ (0, 7, 15, 35, 26), (0, 8, 22, 3, 26), (0, 10, 22, 35, 25), (0, 26, 9, 30, 12).$$

- Their reflected pairs are

$$(0, 2, 5, 9, 10), (0, 10, 12, 17, 18), (0, 5, 9, 15, 16), (0, 9, 11, 18, 19), \\ (0, 13, 8, 16, 17), (0, 22, 17, 28, 29), (0, 4, 13, 25, 27), (0, 20, 8, 23, 25), \\ (0, 35, 11, 24, 27), (0, 37, 21, 7, 10), (0, 18, 7, 14, 20), (0, 24, 2, 15, 21), \\ (0, 32, 11, 19, 26), (0, 23, 4, 18, 26), (0, 31, 3, 15, 25), (0, 23, 3, 27, 12).$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A16 and A17. \square

Proposition 7. *There exists a cyclic $\mathcal{C}(3, 5, 46)$ system.*

Proof. We have $\frac{46 \cdot 45 \cdot 44}{6}$ vertex triples; therefore, we need $46 \cdot 3 \cdot 22$ cycles, which should be generated by 66 base cycles.

- The 22 symmetric base cycles can be taken as

$$(0, i, 46 - 2i, 2i, 46 - i), \quad i = 1, \dots, 22.$$

This is out of scope in Theorem 4, but one can check that these cycles do not cover any difference triplet more than once, cf. Table A18.

- For the first 22 reflected base cycles, we take

$$\begin{aligned} & (0, 1, 5, 8, 10), (0, 1, 6, 8, 17), (0, 1, 7, 11, 18), (0, 1, 8, 10, 23), \\ & (0, 1, 9, 4, 13), (0, 1, 11, 4, 19), (0, 1, 12, 4, 27), (0, 1, 14, 9, 21), \\ & (0, 2, 14, 17, 27), (0, 2, 17, 6, 23), (0, 2, 19, 5, 28), (0, 3, 14, 9, 25), \\ & (0, 3, 16, 25, 30), (0, 3, 18, 40, 21), (0, 3, 20, 35, 26), (0, 4, 22, 35, 17), \\ & (0, 5, 19, 40, 27), (0, 7, 19, 42, 26), (0, 7, 24, 33, 17), (0, 8, 20, 36, 14) \\ & (0, 26, 5, 36, 11), (0, 27, 12, 34, 13). \end{aligned}$$

- Their reflected pairs are

$$\begin{aligned} & (0, 2, 5, 9, 10), (0, 9, 11, 16, 17), (0, 7, 11, 17, 18), (0, 13, 15, 22, 23), \\ & (0, 9, 4, 12, 13), (0, 15, 8, 18, 19), (0, 23, 15, 26, 27), (0, 12, 7, 20, 21), \\ & (0, 10, 13, 25, 27), (0, 17, 6, 21, 23), (0, 23, 9, 26, 28), (0, 16, 11, 22, 25), \\ & (0, 5, 14, 27, 30), (0, 27, 3, 18, 21), (0, 37, 6, 23, 26), (0, 28, 41, 13, 17), \\ & (0, 33, 8, 22, 27), (0, 30, 7, 19, 26), (0, 30, 39, 10, 17), (0, 24, 40, 6, 14), \\ & (0, 21, 6, 31, 11), (0, 25, 1, 32, 13). \end{aligned}$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A18 and A23. \square

Proposition 8. *There exists a cyclic $\mathcal{C}(3, 5, 47)$ system.*

Proof. We have $\frac{47 \cdot 46 \cdot 45}{6}$ vertex triples; therefore, we need $47 \cdot 23 \cdot 3$ cycles, which should be generated by 69 base cycles.

- The 23 symmetric base cycles can be taken as

$$(0, i, 47 - 3i, 3i, 47 - i), \quad i = 1, \dots, 23$$

as guaranteed by Theorem 4.

- For the first 23 reflected base cycles, we take

$$\begin{aligned} & (0, 1, 5, 8, 10), (0, 1, 6, 8, 18), (0, 1, 7, 11, 34), (0, 1, 8, 10, 21), \\ & (0, 1, 9, 4, 15), (0, 1, 11, 8, 35), (0, 1, 19, 3, 26), (0, 2, 11, 6, 22), \\ & (0, 2, 14, 9, 26), (0, 2, 16, 6, 19), (0, 2, 20, 17, 28), (0, 3, 15, 22, 30), \\ & (0, 3, 16, 9, 31), (0, 3, 17, 35, 21), (0, 23, 32, 40, 4), (0, 6, 17, 32, 26), \\ & (0, 6, 20, 36, 16), (0, 23, 30, 39, 6), (0, 8, 26, 13, 22), (0, 25, 5, 33, 10), \\ & (0, 23, 5, 35, 10), (0, 12, 25, 43, 21), (0, 28, 13, 32, 12). \end{aligned}$$

- Their reflected pairs are

$$\begin{aligned} & (0, 2, 5, 9, 10), (0, 10, 12, 17, 18), (0, 23, 27, 33, 34), (0, 11, 13, 20, 21), \\ & (0, 11, 6, 14, 15), (0, 27, 24, 34, 35), (0, 23, 7, 25, 26), (0, 16, 11, 20, 22), \\ & (0, 17, 12, 24, 26), (0, 13, 3, 17, 19), (0, 11, 8, 26, 28), (0, 8, 15, 27, 30), \\ & (0, 22, 15, 28, 31), (0, 33, 4, 18, 21), (0, 11, 19, 28, 4), (0, 41, 9, 20, 26), \\ & (0, 27, 43, 10, 16), (0, 14, 23, 30, 6), (0, 9, 43, 14, 22), (0, 24, 5, 32, 10), \\ & (0, 22, 5, 34, 10), (0, 25, 43, 9, 21), (0, 27, 46, 31, 12). \end{aligned}$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A20 and A21. \square

Proposition 9. *There exists a cyclic $\mathcal{C}(3, 5, 55)$ system.*

Proof. We have $\frac{55 \cdot 54 \cdot 53}{6} = 55 \cdot 477$ vertex triples, for which we need to define $11 \cdot 9 \cdot 53$ cycles. The number of symmetric differences is $\frac{55-1}{2} = 27$.

- We multiply the two base cycles of $\mathcal{C}(3, 5, 5)$ by 11 and take their 11 distinct positions:

$$(i, 11 + i, 22 + i, 33 + i, 44 + i), \quad (i, 22 + i, 44 + i, 11 + i, 33 + i), \quad i = 0, 1, \dots, 10.$$

These cycles cover $110 = 55 \cdot 2$ vertex triples, including all triples belonging to two symmetric differences. There remain $55 \cdot 475$ triples to be covered and 25 symmetric differences.

- The general principles cannot be applied to cover symmetric differences divisible by 5, which here means 5, 10, 15, 20, 25. We cover those with five specially designed base cycles, most of them of the form $(0, 5i, 55 - 2i, 2i, 55 - 5i)$, as follows:

$$(0, 5, 53, 2, 50), \quad (0, 10, 51, 4, 45), \quad (0, 15, 49, 6, 40), \quad (0, 20, 47, 8, 35), \quad (0, 25, 43, 12, 30).$$

These cycles cover $5 \cdot 25$ vertex triples, five of them are symmetric. There remain $55 \cdot 450$ triples to be covered, for which we define 90 base cycles.

(An alternative approach is to multiply the $\mathcal{C}(3, 5, 11)$ system by 5, hence to take the three base cycles $(0, 5, 30, 45, 10)$, $(0, 5, 20, 10, 35)$, and $(0, 20, 45, 35, 50)$; but then, the continuation would of course be different.)

- The remaining 20 symmetric base cycles can be taken as

$$(0, i, 55 - 3i, 3i, 55 - i), \quad i \in \{1, \dots, 27\} \setminus (\{11, 22\} \cup \{5, 10, 15, 20, 25\})$$

as guaranteed by Theorem 4. There remain 70 base cycles to be defined.

- The first half of 70 reflected base cycles is

$$\begin{aligned} &(0, 1, 5, 10, 7), \quad (0, 1, 6, 9, 17), \quad (0, 1, 8, 10, 21), \quad (0, 1, 9, 11, 20), \\ &(0, 1, 10, 14, 25), \quad (0, 1, 11, 5, 16), \quad (0, 1, 12, 5, 22), \quad (0, 1, 13, 3, 30), \\ &(0, 1, 15, 3, 23), \quad (0, 2, 15, 5, 20), \quad (0, 2, 16, 5, 23), \quad (0, 2, 17, 5, 27), \\ &(0, 2, 18, 8, 26), \quad (0, 2, 22, 6, 25), \quad (0, 3, 17, 8, 27), \quad (0, 3, 18, 9, 33), \\ &(0, 3, 20, 14, 34), \quad (0, 3, 21, 7, 36), \quad (0, 4, 19, 11, 24), \quad (0, 4, 22, 9, 29), \\ &(0, 4, 23, 10, 32), \quad (0, 4, 25, 48, 33), \quad (0, 5, 12, 18, 34), \quad (0, 5, 21, 49, 25), \\ &(0, 9, 34, 45, 26), \quad (0, 10, 24, 53, 31), \quad (0, 10, 35, 48, 25), \quad (0, 11, 37, 6, 26), \\ &(0, 12, 27, 43, 17), \quad (0, 12, 34, 6, 23), \quad (0, 12, 35, 7, 25), \quad (0, 13, 28, 50, 36), \\ &\quad (0, 16, 41, 8, 25), \quad (0, 17, 37, 1, 32), \quad (0, 19, 40, 13, 33). \end{aligned}$$

- Their reflected pairs are

$$\begin{aligned} &(0, 52, 2, 6, 7), \quad (0, 8, 11, 16, 17), \quad (0, 11, 13, 20, 21), \quad (0, 9, 11, 19, 20), \\ &(0, 11, 15, 24, 25), \quad (0, 11, 5, 15, 16), \quad (0, 17, 10, 21, 22), \quad (0, 27, 17, 29, 30), \\ &(0, 20, 8, 22, 23), \quad (0, 15, 5, 18, 20), \quad (0, 18, 7, 21, 23), \quad (0, 22, 10, 25, 27), \\ &(0, 18, 8, 24, 26), \quad (0, 19, 3, 23, 25), \quad (0, 19, 10, 24, 27), \quad (0, 24, 15, 30, 33), \\ &(0, 20, 14, 31, 34), \quad (0, 29, 15, 33, 36), \quad (0, 13, 5, 20, 24), \quad (0, 20, 7, 25, 29), \\ &(0, 22, 9, 28, 32), \quad (0, 40, 8, 29, 33), \quad (0, 16, 22, 29, 34), \quad (0, 31, 4, 20, 25), \\ &(0, 36, 47, 17, 26), \quad (0, 33, 7, 21, 31), \quad (0, 32, 45, 15, 25), \quad (0, 20, 44, 15, 26), \\ &(0, 29, 45, 5, 17), \quad (0, 17, 44, 11, 23), \quad (0, 18, 45, 13, 25), \quad (0, 41, 8, 23, 36), \\ &\quad (0, 17, 39, 9, 25), \quad (0, 31, 50, 15, 32), \quad (0, 20, 48, 14, 33). \end{aligned}$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A26 and A27. \square

Proposition 10. *There exists a cyclic $\mathcal{C}(3, 5, 56)$ system.*

Proof. We have $\frac{56 \cdot 55 \cdot 54}{6} = 56 \cdot 55 \cdot 9$ vertex triples, for which we need to define $11 \cdot 9 = 99$ base cycles. The number of symmetric differences is 27. For those 27, we can start with a general formula $(0, i, 56 - 2i, 2i, 56 - i)$ as an approach similar to the one in the case of $v = 46$. However, in the current situation, several adjustments are needed to resolve collisions.

- For odd symmetric differences, we only need to make one little change, for the difference $i = 7$. Therefore, the 14 odd symmetric base cycles are

$$\begin{aligned} & (0, 1, 54, 2, 55), \quad (0, 3, 50, 6, 53), \quad (0, 5, 46, 10, 51), \quad (0, 7, 44, 12, 49), \\ & (0, 9, 38, 18, 47), \quad (0, 11, 34, 22, 45), \quad (0, 13, 30, 26, 43), \quad (0, 15, 26, 30, 41), \\ & (0, 17, 22, 34, 39), \quad (0, 19, 18, 38, 37), \quad (0, 21, 14, 42, 35), \quad (0, 23, 10, 46, 33), \\ & (0, 25, 6, 50, 31), \quad (0, 27, 2, 54, 29). \end{aligned}$$

- The even case is less favorable. Beyond the expected values 8, 16, and 24—which are multiples of $v/7$ —we also need to modify the base cycle for $i = 14$. Hence, we take the following 13 base cycles for the even differences:

$$\begin{aligned} & (0, 2, 52, 4, 54), \quad (0, 4, 48, 8, 52), \quad (0, 6, 44, 12, 50), \quad (0, 8, 42, 14, 48), \\ & (0, 10, 36, 20, 46), \quad (0, 12, 32, 24, 44), \quad (0, 14, 32, 24, 42), \quad (0, 16, 30, 26, 40), \\ & (0, 18, 20, 36, 38), \quad (0, 20, 16, 40, 36), \quad (0, 22, 12, 44, 34), \quad (0, 24, 10, 46, 32), \\ & (0, 26, 4, 52, 30). \end{aligned}$$

- There remain $99 - 27 = 72$ base cycles to be defined. We arrange them in 36 reflected pairs. The first half is

$$\begin{aligned} & (0, 1, 5, 8, 10), \quad (0, 1, 6, 8, 15), \quad (0, 1, 7, 11, 16), \quad (0, 1, 8, 11, 21), \\ & (0, 1, 9, 4, 18), \quad (0, 1, 11, 13, 22), \quad (0, 1, 12, 4, 23), \quad (0, 1, 13, 6, 17), \\ & (0, 1, 14, 3, 27), \quad (0, 1, 24, 3, 28), \quad (0, 1, 25, 8, 31), \quad (0, 2, 14, 17, 28), \\ & (0, 2, 15, 6, 21), \quad (0, 2, 16, 8, 25), \quad (0, 2, 17, 20, 39), \quad (0, 2, 22, 5, 26), \\ & (0, 3, 16, 8, 29), \quad (0, 3, 17, 22, 35), \quad (0, 3, 19, 9, 26), \quad (0, 3, 21, 43, 25), \\ & (0, 4, 19, 26, 32), \quad (0, 5, 11, 24, 31), \quad (0, 5, 21, 12, 33), \quad (0, 6, 22, 49, 27), \\ & (0, 7, 36, 48, 18), \quad (0, 9, 33, 50, 23), \quad (0, 10, 31, 48, 19), \quad (0, 11, 35, 48, 20), \\ & (0, 11, 36, 51, 27), \quad (0, 12, 28, 5, 25), \quad (0, 14, 29, 50, 20), \quad (0, 14, 37, 7, 26), \\ & (0, 14, 43, 10, 25), \quad (0, 15, 34, 5, 23), \quad (0, 16, 33, 2, 24), \quad (0, 16, 35, 7, 25). \end{aligned}$$

- Their reflected pairs are

$$\begin{aligned} & (0, 2, 5, 9, 10), \quad (0, 7, 9, 14, 15), \quad (0, 5, 9, 15, 16), \quad (0, 10, 13, 20, 21), \\ & (0, 14, 9, 17, 18), \quad (0, 9, 11, 21, 22), \quad (0, 19, 11, 22, 23), \quad (0, 11, 4, 16, 17), \\ & (0, 24, 13, 26, 27), \quad (0, 25, 4, 27, 28), \quad (0, 23, 6, 30, 31), \quad (0, 11, 14, 26, 28), \\ & (0, 15, 6, 19, 21), \quad (0, 17, 9, 23, 25), \quad (0, 19, 22, 37, 39), \quad (0, 21, 4, 24, 26), \\ & (0, 21, 13, 26, 29), \quad (0, 13, 18, 32, 35), \quad (0, 17, 7, 23, 26), \quad (0, 38, 4, 22, 25), \\ & (0, 6, 13, 28, 32), \quad (0, 7, 20, 26, 31), \quad (0, 21, 12, 28, 33), \quad (0, 34, 5, 21, 27), \\ & (0, 26, 38, 11, 18), \quad (0, 29, 46, 14, 23), \quad (0, 27, 44, 9, 19), \quad (0, 28, 41, 9, 20), \\ & (0, 32, 47, 16, 27), \quad (0, 20, 53, 13, 25), \quad (0, 26, 47, 6, 20), \quad (0, 19, 45, 12, 26), \\ & (0, 15, 38, 11, 25), \quad (0, 18, 45, 8, 23), \quad (0, 22, 47, 8, 24), \quad (0, 18, 46, 9, 25). \end{aligned}$$

These base cycles generate a system with the required number of 5-cycles and cover each vertex triple exactly once, as it can be verified from the data in Tables A28 and A29. \square

5.2. Cyclic 2-Split Systems of Orders 32, 40, 50, and 52

In this subsection, we present constructions that are not only cyclic but also 2-split. The cases of $v = 32$, $v = 40$, $v = 50$, and $v = 52$ are considered. For $v = 40$ and $v = 52$, no constructions have been known so far. Although the existence of a cyclic $\mathcal{C}(3, 5, 32)$ and a 2-split $\mathcal{C}(3, 5, 50)$ was known, the former was not 2-split and the latter was not cyclic..

For a 2-split system, it is convenient to write $2v$ for its order and to assume that it is built on the vertex-disjoint union of two subsystems of order v each. Throughout this subsection, we assume that the vertex sets of those subsystems are

$$X_1 = \{0, 2, \dots, 2v - 2\}, \quad X_2 = \{1, 3, \dots, 2v - 1\}.$$

Then, one can start with the base blocks of a cyclic $\mathcal{C}(3, 5, v)$ and multiply all its elements by 2, hence obtaining a cyclic subsystem induced by X_1 . Moreover, rotation by 1 yields an induced cyclic subsystem on X_2 .

Proposition 11. *There exists a cyclic 2-split $\mathcal{C}(3, 5, 32)$ system.*

Proof. We have $\frac{32 \cdot 31 \cdot 30}{6}$ vertex triples; therefore, we need $32 \cdot 31$ cycles that should be generated by 31 base cycles.

- The seven base cycles obtained from a cyclic $\mathcal{C}(3, 5, 16)$ are

$$(0, 2, 18, 14, 30), (0, 28, 26, 6, 4), (0, 6, 20, 12, 26), (0, 24, 30, 2, 8), \\ (0, 22, 30, 2, 10), (0, 20, 2, 30, 12), (0, 18, 8, 24, 14).$$

All these cycles are symmetric. They are obtained from the system given in [10] by rotating the base cycles in a way that 0 is the center of each symmetric triangle.

- The other eight symmetric base cycles are

$$(0, 1, 30, 2, 31), (0, 3, 26, 6, 29), (0, 5, 22, 10, 27), (0, 7, 18, 14, 25), \\ (0, 9, 14, 18, 23), (0, 11, 10, 22, 21), (0, 13, 6, 26, 19), (0, 15, 2, 30, 17).$$

- We take the following eight base cycles as the first half of reflected crossing cycles:

$$(0, 1, 5, 8, 13), (0, 1, 6, 3, 14), (0, 1, 8, 7, 16), (0, 1, 10, 29, 15), \\ (0, 3, 10, 30, 19), (0, 3, 13, 22, 15), (0, 5, 16, 24, 7), (0, 6, 17, 2, 11).$$

- Their reflected base cycles are

$$(0, 5, 8, 12, 13), (0, 11, 8, 13, 14), (0, 9, 8, 15, 16), (0, 18, 5, 14, 15), \\ (0, 21, 9, 16, 19), (0, 25, 2, 12, 15), (0, 15, 23, 2, 7), (0, 9, 26, 5, 11).$$

Figure 5 exhibits one base cycle derived from $\mathcal{C}(3, 5, 16)$ in red and one crossing base cycle in blue.

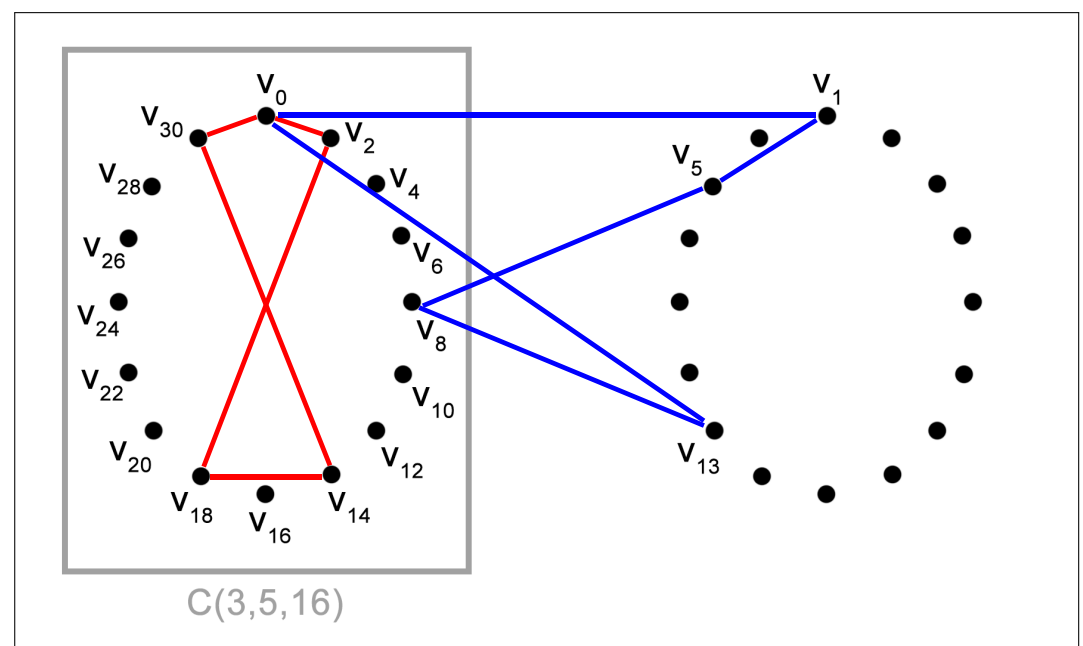


Figure 5. $\mathcal{C}(3, 5, 32)$: base cycles $(0, 2, 18, 14, 30)$ and $(0, 1, 5, 8, 13)$.

The set of base cycles defined above generates a system with the required number of 5-cycles and covers each vertex triple exactly once, as it can be verified from the data in Tables A8 and A9. \square

Proposition 12. *There exists a cyclic 2-split $\mathcal{C}(3, 5, 40)$ system.*

Proof. We have $\frac{40 \cdot 39 \cdot 38}{6}$ vertex triples, from which the two $\mathcal{C}(3, 5, 20)$ subsystems together cover $\frac{40 \cdot 19 \cdot 18}{6}$ triples. Hence, we need to cover the remaining $\frac{40 \cdot 30 \cdot 38}{6}$ triples, which therefore need $40 \cdot 38$ cycles, requiring 38 additional base cycles. For the sake of completeness, we also list the base cycles that generate the $\mathcal{C}(3, 5, 20)$ subsystem and present them in a way that reflects the iterated doubling $5 \rightarrow 10 \rightarrow 20 \rightarrow 40$.

- We take $\frac{1}{5} \binom{5}{3} = 2$ base cycles, with the cyclic $\mathcal{C}(3, 5, 5)$ multiplied by 8:

$$(0, 8, 16, 24, 32), \quad (0, 16, 32, 8, 24);$$

- Furthermore, we take $\frac{1}{5} \left(\binom{10}{3} - 2 \cdot \binom{5}{3} \right) \cdot \frac{1}{10} = 2$ base cycles of a cyclic $\mathcal{C}(3, 5, 10)$ from [2], multiplied by 4:

$$(0, 4, 8, 20, 24), \quad (0, 8, 20, 16, 28);$$

- We also take $\frac{1}{5} \left(\binom{20}{3} - 2 \cdot \binom{10}{3} \right) \cdot \frac{1}{20} = 9$ base cycles of a cyclic $\mathcal{C}(3, 5, 20)$ —five symmetric cycles and two reflected pairs—from [10], multiplied by 2:

$$\begin{aligned} (0, 2, 36, 4, 38), \quad (0, 6, 24, 16, 34), \quad (0, 6, 14, 20, 10), \quad (0, 2, 16, 12, 26), \\ (0, 34, 10, 4, 22), \quad (0, 2, 12, 28, 10), \quad (0, 30, 14, 32, 2), \quad (0, 18, 32, 12, 14), \\ (0, 20, 18, 32, 14). \end{aligned}$$

- The other ten symmetric base cycles are

$$\begin{aligned} (0, 1, 6, 34, 39), \quad (0, 3, 18, 22, 37), \quad (0, 5, 2, 38, 35), \quad (0, 7, 2, 38, 33), \\ (0, 9, 14, 26, 31), \quad (0, 11, 26, 14, 29), \quad (0, 13, 38, 2, 27), \quad (0, 15, 10, 30, 25), \\ (0, 17, 22, 18, 23), \quad (0, 19, 34, 6, 21). \end{aligned}$$

Note that a little adjustment was necessary for the cycle containing the symmetric difference $(5, 5, 10)$ —the cycle $(0, 5, 2, 38, 35)$ —which does not follow the general arithmetic pattern of the others.

- We take the following 14 base cycles as the first half of reflected crossing cycles:

$$\begin{aligned} (0, 1, 3, 10, 9), \quad (0, 1, 4, 9, 15), \quad (0, 1, 8, 11, 17), \quad (0, 1, 11, 14, 23), \\ (0, 1, 12, 3, 16), \quad (0, 1, 13, 24, 21), \quad (0, 1, 14, 3, 22), \quad (0, 3, 14, 7, 26), \\ (0, 5, 16, 36, 19), \quad (0, 5, 21, 36, 13), \quad (0, 7, 16, 35, 22), \quad (0, 7, 20, 31, 13), \\ (0, 7, 24, 9, 17), \quad (0, 10, 29, 2, 19). \end{aligned}$$

- Their reflected base cycles are

$$\begin{aligned} (0, 39, 6, 8, 9), \quad (0, 6, 11, 14, 15), \quad (0, 6, 9, 16, 17), \quad (0, 9, 12, 22, 23), \\ (0, 13, 4, 15, 16), \quad (0, 37, 8, 20, 21), \quad (0, 19, 8, 21, 22), \quad (0, 19, 12, 23, 26), \\ (0, 23, 3, 14, 19), \quad (0, 17, 32, 8, 13), \quad (0, 27, 6, 15, 22), \quad (0, 22, 33, 6, 13), \\ (0, 8, 33, 10, 17), \quad (0, 17, 30, 9, 19). \end{aligned}$$

Hence, the number of base cycles supplementing the two copies of $\mathcal{C}(3, 5, 20)$ is $10 + 14 + 14 = 38$ as needed. This set of base cycles generates a system with the required number of 5-cycles and covers each vertex triple exactly once, as it can be verified from the data in Tables A14 and A15. \square

Proposition 13. *There exists a cyclic 2-split $\mathcal{C}(3, 5, 50)$ system.*

Proof. We have $\frac{50 \cdot 49 \cdot 48}{6} = 50 \cdot 392$ vertex triples, including 24 symmetric differences.

- We take the 20 base cycles of $\mathcal{C}(3, 5, 25)$, multiplied by 2; for the sake of completeness, we also list them here:

$$\begin{aligned} & (0, 2, 44, 6, 48), \quad (0, 4, 38, 12, 46), \quad (0, 6, 32, 18, 44), \quad (0, 8, 26, 24, 42), \\ & (0, 10, 20, 30, 40), \quad (0, 12, 14, 36, 38), \quad (0, 14, 8, 42, 36), \quad (0, 16, 2, 48, 34), \\ & (0, 18, 46, 4, 32), \quad (0, 20, 40, 10, 30), \quad (0, 22, 34, 16, 28), \quad (0, 24, 28, 22, 26), \\ & (0, 2, 10, 22, 20), \quad (0, 4, 10, 24, 20), \quad (0, 6, 26, 40, 16), \quad (0, 8, 28, 40, 18), \\ & (0, 48, 10, 18, 20), \quad (0, 46, 10, 16, 20), \quad (0, 26, 40, 10, 16), \quad (0, 28, 40, 10, 18). \end{aligned}$$

These cover $2 \cdot \binom{25}{3} = 50 \cdot 92$ vertex triples. Hence, there remain $50 \cdot 300$ vertex triples to be covered, which need $50 \cdot 60$ cycles, that means 60 base cycles. The number of symmetric differences not covered so far is $24 - 12 = 12$.

- To define the 12 base cycles for odd symmetric differences, the “2i-rule” is almost perfect, with just one little adjustment for $i = 5$. Namely, we take

$$(0, i, 50 - 2i, 2i, 50 - i), \quad i \in \{1, 3\} \cup \{7, 9, \dots, 23\}$$

and $(0, 5, 38, 12, 45)$ for $i = 5$.

- For the remaining $60 - 12 = 48$ base cycles, we define 24 reflected pairs. For the first half, we take

$$\begin{aligned} & (0, 1, 5, 8, 13), \quad (0, 1, 6, 3, 14), \quad (0, 1, 7, 12, 21), \quad (0, 1, 8, 3, 15), \\ & (0, 1, 9, 16, 19), \quad (0, 1, 10, 3, 16), \quad (0, 1, 11, 14, 29), \quad (0, 1, 12, 3, 20), \\ & (0, 1, 23, 4, 25), \quad (0, 2, 15, 6, 25), \quad (0, 2, 17, 6, 29), \quad (0, 3, 17, 34, 27), \\ & (0, 3, 21, 8, 25), \quad (0, 3, 23, 10, 29), \quad (0, 4, 33, 16, 11), \quad (0, 5, 21, 40, 29), \\ & (0, 5, 23, 34, 9), \quad (0, 5, 31, 4, 19), \quad (0, 6, 35, 26, 15), \quad (0, 9, 22, 39, 16), \\ & (0, 9, 24, 2, 33), \quad (0, 11, 30, 43, 18), \quad (0, 13, 36, 8, 23), \quad (0, 15, 32, 5, 25). \end{aligned}$$

- The reflected pairs of these cycles are

$$\begin{aligned} & (0, 5, 8, 12, 13), \quad (0, 11, 8, 13, 14), \quad (0, 9, 14, 20, 21), \quad (0, 12, 7, 14, 15), \\ & (0, 3, 10, 18, 19), \quad (0, 13, 6, 15, 16), \quad (0, 15, 18, 28, 29), \quad (0, 17, 8, 19, 20), \\ & (0, 21, 2, 24, 25), \quad (0, 19, 10, 23, 25), \quad (0, 23, 12, 27, 29), \quad (0, 43, 10, 24, 27), \\ & (0, 17, 4, 22, 25), \quad (0, 19, 6, 26, 29), \quad (0, 45, 28, 7, 11), \quad (0, 39, 8, 24, 29), \\ & (0, 25, 36, 4, 9), \quad (0, 15, 38, 14, 19), \quad (0, 39, 30, 9, 15), \quad (0, 27, 44, 7, 16), \\ & (0, 31, 9, 24, 33), \quad (0, 25, 38, 7, 18), \quad (0, 15, 37, 10, 23), \quad (0, 20, 43, 10, 25). \end{aligned}$$

This set of base cycles generates a system with the required number of 5-cycles and covers each vertex triple exactly once, as it can be verified from the data in Tables A22 and A23. \square

Proposition 14. *There exists a cyclic 2-split $\mathcal{C}(3, 5, 52)$ system.*

Proof. We have $\frac{52 \cdot 51 \cdot 50}{6}$ vertex triples; therefore, we need $52 \cdot 17 \cdot 5$ cycles, which should be generated by 85 base cycles.

- The 20 base cycles of a cyclic $\mathcal{C}(3, 5, 26)$ system are multiplied by 2:

$$\begin{aligned} & (0, 2, 48, 4, 50), \quad (0, 6, 40, 12, 46), \quad (0, 10, 32, 20, 42), \quad (0, 14, 24, 28, 38), \\ & (0, 18, 16, 36, 34), \quad (0, 22, 8, 44, 30), \quad (0, 4, 44, 8, 48), \quad (0, 8, 36, 16, 44), \\ & (0, 12, 44, 8, 40), \quad (0, 16, 42, 10, 36), \quad (0, 20, 50, 2, 32), \quad (0, 24, 46, 6, 28), \\ & (0, 2, 16, 34, 26), \quad (0, 2, 24, 50, 12), \quad (0, 24, 40, 46, 4), \quad (0, 6, 14, 4, 38), \\ & (0, 44, 10, 24, 26), \quad (0, 14, 40, 10, 12), \quad (0, 10, 16, 32, 4), \quad (0, 34, 24, 32, 38). \end{aligned}$$

These include 12 symmetric cycles; the other eight cycles are in four reflected pairs.

- The further 13 symmetric base cycles are

$$\begin{aligned}
 &(0, 1, 50, 2, 51), \quad (0, 3, 46, 6, 49), \quad (0, 5, 42, 10, 47), \quad (0, 7, 38, 14, 45), \\
 &(0, 9, 34, 18, 43), \quad (0, 11, 30, 22, 41), \quad (0, 13, 28, 24, 39), \quad (0, 15, 22, 30, 37), \\
 &(0, 17, 18, 34, 35), \quad (0, 19, 14, 38, 33), \quad (0, 21, 10, 42, 31), \quad (0, 23, 6, 46, 29), \\
 &\quad (0, 25, 2, 50, 27).
 \end{aligned}$$

As we noted above, the even symmetric cycles of $\mathcal{C}(3, 5, 26)$ were irregular at several points although the odd cycles had a regular behavior. A little adjustment also occurs here, in the cycle containing the symmetric difference $(13, 13, 26)$ —the cycle $(0, 13, 28, 24, 39)$ —which does not follow the general arithmetic pattern of the others.

- We take the following 26 base cycles as the first half of reflected crossing cycles:

$$\begin{aligned}
 &(0, 1, 5, 8, 13), \quad (0, 1, 6, 3, 14), \quad (0, 1, 7, 12, 19), \quad (0, 1, 8, 3, 20), \\
 &(0, 1, 9, 18, 11), \quad (0, 1, 10, 5, 21), \quad (0, 1, 12, 3, 16), \quad (0, 1, 15, 6, 25), \\
 &(0, 1, 22, 3, 26), \quad (0, 1, 23, 10, 29), \quad (0, 2, 13, 6, 31), \quad (0, 2, 15, 8, 35), \\
 &(0, 3, 10, 19, 31), \quad (0, 3, 13, 20, 33), \quad (0, 3, 15, 30, 21), \quad (0, 3, 18, 38, 27), \\
 &(0, 25, 12, 36, 3), \quad (0, 23, 12, 38, 3), \quad (0, 4, 21, 46, 31), \quad (0, 5, 18, 41, 25), \\
 &(0, 5, 27, 10, 23), \quad (0, 7, 30, 41, 24), \quad (0, 9, 32, 47, 24), \quad (0, 13, 30, 44, 21), \\
 &\quad (0, 13, 34, 8, 27), \quad (0, 15, 31, 48, 33).
 \end{aligned}$$

- Their reflected base cycle pairs are

$$\begin{aligned}
 &(0, 5, 8, 12, 13), \quad (0, 11, 8, 13, 14), \quad (0, 7, 12, 18, 19), \quad (0, 17, 12, 19, 20), \\
 &(0, 45, 2, 10, 11), \quad (0, 16, 11, 20, 21), \quad (0, 13, 4, 15, 16), \quad (0, 19, 10, 24, 25), \\
 &(0, 23, 4, 25, 26), \quad (0, 19, 6, 28, 29), \quad (0, 25, 18, 29, 31), \quad (0, 27, 20, 33, 35), \\
 &(0, 12, 21, 28, 31), \quad (0, 13, 20, 30, 33), \quad (0, 43, 6, 18, 21), \quad (0, 41, 9, 24, 27), \\
 &(0, 19, 43, 30, 3), \quad (0, 17, 43, 32, 3), \quad (0, 37, 10, 27, 31), \quad (0, 36, 7, 20, 25), \\
 &(0, 13, 48, 18, 23), \quad (0, 35, 46, 17, 24), \quad (0, 29, 44, 15, 24), \quad (0, 29, 43, 8, 21), \\
 &\quad (0, 19, 45, 14, 27), \quad (0, 37, 2, 18, 33).
 \end{aligned}$$

This set of base cycles generates a system with the required number of 5-cycles and covers each vertex triple exactly once, as it can be verified from the data in Tables A24 and A25. \square

6. Conclusions

In this paper, we further developed the theory of decompositions of complete 3-uniform hypergraphs into 5-cycles. We introduced new recursive rules to generate larger systems from known smaller ones and designed many initial constructions from which new systems can be built.

As formulated in Conjecture 1, we expect that, for all feasible orders except $v = 7$, there exists a cyclic 3-uniform 5-cycle system. Analogously, we expect that, for all feasible orders, there exists a more structured 3-uniform 5-cycle system, namely a cyclic 2-split system, with no exceptions at all.

With regards to our conditional recursion $v \rightarrow 8v$ of Section 3, it is an interesting new problem to analyze for which values of v does the hypergraph $\mathcal{H}_{\langle 4 \times v \rangle}$ admit an edge decomposition into 5-cycles. Perhaps the corresponding divisibility conditions will be sufficient for this type of systems too.

We presented cyclic hypercycle systems $\mathcal{C}(3, 5, v)$ of orders $v = 25, 26, 31, 35, 37, 41, 46, 47, 55$, and 56 ; a highly symmetric construction for $v = 40$; and cyclic 2-split constructions of orders $32, 40, 50$, and 52 . These systems were determined using computer-aided methods. As a consequence, we obtain that all orders $v \leq 60$ permitted by the divisibility conditions admit a $\mathcal{C}(3, 5, v)$ system.

The recursive constructions of Section 3 establish the existence of 2-split $\mathcal{C}(3, 5, v)$ systems for many previously unsolved cases. The consequences of the $v \rightarrow 10v$ step result in an infinite set of new solutions, including $v = 70, 160, 200, 260, 310, 320, 350, 370, 460, 470, 520, 550, 560, 700, 800, 820, 1000, \dots$. Moreover, some previously solved cases, e.g., $v = 100, 110, 400, 410$, and 500 , are now proven to admit 2-split systems, which are suitable for serving as initial structures for further recursive operations of [10].

A useful tool in these constructions is the auxiliary system over $\mathcal{H}_{(4 \times 5)}$, found in Section 2, for which we expect further applications in the future, too. One nice structure derived from it is the $\mathcal{C}(3, 5, 40)$ system of order 40 that benefits from lots of symmetries and that is built over a Steiner quadruple system using as few as seven generating cycles.

Combinatorial designs have a huge number of applications in various fields, e.g., in statistical experimental design, coding theory, optical networks, and even in music composition. Quoting from the historical introduction in the *Handbook of Combinatorics* ([17], p. 11), every passing year sees thousands of new published papers employing the results and techniques of combinatorial design theory. Concerning the subject of the current paper, it would be very interesting to see that hypercycle systems can also be applied to improve the efficiency of methods in the abovementioned fields. In this direction, one of our referees proposed that the constructed systems could for example be implemented in turbo coding. The connection between block design and block codes is well known, with several notable results, for example, Steiner systems and Golay codes [18], the application of finite geometries to LDPC codes (low-density parity-check codes) [19], or a more recently published link between Steiner systems and rank metric codes [20]. It is worth mentioning that a combination of turbo coding and balanced incomplete block design was also proposed in [21] for a digital fingerprinting system. As we have shown in this paper, hypercycle and Steiner systems are closely related designs; hence, this may turn out to be an exciting track of research in the future for us. We aim to extend our work and to investigate the cases of different hypercycle lengths to make the constructions more suitable for application.

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Appendix A. Auxiliary Tables

The number of triplet types is quadratic when it is expressed as a function of the number of vertices, and it gets harder and harder to check whether all triplets are included in the constructed cycles as v grows. The aim of including the supplementary material presented in this Appendix is to facilitate validation of the cycle systems for which the cycles were only listed in the text of the paper.

We present additional data in two types of tables regarding cyclic constructions, calculating the triplet types. The process of calculating the triplet types for a cycle is detailed in Table A1, taking the base cycle $(0, 1, 24, 2, 25)$ as an example from Table A5a. For each vertex triple covered by the cycle in question, the distance between any two vertices is calculated according to the following formula:

$$dist_{1,2} = \begin{cases} |x_1 - x_2|, & \text{if } |x_1 - x_2| \leq \lfloor v/2 \rfloor, \\ v - |x_1 - x_2|, & \text{otherwise.} \end{cases}$$

After that, the vertex pair with the smallest distance is considered for reference. The vertex triple is then rearranged (if necessary) in the way that this reference pair is put in the smaller arc of the cyclic order $0, 1, \dots, v-1, 0$ and the third vertex of the vertex triple is put last. (In about $2/3$ of symmetric difference triplets, the smaller distance occurs twice. Then, the vertex at equal distance from the other two is viewed as the middle vertex.) Finally, the triple is rotated so that the first vertex is labelled 1. The ordered 3-tuple obtained in this way is the triplet type associated with the vertex triple.

A set of base cycles generates a $\mathcal{C}(3, 5, v)$ properly if and only if all triplet types occur precisely once.

The tables are arranged in increasing order of v . For each v , the table of first kind (split into subtables if v is not small) lists the base cycles and calculates the three distances for each of the five edges in every cycle. After that, the tables of second kind for the same v compute the triplet types determined by the edges of each cycle. For symmetric base cycles, the symmetric differences and half of the reflected differences are emphasized with colored background. In the case of reflected base cycles, the five difference triplets in one representative cycle and its reflected pair are highlighted with a colored background.

Table A1. Example base cycle (0, 1, 24, 2, 25) from the $v = 26$ case (detailed in Table A5a).

Ind. (mod5)	abs(Distance)			Real Distance			d	First 2	Direction	Triple	Type
(x_i, x_{i+1}, x_{i+2})	$x_i - x_{i+1}$	$x_{i+1} - x_{i+2}$	$x_{i+2} - x_i$	$x_i - x_{i+1}$	$x_{i+1} - x_{i+2}$	$x_{i+2} - x_i$					
(0,1,24)	1	23	24	1	3	2	1	0 1	right	0 1 24	1 2 25
(1,24,2)	23	22	1	3	4	1	1	2 1	left	1 2 24	1 2 24
(24,2,25)	22	23	1	4	3	1	1	25 24	left	24 25 2	1 2 5
(2,25,0)	23	25	2	3	1	2	1	25 0	right	25 0 2	1 2 4
(25,0,1)	25	1	24	1	1	2	1	25 0	right	25 0 1	1 2 3

Table A2. Cyclic $\mathcal{C}(3, 5, 25)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of the reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example. With red, we highlighted the special base cycles that only have $v/5$ different positions, as $i \mapsto i + v/5$ results in an automorphism of these cycles.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles																			
0	1	22	3	24	1	4	3	4	6	2	6	4	2	4	1	3	1	1	2
0	2	19	6	23	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	3	16	9	22	3	12	9	12	7	6	7	12	6	12	3	9	3	3	6
0	4	13	12	21	4	9	12	9	1	8	1	9	8	9	4	12	4	4	8
0	5	10	15	20	5	5	10	5	5	10	5	5	10	5	5	10	5	5	10
0	6	7	18	19	6	1	7	1	11	12	11	1	12	1	6	7	6	6	12
0	7	4	21	18	7	3	4	3	8	11	8	3	11	3	7	4	7	7	11
0	8	1	24	17	8	7	1	7	2	9	2	7	9	7	8	1	8	8	9
0	9	23	2	16	9	11	2	11	4	7	4	11	7	11	9	2	9	9	7
0	10	20	5	15	10	10	5	10	10	5	10	10	5	10	10	5	10	10	5
0	11	17	8	14	11	6	8	6	9	3	9	6	3	6	11	8	11	11	3
0	12	14	11	13	12	2	11	2	3	1	3	2	1	2	12	11	12	12	1
Reflected cycles 1st half																			
0	1	5	11	10	1	4	5	4	6	10	6	1	5	1	10	11	10	1	9
0	2	5	12	10	2	3	5	3	7	10	7	2	5	2	10	12	10	2	8
0	3	13	20	8	3	10	12	10	7	8	7	12	5	12	8	5	8	3	5
0	4	14	20	9	4	10	11	10	6	9	6	11	5	11	9	5	9	4	5
Reflected cycles 2nd half																			
0	24	5	9	10	1	6	5	6	4	10	4	1	5	1	10	9	10	1	11
0	23	5	8	10	2	7	5	7	3	10	3	2	5	2	10	8	10	2	12
0	13	20	5	8	12	7	5	7	10	8	10	3	12	3	8	5	8	12	5
0	14	20	5	9	11	6	5	6	10	9	10	4	11	4	9	5	9	11	5

Table A3. $\mathcal{C}(3, 5, 25)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,22,3,24)	1	0 1 22	1 2 23	2	1 3 22	1 3 22	2	22 24 3	1 3 7	1	24 0 3	1 2 5	1	24 0 1	1 2 3	
(0,2,19,6,23)	2	0 2 19	1 3 20	4	2 6 19	1 5 18	4	19 23 6	1 5 13	2	23 0 6	1 3 9	2	23 0 2	1 3 5	
(0,3,16,9,22)	3	0 3 16	1 4 17	6	3 9 16	1 7 14	6	16 22 9	1 7 19	3	22 0 9	1 4 13	3	22 0 3	1 4 7	
(0,4,13,12,21)	4	0 4 13	1 5 14	1	12 13 4	1 2 18	1	12 13 21	1 2 10	4	21 0 12	1 5 17	4	21 0 4	1 5 9	
(0,5,10,15,20)	5	0 5 10	1 6 11	5	5 10 15	1 6 11	5	10 15 20	1 6 11	5	15 20 0	1 6 11	5	20 0 5	1 6 11	
(0,6,7,18,19)	1	6 7 0	1 2 20	1	6 7 18	1 2 13	1	18 19 7	1 2 15	1	18 19 0	1 2 8	6	19 0 6	1 7 13	
(0,7,4,21,18)	3	4 7 0	1 4 22	3	4 7 21	1 4 18	3	18 21 4	1 4 12	3	18 21 0	1 4 8	7	18 0 7	1 8 15	
(0,8,1,24,17)	1	0 1 8	1 2 9	2	24 1 8	1 3 10	2	24 1 17	1 3 19	1	24 0 17	1 2 19	8	17 0 8	1 9 17	
(0,9,23,2,16)	2	23 0 9	1 3 12	4	23 2 9	1 5 12	4	23 2 16	1 5 19	2	0 2 16	1 3 17	7	9 16 0	1 8 17	
(0,10,20,5,15)	5	20 0 10	1 6 16	5	5 10 20	1 6 16	5	15 20 5	1 6 16	5	0 5 15	1 6 16	5	10 15 0	1 6 16	
(0,11,17,8,14)	6	11 17 0	1 7 15	3	8 11 17	1 4 10	3	14 17 8	1 4 20	6	8 14 0	1 7 18	3	11 14 0	1 4 15	
(0,12,14,11,13)	2	12 14 0	1 3 14	1	11 12 14	1 2 4	1	13 14 11	1 2 24	2	11 13 0	1 3 15	1	12 13 0	1 2 14	

(b) First half of the reflected base cycle pairs.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,5,11,10)	1	0 1 5	1 2 6	4	1 5 11	1 5 11	1	10 11 5	1 2 21	1	10 11 0	1 2 16	1	0 1 10	1 2 11	
(0,2,5,12,10)	2	0 2 5	1 3 6	3	2 5 12	1 4 11	2	10 12 5	1 3 21	2	10 12 0	1 3 16	2	0 2 10	1 3 11	
(0,3,13,20,8)	3	0 3 13	1 4 14	7	13 20 3	1 8 16	5	8 13 20	1 6 13	5	20 0 8	1 6 14	3	0 3 8	1 4 9	
(0,4,14,20,9)	4	0 4 14	1 5 15	6	14 20 4	1 7 16	5	9 14 20	1 6 12	5	20 0 9	1 6 15	4	0 4 9	1 5 10	

(c) Second half of the reflected base cycle pairs.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,24,5,9,10)	1	24 0 5	1 2 7	4	5 9 24	1 5 20	1	9 10 5	1 2 22	1	9 10 0	1 2 17	1	24 0 10	1 2 12	
(0,23,5,8,10)	2	23 0 5	1 3 8	3	5 8 23	1 4 19	2	8 10 5	1 3 23	2	8 10 0	1 3 18	2	23 0 10	1 3 13	
(0,13,20,5,8)	5	20 0 13	1 6 19	7	13 20 5	1 8 18	3	5 8 20	1 4 16	3	5 8 0	1 4 21	5	8 13 0	1 6 18	
(0,14,20,5,9)	5	20 0 14	1 6 20	6	14 20 5	1 7 17	4	5 9 20	1 5 16	4	5 9 0	1 5 21	5	9 14 0	1 6 17	

Table A4. Cyclic $\mathcal{C}(3, 5, 26)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles																			
0	1	24	2	25	1	3	2	3	4	1	4	3	1	3	1	2	1	1	2
0	3	20	6	23	3	9	6	9	12	3	12	9	3	9	3	6	3	3	6
0	5	16	10	21	5	11	10	11	6	5	6	11	5	11	5	10	5	5	10
0	7	12	14	19	7	5	12	5	2	7	2	5	7	5	7	12	7	7	12
0	9	8	18	17	9	1	8	1	10	9	10	1	9	1	9	8	9	9	8
0	11	4	22	15	11	7	4	7	8	11	8	7	11	7	11	4	11	11	4
0	2	22	4	24	2	6	4	6	8	2	8	6	2	6	2	4	2	2	4
0	4	18	8	22	4	12	8	12	10	4	10	12	4	12	4	8	4	4	8
0	6	22	4	20	6	10	4	10	8	2	8	10	2	10	6	4	6	6	12
0	8	21	5	18	8	13	5	13	10	3	10	13	3	13	8	5	8	8	10
0	10	25	1	16	10	11	1	11	2	9	2	11	9	11	10	1	10	10	6
0	12	23	3	14	12	11	3	11	6	9	6	11	9	11	12	3	12	12	2
Reflected cycles 1st half																			
0	1	8	17	13	1	7	8	7	9	10	9	4	5	4	13	9	13	1	12
0	1	12	25	6	1	11	12	11	13	2	13	7	6	7	6	1	6	1	5
0	12	20	23	2	12	8	6	8	3	11	3	5	8	5	2	3	2	12	10
0	3	7	2	19	3	4	7	4	5	1	5	9	12	9	7	2	7	3	10
Reflected cycles 2nd half																			
0	22	5	12	13	4	9	5	9	7	10	7	1	8	1	13	12	13	4	9
0	7	20	5	6	7	13	6	13	11	2	11	1	12	1	6	5	6	7	1
0	5	8	16	2	5	3	8	3	8	11	8	12	6	12	2	10	2	5	3
0	17	12	16	19	9	5	12	5	4	1	4	3	7	3	7	10	7	9	2

Table A5. $\mathcal{C}(3, 5, 26)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.															
Cycles	(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,24,2,25)	1	0 1 24	1 2 25	1	1 2 24	1 2 24	1	24 25 2	1 2 5	1	25 0 2	1 2 4	1	25 0 1	1 2 3
(0,2,22,4,24)	2	0 2 22	1 3 23	2	2 4 22	1 3 21	2	22 24 4	1 3 9	2	24 0 4	1 3 7	2	24 0 2	1 3 5
(0,3,20,6,23)	3	0 3 20	1 4 21	3	3 6 20	1 4 18	3	20 23 6	1 4 13	3	23 0 6	1 4 10	3	23 0 3	1 4 7
(0,4,18,8,22)	4	0 4 18	1 5 19	4	4 8 18	1 5 15	4	18 22 8	1 5 17	4	22 0 8	1 5 13	4	22 0 4	1 5 9
(0,5,16,10,21)	5	0 5 16	1 6 17	5	5 10 16	1 6 12	5	16 21 10	1 6 21	5	21 0 10	1 6 16	5	21 0 5	1 6 11
(0,6,22,4,20)	4	22 0 6	1 5 11	2	4 6 22	1 3 19	2	20 22 4	1 3 11	4	0 4 20	1 5 21	6	20 0 6	1 7 13
(0,7,12,14,19)	5	7 12 0	1 6 20	2	12 14 7	1 3 22	2	12 14 19	1 3 8	5	14 19 0	1 6 13	7	19 0 7	1 8 15
(0,8,21,5,18)	5	21 0 8	1 6 14	3	5 8 21	1 4 17	3	18 21 5	1 4 14	5	0 5 18	1 6 19	8	18 0 8	1 9 17
(0,9,8,18,17)	1	8 9 0	1 2 19	1	8 9 18	1 2 11	1	17 18 8	1 2 18	1	17 18 0	1 2 10	8	9 17 0	1 9 18
(0,10,25,1,16)	1	25 0 10	1 2 12	2	25 1 10	1 3 12	2	25 1 16	1 3 18	1	0 1 16	1 2 17	6	10 16 0	1 7 17
(0,11,4,22,15)	4	0 4 11	1 5 12	7	4 11 22	1 8 19	7	15 22 4	1 8 16	4	22 0 15	1 5 20	4	11 15 0	1 5 16
(0,12,23,3,14)	3	23 0 12	1 4 16	6	23 3 12	1 7 16	6	23 3 14	1 7 18	3	0 3 14	1 4 15	2	12 14 0	1 3 15
(b) First half of the reflected base cycle pairs.															
Cycles	(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,8,17,13)	1	0 1 8	1 2 9	7	1 8 17	1 8 17	4	13 17 8	1 5 22	4	13 17 0	1 5 14	1	0 1 13	1 2 14
(0,1,12,25,6)	1	0 1 12	1 2 13	2	25 1 12	1 3 14	6	6 12 25	1 7 20	1	25 0 6	1 2 8	1	0 1 6	1 2 7
(0,12,20,23,2)	6	20 0 12	1 7 19	3	20 23 12	1 4 19	3	20 23 2	1 4 9	2	0 2 23	1 3 24	2	0 2 12	1 3 13
(0,3,7,2,19)	3	0 3 7	1 4 8	1	2 3 7	1 2 6	5	2 7 19	1 6 18	2	0 2 19	1 3 20	3	0 3 19	1 4 20
(c) Second half of the reflected base cycle pairs.															
Cycles	(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,22,5,12,13)	4	22 0 5	1 5 10	7	5 12 22	1 8 18	1	12 13 5	1 2 20	1	12 13 0	1 2 15	4	22 0 13	1 5 18
(0,7,20,5,6)	6	20 0 7	1 7 14	2	5 7 20	1 3 16	1	5 6 20	1 2 16	1	5 6 0	1 2 22	1	6 7 0	1 2 21
(0,5,8,16,2)	3	5 8 0	1 4 22	3	5 8 16	1 4 12	6	2 8 16	1 7 15	2	0 2 16	1 3 17	2	0 2 5	1 3 6
(0,17,12,16,19)	5	12 17 0	1 6 15	1	16 17 12	1 2 23	3	16 19 12	1 4 23	3	16 19 0	1 4 11	2	17 19 0	1 3 10

Table A6. Cyclic $\mathcal{C}(3, 5, 31)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles																			
0	1	28	3	30	1	4	3	4	6	2	6	4	2	4	1	3	1	1	2
0	2	25	6	29	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	3	22	9	28	3	12	9	12	13	6	13	12	6	12	3	9	3	3	6
0	4	19	12	27	4	15	12	15	7	8	7	15	8	15	4	12	4	4	8
0	5	16	15	26	5	11	15	11	1	10	1	11	10	11	5	15	5	5	10
0	6	13	18	25	6	7	13	7	5	12	5	7	12	7	6	13	6	6	12
0	7	10	21	24	7	3	10	3	11	14	11	3	14	3	7	10	7	7	14
0	8	7	24	23	8	1	7	1	14	15	14	1	15	1	8	7	8	8	15
0	9	4	27	22	9	5	4	5	8	13	8	5	13	5	9	4	9	9	13
0	10	1	30	21	10	9	1	9	2	11	2	9	11	9	10	1	10	10	11
0	11	29	2	20	11	13	2	13	4	9	4	13	9	13	11	2	11	11	9
0	12	26	5	19	12	14	5	14	10	7	10	14	7	14	12	5	12	12	7
0	13	23	8	18	13	10	8	10	15	5	15	10	5	10	13	8	13	13	5
0	14	20	11	17	14	6	11	6	9	3	9	6	3	6	14	11	14	14	3
0	15	17	14	16	15	2	14	2	3	1	3	2	1	2	15	14	15	15	1

Table A6. Cont.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Reflected cycles 1st half																			
0	1	5	8	20	1	4	5	4	3	7	3	12	15	12	11	8	11	1	12
0	1	7	17	9	1	6	7	6	10	15	10	8	2	8	9	14	9	1	8
0	1	13	15	18	1	12	13	12	2	14	2	3	5	3	13	15	13	1	14
0	2	9	22	7	2	7	9	7	13	11	13	15	2	15	7	9	7	2	5
0	13	27	23	3	13	14	4	14	4	10	4	11	7	11	3	8	3	13	10
0	15	6	25	4	15	9	6	9	12	10	12	10	2	10	4	6	4	15	11
0	5	11	6	14	5	6	11	6	5	1	5	8	3	8	14	6	14	5	9
Reflected cycles 2nd half																			
0	12	15	19	20	12	3	15	3	4	7	4	1	5	1	11	12	11	12	8
0	23	2	8	9	8	10	2	10	6	15	6	1	7	1	9	8	9	8	14
0	3	5	17	18	3	2	5	2	12	14	12	1	13	1	13	14	13	3	15
0	16	29	5	7	15	13	2	13	7	11	7	2	9	2	7	5	7	15	9
0	11	7	21	3	11	4	7	4	14	10	14	13	4	13	3	10	3	11	8
0	10	29	20	4	10	12	2	12	9	10	9	15	6	15	4	11	4	10	6
0	8	3	9	14	8	5	3	5	6	1	6	5	11	5	14	9	14	8	6

Table A7. $C(3, 5, 31)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.																
Cycles		(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,28,3,30)	1	0 1 28	1 2 29	2	1 3 28	1 3 28	2	28 30 3	1 3 7	1	30 0 3	1 2 5	1	30 0 1	1 2 3	
(0,2,25,6,29)	2	0 2 25	1 3 26	4	2 6 25	1 5 24	4	25 29 6	1 5 13	2	29 0 6	1 3 9	2	29 0 2	1 3 5	
(0,3,22,9,28)	3	0 3 22	1 4 23	6	3 9 22	1 7 20	6	22 28 9	1 7 19	3	28 0 9	1 4 13	3	28 0 3	1 4 7	
(0,4,19,12,27)	4	0 4 19	1 5 20	7	12 19 4	1 8 24	7	12 19 27	1 8 16	4	27 0 12	1 5 17	4	27 0 4	1 5 9	
(0,5,16,15,26)	5	0 5 16	1 6 17	1	15 16 5	1 2 22	1	15 16 26	1 2 12	5	26 0 15	1 6 21	5	26 0 5	1 6 11	
(0,6,13,18,25)	6	0 6 13	1 7 14	5	13 18 6	1 6 25	5	13 18 25	1 6 13	6	25 0 18	1 7 25	6	25 0 6	1 7 13	
(0,7,10,21,24)	3	7 10 0	1 4 25	3	7 10 21	1 4 15	3	21 24 10	1 4 21	3	21 24 0	1 4 11	7	24 0 7	1 8 15	
(0,8,7,24,23)	1	7 8 0	1 2 25	1	7 8 24	1 2 18	1	23 24 7	1 2 16	1	23 24 0	1 2 9	8	23 0 8	1 9 17	
(0,9,4,27,22)	4	0 4 9	1 5 10	5	4 9 27	1 6 24	5	22 27 4	1 6 14	4	27 0 22	1 5 27	9	22 0 9	1 10 19	
(0,10,1,30,21)	1	0 1 10	1 2 11	2	30 1 10	1 3 12	2	30 1 21	1 3 23	1	30 0 21	1 2 23	10	21 0 10	1 11 21	
(0,11,29,2,20)	2	29 0 11	1 3 14	4	29 2 11	1 5 14	4	29 2 20	1 5 23	2	0 2 20	1 3 21	9	11 20 0	1 10 21	
(0,12,26,5,19)	5	26 0 12	1 6 18	7	5 12 26	1 8 22	7	19 26 5	1 8 18	5	0 5 19	1 6 20	7	12 19 0	1 8 20	
(0,13,23,8,18)	8	23 0 13	1 9 22	5	8 13 23	1 6 16	5	18 23 8	1 6 22	8	0 8 18	1 9 19	5	13 18 0	1 6 19	
(0,14,20,11,17)	6	14 20 0	1 7 18	3	11 14 20	1 4 10	3	17 20 11	1 4 26	6	11 17 0	1 7 21	3	14 17 0	1 4 18	
(0,15,17,14,16)	2	15 17 0	1 3 17	1	14 15 17	1 2 4	1	16 17 14	1 2 30	2	14 16 0	1 3 18	1	15 16 0	1 2 17	
(b) First half of the reflected base cycle pairs.																
Cycles		(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,5,8,20)	1	0 1 5	1 2 6	3	5 8 1	1 4 28	3	5 8 20	1 4 16	8	0 8 20	1 9 21	1	0 1 20	1 2 21	
(0,1,7,17,9)	1	0 1 7	1 2 8	6	1 7 17	1 7 17	2	7 9 17	1 3 11	8	9 17 0	1 9 23	1	0 1 9	1 2 10	
(0,1,13,15,18)	1	0 1 13	1 2 14	2	13 15 1	1 3 20	2	13 15 18	1 3 6	3	15 18 0	1 4 17	1	0 1 18	1 2 19	
(0,2,9,22,7)	2	0 2 9	1 3 10	7	2 9 22	1 8 21	2	7 9 22	1 3 16	7	0 7 22	1 8 23	2	0 2 7	1 3 8	
(0,13,27,23,3)	4	27 0 13	1 5 18	4	23 27 13	1 5 22	4	23 27 3	1 5 12	3	0 3 23	1 4 24	3	0 3 13	1 4 14	
(0,15,6,25,4)	6	0 6 15	1 7 16	9	6 15 25	1 10 20	2	4 6 25	1 3 22	4	0 4 25	1 5 26	4	0 4 15	1 5 16	
(0,5,11,6,14)	5	0 5 11	1 6 12	1	5 6 11	1 2 7	3	11 14 6	1 4 27	6	0 6 14	1 7 15	5	0 5 14	1 6 15	
(c) Second half of the reflected base cycle pairs.																
Cycles		(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,12,15,19,20)	3	12 15 0	1 4 20	3	12 15 19	1 4 8	1	19 20 15	1 2 28	1	19 20 0	1 2 13	8	12 20 0	1 9 20	
(0,23,2,8,9)	2	0 2 23	1 3 24	6	2 8 23	1 7 22	1	8 9 2	1 2 26	1	8 9 0	1 2 24	8	23 0 9	1 9 18	
(0,3,5,17,18)	2	3 5 0	1 3 29	2	3 5 17	1 3 15	1	17 18 5	1 2 20	1	17 18 0	1 2 15	3	0 3 18	1 4 19	
(0,16,29,5,7)	2	29 0 16	1 3 19	7	29 5 16	1 8 19	2	5 7 29	1 3 25	2	5 7 0	1 3 27	7	0 7 16	1 8 17	
(0,11,7,21,3)	4	7 11 0	1 5 25	4	7 11 21	1 5 15	4	3 7 21	1 5 19	3	0 3 21	1 4 22	3	0 3 11	1 4 12	
(0,10,29,20,4)	2	29 0 10	1 3 13	9	20 29 10	1 10 22	6	29 4 20	1 7 23	4	0 4 20	1 5 21	4	0 4 10	1 5 11	
(0,8,3,9,14)	3	0 8 3	1 4 9	1	8 9 3	1 2 27	5	9 14 3	1 6 26	5	9 14 0	1 6 23	6	8 14 0	1 7 24	

Table A8. Cyclic split $\mathcal{C}(3, 5, 32)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles for odd values																			
0	1	30	2	31	1	3	2	3	4	1	4	3	1	3	1	2	1	1	2
0	3	26	6	29	3	9	6	9	12	3	12	9	3	9	3	6	3	3	6
0	5	22	10	27	5	15	10	15	12	5	12	15	5	15	5	10	5	5	10
0	7	18	14	25	7	11	14	11	4	7	4	11	7	11	7	14	7	7	14
0	9	14	18	23	9	5	14	5	4	9	4	5	9	5	9	14	9	9	14
0	11	10	22	21	11	1	10	1	12	11	12	1	11	1	11	10	11	11	10
0	13	6	26	19	13	7	6	7	12	13	12	7	13	7	13	6	13	13	6
0	15	2	30	17	15	13	2	13	4	15	4	13	15	13	15	2	15	15	2
Symmetric base cycles for even values: base cycles of $\mathcal{C}(3, 5, 16)$ with vertex indices multiplied by two																			
0	2	18	14	30	2	16	14	16	4	12	4	16	12	16	2	14	2	2	4
0	28	26	6	4	4	2	6	2	12	10	12	2	10	2	4	6	4	4	8
0	6	20	12	26	6	14	12	14	8	6	8	14	6	14	6	12	6	6	12
0	24	30	2	8	8	6	2	6	4	10	4	6	10	6	8	2	8	8	16
0	22	30	2	10	10	8	2	8	4	12	4	8	12	8	10	2	10	10	12
0	20	2	30	12	12	14	2	14	4	10	4	14	10	14	12	2	12	12	8
0	18	8	24	14	14	10	8	10	16	6	16	10	6	10	14	8	14	14	4
Reflected cycles 1st half																			
0	1	5	8	13	1	4	5	4	3	7	3	5	8	5	13	8	13	1	12
0	1	6	3	14	1	5	6	5	3	2	3	11	8	11	14	3	14	1	13
0	1	8	7	16	1	7	8	7	1	6	1	9	8	9	16	7	16	1	15
0	1	10	29	15	1	9	10	9	13	4	13	14	5	14	15	3	15	1	14
0	3	10	30	19	3	7	10	7	12	5	12	11	9	11	13	2	13	3	16
0	3	13	22	15	3	10	13	10	9	13	9	7	2	7	15	10	15	3	12
0	5	16	24	7	5	11	16	11	8	13	8	15	9	15	7	8	7	5	2
0	6	17	2	11	6	11	15	11	15	4	15	9	6	9	11	2	11	6	5
Reflected cycles 2nd half																			
0	5	8	12	13	5	3	8	3	4	7	4	1	5	1	13	12	13	5	8
0	11	8	13	14	11	3	8	3	5	2	5	1	6	1	14	13	14	11	3
0	9	8	15	16	9	1	8	1	7	6	7	1	8	1	16	15	16	9	7
0	18	5	14	15	14	13	5	13	9	4	9	1	10	1	15	14	15	14	3
0	21	9	16	19	11	12	9	12	7	5	7	3	10	3	13	16	13	11	2
0	25	2	12	15	7	9	2	9	10	13	10	3	13	3	15	12	15	7	10
0	15	23	2	7	15	8	9	8	11	13	11	5	16	5	7	2	7	15	8
0	9	26	5	11	9	15	6	15	11	4	11	6	15	6	11	5	11	9	2

Table A9. $\mathcal{C}(3, 5, 32)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles for odd values.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,30,2,31)	1	0 1 30	1 2 31	1	1 2 30	1 2 30	1	30 31 2	1 2 5	1	31 0 2	1 2 4	1	31 0 1	1 2 3	
(0,3,26,6,29)	3	0 3 26	1 4 27	3	3 6 26	1 4 24	3	26 29 6	1 4 13	3	29 0 6	1 4 10	3	29 0 3	1 4 7	
(0,5,22,10,27)	5	0 5 22	1 6 23	5	5 10 22	1 6 18	5	22 27 10	1 6 21	5	27 0 10	1 6 16	5	27 0 5	1 6 11	
(0,7,18,14,25)	7	0 7 18	1 8 19	4	14 18 7	1 5 26	4	14 18 25	1 5 12	7	25 0 14	1 8 22	7	25 0 7	1 8 15	
(0,9,14,18,23)	5	9 14 0	1 6 24	4	14 18 9	1 5 28	4	14 18 23	1 5 10	5	18 23 0	1 6 15	9	23 0 9	1 10 19	
(0,11,10,22,21)	1	10 11 0	1 2 23	1	10 11 22	1 2 13	1	21 22 10	1 2 22	1	21 22 0	1 2 12	10	11 21 0	1 11 22	
(0,13,6,26,19)	6	0 6 13	1 7 14	7	6 13 26	1 8 21	7	19 26 6	1 8 20	6	26 0 19	1 7 26	6	13 19 0	1 7 20	
(0,15,2,30,17)	2	0 2 15	1 3 16	4	30 2 15	1 5 18	4	30 2 17	1 5 20	2	30 0 17	1 3 20	2	15 17 0	1 3 18	
(b) Symmetric base cycles for even values: base cycles of $\mathcal{C}(3, 5, 26)$ with vertex indices multiplied by two.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,2,18,14,30)	2	0 2 18	1 3 19	4	14 18 2	1 5 21	4	14 18 30	1 5 17	2	30 0 14	1 3 17	2	30 0 2	1 3 5	
(0,28,26,6,4)	2	26 28 0	1 3 7	2	26 28 6	1 3 13	2	4 6 26	1 3 23	2	4 6 0	1 3 29	4	28 0 4	1 5 9	
(0,6,20,12,26)	6	0 6 20	1 7 21	6	6 12 20	1 7 15	6	20 26 12	1 7 25	6	26 0 12	1 7 19	6	26 0 6	1 7 13	
(0,24,30,2,8)	2	30 0 24	1 3 27	4	30 2 24	1 5 27	4	30 2 8	1 5 11	2	0 2 8	1 3 9	8	24 0 8	1 9 17	
(0,22,30,2,10)	2	30 0 22	1 3 25	4	30 2 22	1 5 25	4	30 2 10	1 5 13	2	0 2 10	1 3 11	10	22 0 10	1 11 21	
(0,20,2,30,12)	2	0 2 20	1 3 21	4	30 2 20	1 5 23	4	30 2 12	1 5 15	2	30 0 12	1 3 15	8	12 20 0	1 9 21	
(0,18,8,24,14)	8	0 8 18	1 9 19	6	18 24 8	1 7 23	6	8 14 24	1 7 17	8	24 0 14	1 9 23	4	14 18 0	1 5 19	
(c) First half of the reflected base cycle pairs.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,5,8,13)	1	0 1 5	1 2 6	3	5 8 1	1 4 29	3	5 8 13	1 4 9	5	8 13 0	1 6 25	1	0 1 13	1 2 14	
(0,1,6,3,14)	1	0 1 6	1 2 7	2	1 3 6	1 3 6	3	3 6 14	1 4 12	3	0 3 14	1 4 15	1	0 1 14	1 2 15	
(0,1,8,7,16)	1	0 1 8	1 2 9	1	7 8 1	1 2 27	1	7 8 16	1 2 10	7	0 7 16	1 8 17	1	0 1 16	1 2 17	
(0,1,10,29,15)	1	0 1 10	1 2 11	4	29 1 10	1 5 14	5	10 15 29	1 6 20	3	29 0 15	1 4 19	1	0 1 15	1 2 16	
(0,3,10,30,19)	3	0 3 10	1 4 11	5	30 3 10	1 6 13	9	10 19 30	1 10 21	2	30 0 19	1 3 22	3	0 3 19	1 4 20	
(0,3,13,22,15)	3	0 3 13	1 4 14	9	13 22 3	1 10 23	2	13 15 22	1 3 10	7	15 22 0	1 8 18	3	0 3 15	1 4 16	
(0,5,16,24,7)	5	0 5 16	1 6 17	8	16 24 5	1 9 22	8	16 24 7	1 9 24	7	0 7 24	1 8 25	2	5 7 0	1 3 28	
(0,6,17,2,11)	6	0 6 17	1 7 18	4	2 6 17	1 5 16	6	11 17 2	1 7 24	2	0 2 11	1 3 12	5	6 11 0	1 6 27	
(d) Second half of the reflected base cycle pairs.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,5,8,12,13)	3	5 8 0	1 4 28	3	5 8 12	1 4 8	1	12 13 8	1 2 29	1	12 13 0	1 2 21	5	0 5 13	1 6 14	
(0,11,8,13,14)	3	8 11 0	1 4 25	2	11 13 8	1 3 30	1	13 14 8	1 2 28	1	13 14 0	1 2 20	3	11 14 0	1 4 22	
(0,9,8,15,16)	1	8 9 0	1 2 25	1	8 9 15	1 2 8	1	15 16 8	1 2 26	1	15 16 0	1 2 18	7	9 16 0	1 8 24	
(0,18,5,14,15)	5	0 5 18	1 6 19	4	14 18 5	1 5 24	1	14 15 5	1 2 24	1	14 15 0	1 2 19	3	15 18 0	1 4 18	
(0,21,9,16,19)	9	0 9 21	1 10 22	5	16 21 9	1 6 26	3	16 19 9	1 4 26	3	16 19 0	1 4 17	2	19 21 0	1 3 14	
(0,25,2,12,15)	2	0 2 25	1 3 26	9	25 2 12	1 10 20	3	12 15 2	1 4 23	3	12 15 0	1 4 21	7	25 0 15	1 8 23	
(0,15,23,2,7)	8	15 23 0	1 9 18	8	15 23 2	1 9 20	5	2 7 23	1 6 22	2	0 2 7	1 3 8	7	0 7 15	1 8 16	
(0,9,26,5,11)	6	26 0 9	1 7 16	4	5 9 26	1 5 22	6	5 11 26	1 7 22	5	0 5 11	1 6 12	2	9 11 0	1 3 24	

Table A10. Cyclic $\mathcal{C}(3, 5, 35)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. The bright green cells mark the base cycle vertices where the original +3 rule had to be modified for v_4 (which means a modification for v_3 as well). In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example. With red, we highlighted the special base cycles that only have $v/5$ different positions, as $i \mapsto i + v/5$ results in an automorphism of these cycles.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles																			
0	1	32	3	34	1	4	3	4	6	2	6	4	2	4	1	3	1	1	2
0	2	29	6	33	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	3	26	9	32	3	12	9	12	17	6	17	12	6	12	3	9	3	3	6
0	4	23	12	31	4	16	12	16	11	8	11	16	8	16	4	12	4	4	8
0	5	33	2	30	5	7	2	7	4	3	4	7	3	7	5	2	5	5	10
0	6	17	18	29	6	11	17	11	1	12	1	11	12	11	6	17	6	6	12
0	7	14	21	28	7	7	14	7	7	14	7	7	14	7	7	14	7	7	14
0	8	11	24	27	8	3	11	3	13	16	13	3	16	3	8	11	8	8	16
0	9	8	27	26	9	1	8	1	16	17	16	1	17	1	9	8	9	9	17
0	10	31	4	25	10	14	4	14	8	6	8	14	6	14	10	4	10	10	15
0	11	2	33	24	11	9	2	9	4	13	4	9	13	9	11	2	11	11	13
0	12	34	1	23	12	13	1	13	2	11	2	13	11	13	12	1	12	12	11
0	13	31	4	22	13	17	4	17	8	9	8	17	9	17	13	4	13	13	9
0	14	28	7	21	14	14	7	14	14	7	14	14	7	14	14	7	14	14	7
0	15	29	6	20	15	14	6	14	12	9	12	14	9	14	15	6	15	15	5
0	16	22	13	19	16	6	13	6	9	3	9	6	3	6	16	13	16	16	3
0	17	19	16	18	17	2	16	2	3	1	3	2	1	2	17	16	17	17	1
Reflected cycles 1st half																			
0	1	5	10	7	1	4	5	4	5	9	5	3	2	3	7	10	7	1	6
0	1	6	9	20	1	5	6	5	3	8	3	11	14	11	15	9	15	1	16
0	1	8	10	22	1	7	8	7	2	9	2	12	14	12	13	10	13	1	14
0	1	10	16	21	1	9	10	9	6	15	6	5	11	5	14	16	14	1	15
0	2	10	22	17	2	8	10	8	12	15	12	5	7	5	17	13	17	2	15
0	3	15	31	17	3	12	15	12	16	7	16	14	2	14	17	4	17	3	14
0	4	11	24	19	4	7	11	7	13	15	13	5	8	5	16	11	16	4	15
0	4	24	34	10	4	15	11	15	10	5	10	11	14	11	10	1	10	4	6
0	5	14	33	23	5	9	14	9	16	7	16	10	9	10	12	2	12	5	17
0	6	16	33	13	6	10	16	10	17	8	17	15	3	15	13	2	13	6	7
0	7	15	28	18	7	8	15	8	13	14	13	10	3	10	17	7	17	7	11
Reflected cycles 2nd half																			
0	32	2	6	7	3	5	2	5	4	9	4	1	5	1	7	6	7	3	10
0	11	14	19	20	11	3	14	3	5	8	5	1	6	1	15	16	15	11	9
0	12	14	21	22	12	2	14	2	7	9	7	1	8	1	13	14	13	12	10
0	5	11	20	21	5	6	11	6	9	15	9	1	10	1	14	15	14	5	16
0	30	7	15	17	5	12	7	12	8	15	8	2	10	2	17	15	17	5	13
0	21	2	14	17	14	16	2	16	12	7	12	3	15	3	17	14	17	14	4
0	30	8	15	19	5	13	8	13	7	15	7	4	11	4	16	15	16	5	11
0	25	9	18	23	10	16	9	16	9	7	9	5	14	5	12	17	12	10	2
0	15	32	7	13	15	17	3	17	10	8	10	6	16	6	13	7	13	15	2
0	25	3	11	18	10	13	3	13	8	14	8	7	15	7	17	11	17	10	7

Table A11. $\mathcal{C}(3, 5, 35)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,32,3,34)	1	0 1 32	1 2 33	2	1 3 32	1 3 32	2	32 34 3	1 3 7	1	34 0 3	1 2 5	1	34 0 1	1 2 3
(0,2,29,6,33)	2	0 2 29	1 3 30	4	2 6 29	1 5 28	4	29 33 6	1 5 13	2	33 0 6	1 3 9	2	33 0 2	1 3 5
(0,3,26,9,32)	3	0 3 26	1 4 27	6	3 9 26	1 7 24	6	26 32 9	1 7 19	3	32 0 9	1 4 13	3	32 0 3	1 4 7
(0,4,23,12,31)	4	0 4 23	1 5 24	8	4 12 23	1 9 20	8	23 31 12	1 9 25	4	31 0 12	1 5 17	4	31 0 4	1 5 9
(0,5,33,2,30)	2	33 0 5	1 3 8	3	2 5 33	1 4 32	3	30 33 2	1 4 8	2	0 2 30	1 3 31	5	30 0 5	1 6 11
(0,6,17,18,29)	6	0 6 17	1 7 18	1	17 18 6	1 2 25	1	17 18 29	1 2 13	6	29 0 18	1 7 25	6	29 0 6	1 7 13
(0,7,14,21,28)	7	0 7 14	1 8 15	7	7 14 21	1 8 15	7	14 21 28	1 8 15	7	21 28 0	1 8 15	7	28 0 7	1 8 15
(0,8,11,24,27)	3	8 11 0	1 4 28	3	8 11 24	1 4 17	3	24 27 11	1 4 23	3	24 27 0	1 4 12	8	27 0 8	1 9 17
(0,9,8,27,26)	1	8 9 0	1 2 28	1	8 9 27	1 2 20	1	26 27 8	1 2 18	1	26 27 0	1 2 10	9	26 0 9	1 10 19
(0,10,31,4,25)	4	31 0 10	1 5 15	6	4 10 31	1 7 28	6	25 31 4	1 7 15	4	0 4 25	1 5 26	10	25 0 10	1 11 21
(0,11,2,33,24)	2	0 2 11	1 3 12	4	33 2 11	1 5 14	4	33 2 24	1 5 27	2	33 0 24	1 3 27	11	24 0 11	1 12 23
(0,12,34,1,23)	1	34 0 12	1 2 14	2	34 1 12	1 3 14	2	34 1 23	1 3 25	1	0 1 23	1 2 24	11	12 23 0	1 12 24
(0,13,31,4,22)	4	31 0 13	1 5 18	8	31 4 13	1 9 18	8	31 4 22	1 9 27	4	0 4 22	1 5 23	9	13 22 0	1 10 23
(0,14,28,7,21)	7	28 0 14	1 8 22	7	7 14 28	1 8 22	7	21 28 7	1 8 22	7	0 7 21	1 8 22	7	14 21 0	1 8 22
(0,15,29,6,20)	6	29 0 15	1 7 22	9	6 15 29	1 10 24	9	20 29 6	1 10 22	6	0 6 20	1 7 21	5	15 20 0	1 6 21
(0,16,22,13,19)	6	16 22 0	1 7 20	3	13 16 22	1 4 10	3	19 22 13	1 4 30	6	13 19 0	1 7 23	3	16 19 0	1 4 20
(0,17,19,16,18)	2	17 19 0	1 3 19	1	16 17 19	1 2 4	1	18 19 16	1 2 34	2	16 18 0	1 3 20	1	17 18 0	1 2 19
(b) First half of the reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,5,10,7)	1	0 1 5	1 2 6	4	1 5 10	1 5 10	2	5 7 10	1 3 6	3	7 10 0	1 4 29	1	0 1 7	1 2 8
(0,1,6,9,20)	1	0 1 6	1 2 7	3	6 9 1	1 4 31	3	6 9 20	1 4 15	9	0 9 20	1 10 21	1	0 1 20	1 2 21
(0,1,8,10,22)	1	0 1 8	1 2 9	2	8 10 1	1 3 29	2	8 10 22	1 3 15	10	0 10 22	1 11 23	1	0 1 22	1 2 23
(0,1,10,16,21)	1	0 1 10	1 2 11	6	10 16 1	1 7 27	5	16 21 10	1 6 30	5	16 21 0	1 6 20	1	0 1 21	1 2 22
(0,2,10,22,17)	2	0 2 10	1 3 11	8	2 10 22	1 9 21	5	17 22 10	1 6 29	5	17 22 0	1 6 19	2	0 2 17	1 3 18
(0,3,15,31,17)	3	0 3 15	1 4 16	7	31 3 15	1 8 20	2	15 17 31	1 3 17	4	31 0 17	1 5 22	3	0 3 17	1 4 18
(0,4,11,24,19)	4	0 4 11	1 5 12	7	4 11 24	1 8 21	5	19 24 11	1 6 28	5	19 24 0	1 6 17	4	0 4 19	1 5 20
(0,4,24,34,10)	4	0 4 24	1 5 25	5	34 4 24	1 6 26	10	24 34 10	1 11 22	1	34 0 10	1 2 12	4	0 4 10	1 5 11
(0,5,14,33,23)	5	0 5 14	1 6 15	7	33 5 14	1 8 17	9	14 23 33	1 10 20	2	33 0 23	1 3 26	5	0 5 23	1 6 24
(0,6,16,33,13)	6	0 6 16	1 7 17	8	33 6 16	1 9 19	3	13 16 33	1 4 21	2	33 0 13	1 3 16	6	0 6 13	1 7 14
(0,7,15,28,18)	7	0 7 15	1 8 16	8	7 15 28	1 9 22	3	15 18 28	1 4 14	7	28 0 18	1 8 26	7	0 7 18	1 8 19
(c) Second half of the reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,32,2,6,7)	2	0 2 32	1 3 33	4	2 6 32	1 5 31	1	6 7 2	1 2 32	1	6 7 0	1 2 30	3	32 0 7	1 4 11
(0,11,14,19,20)	3	11 14 0	1 4 25	3	11 14 19	1 4 9	1	19 20 14	1 2 31	1	19 20 0	1 2 17	9	11 20 0	1 10 25
(0,12,14,21,22)	2	12 14 0	1 3 24	2	12 14 21	1 3 10	1	21 22 14	1 2 29	1	21 22 0	1 2 15	10	12 22 0	1 11 24
(0,5,11,20,21)	5	0 5 11	1 6 12	6	5 11 20	1 7 16	1	20 21 11	1 2 27	1	20 21 0	1 2 16	5	0 5 21	1 6 22
(0,30,7,15,17)	5	30 0 7	1 6 13	8	7 15 30	1 9 24	2	15 17 7	1 3 28	2	15 17 0	1 3 21	5	30 0 17	1 6 23
(0,21,2,14,17)	2	0 2 21	1 3 22	7	14 21 2	1 8 24	3	14 17 2	1 4 24	3	14 17 0	1 4 22	4	17 21 0	1 5 19
(0,30,8,15,19)	5	30 0 8	1 6 14	7	8 15 30	1 8 23	4	15 19 8	1 5 29	4	15 19 0	1 5 21	5	30 0 19	1 6 25
(0,11,21,6,10)	10	11 21 0	1 11 25	5	6 11 21	1 6 16	4	6 10 21	1 5 16	4	6 10 0	1 5 30	1	10 11 0	1 2 26
(0,25,9,18,23)	9	0 25 9	1 10 26	7	18 25 9	1 8 27	5	18 23 9	1 6 27	5	18 23 0	1 6 18	2	23 25 0	1 3 13
(0,15,32,7,13)	3	32 0 15	1 4 19	8	7 15 32	1 9 26	6	7 13 32	1 7 26	6	7 13 0	1 7 29	2	13 15 0	1 3 23
(0,25,3,11,18)	3	0 3 25	1 4 26	8	3 11 25	1 9 23	7	11 18 3	1 8 28	7	11 18 0	1 8 25	7	18 25 0	1 8 18

Table A12. Cyclic $C(3, 5, 37)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles																			
0	1	34	3	36	1	4	3	4	6	2	6	4	2	4	1	3	1	1	2
0	2	31	6	35	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	3	28	9	34	3	12	9	12	18	6	18	12	6	12	3	9	3	3	6
0	4	25	12	33	4	16	12	16	13	8	13	16	8	16	4	12	4	4	8
0	5	22	15	32	5	17	15	17	7	10	7	17	10	17	5	15	5	5	10
0	6	19	18	31	6	13	18	13	1	12	1	13	12	13	6	18	6	6	12
0	7	16	21	30	7	9	16	9	5	14	5	9	14	9	7	16	7	7	14
0	8	13	24	29	8	5	13	5	11	16	11	5	16	5	8	13	8	8	16
0	9	10	27	28	9	1	10	1	17	18	17	1	18	1	9	10	9	9	18
0	10	7	30	27	10	3	7	3	14	17	14	3	17	3	10	7	10	10	17
0	11	4	33	26	11	7	4	7	8	15	8	7	15	7	11	4	11	11	15
0	12	1	36	25	12	11	1	11	2	13	2	11	13	11	12	1	12	12	13
0	13	35	2	24	13	15	2	15	4	11	4	15	11	15	13	2	13	13	11
0	14	32	5	23	14	18	5	18	10	9	10	18	9	18	14	5	14	14	9
0	15	29	8	22	15	14	8	14	16	7	16	14	7	14	15	8	15	15	7
0	16	26	11	21	16	10	11	10	15	5	15	10	5	10	16	11	16	16	5
0	17	23	14	20	17	6	14	6	9	3	9	6	3	6	17	14	17	17	3
0	18	20	17	19	18	2	17	2	3	1	3	2	1	2	18	17	18	18	1
Reflected cycles 1st half																			
0	1	5	8	27	1	4	5	4	3	7	3	18	15	18	10	8	10	1	11
0	1	6	3	14	1	5	6	5	3	2	3	11	8	11	14	3	14	1	13
0	1	7	11	29	1	6	7	6	4	10	4	18	15	18	8	11	8	1	9
0	1	8	3	15	1	7	8	7	5	2	5	12	7	12	15	3	15	1	14
0	17	25	22	1	17	8	12	8	3	5	3	16	13	16	1	15	1	17	16
0	2	9	15	25	2	7	9	7	6	13	6	10	16	10	12	15	12	2	14
0	2	17	30	12	2	15	17	15	13	9	13	18	5	18	12	7	12	2	10
0	18	14	28	2	18	4	14	4	14	10	14	11	12	11	2	9	2	18	16
0	4	13	29	9	4	9	13	9	16	12	16	17	4	17	9	8	9	4	5
0	5	11	26	17	5	6	11	6	15	16	15	9	6	9	17	11	17	5	12
0	6	14	4	17	6	8	14	8	10	2	10	13	3	13	17	4	17	6	11
0	7	20	34	18	7	13	17	13	14	10	14	16	2	16	18	3	18	7	11
Reflected cycles 2nd half																			
0	19	22	26	27	18	3	15	3	4	7	4	1	5	1	10	11	10	18	8
0	11	8	13	14	11	3	8	3	5	2	5	1	6	1	14	13	14	11	3
0	18	22	28	29	18	4	15	4	6	10	6	1	7	1	8	9	8	18	11
0	12	7	14	15	12	5	7	5	7	2	7	1	8	1	15	14	15	12	3
0	16	13	21	1	16	3	13	3	8	5	8	17	12	17	1	16	1	16	15
0	10	16	23	25	10	6	16	6	7	13	7	2	9	2	12	14	12	10	15
0	19	32	10	12	18	13	5	13	15	9	15	2	17	2	12	10	12	18	7
0	11	25	21	2	11	14	12	14	4	10	4	18	14	18	2	16	2	11	9
0	17	33	5	9	17	16	4	16	9	12	9	4	13	4	9	5	9	17	8
0	28	6	12	17	9	15	6	15	6	16	6	5	11	5	17	12	17	9	11
0	13	3	11	17	13	10	3	10	8	2	8	6	14	6	17	11	17	13	4
0	21	35	11	18	16	14	2	14	13	10	13	7	17	7	18	11	18	16	3

Table A13. $\mathcal{C}(3, 5, 37)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,34,3,36)	1	0 1 34	1 2 35	2	1 3 34	1 3 34	2	34 36 3	1 3 7	1	36 0 3	1 2 5	1	36 0 1	1 2 3
(0,2,31,6,35)	2	0 2 31	1 3 32	4	2 6 31	1 5 30	4	31 35 6	1 5 13	2	35 0 6	1 3 9	2	35 0 2	1 3 5
(0,3,28,9,34)	3	0 3 28	1 4 29	6	3 9 28	1 7 26	6	28 34 9	1 7 19	3	34 0 9	1 4 13	3	34 0 3	1 4 7
(0,4,25,12,33)	4	0 4 25	1 5 26	8	4 12 25	1 9 22	8	25 33 12	1 9 25	4	33 0 12	1 5 17	4	33 0 4	1 5 9
(0,5,22,15,32)	5	0 5 22	1 6 23	7	15 22 5	1 8 28	7	15 22 32	1 8 18	5	32 0 15	1 6 21	5	32 0 5	1 6 11
(0,6,19,18,31)	6	0 6 19	1 7 20	1	18 19 6	1 2 26	1	18 19 31	1 2 14	6	31 0 18	1 7 25	6	31 0 6	1 7 13
(0,7,16,21,30)	7	0 7 16	1 8 17	5	16 21 7	1 6 29	5	16 21 30	1 6 15	7	30 0 21	1 8 29	7	30 0 7	1 8 15
(0,8,13,24,29)	5	8 13 0	1 6 30	5	8 13 24	1 6 17	5	24 29 13	1 6 27	5	24 29 0	1 6 14	8	29 0 8	1 9 17
(0,9,10,27,28)	1	9 10 0	1 2 29	1	9 10 27	1 2 19	1	27 28 10	1 2 21	1	27 28 0	1 2 11	9	28 0 9	1 10 19
(0,10,7,30,27)	3	7 10 0	1 4 31	3	7 10 30	1 4 24	3	27 30 7	1 4 18	3	27 30 0	1 4 11	10	27 0 10	1 11 21
(0,11,4,33,26)	4	0 4 11	1 5 12	7	4 11 33	1 8 30	7	26 33 4	1 8 16	4	33 0 26	1 5 31	11	26 0 11	1 12 23
(0,12,1,36,25)	1	0 1 12	1 2 13	2	36 1 12	1 3 14	2	36 1 25	1 3 27	1	36 0 25	1 2 27	12	25 0 12	1 13 25
(0,13,35,2,24)	2	35 0 13	1 3 16	4	35 2 13	1 5 16	4	35 2 24	1 5 27	2	0 2 24	1 3 25	11	13 24 0	1 12 25
(0,14,32,5,23)	5	32 0 14	1 6 20	9	5 14 32	1 10 28	9	23 32 5	1 10 20	5	0 5 23	1 6 24	9	14 23 0	1 10 24
(0,15,29,8,22)	8	29 0 15	1 9 24	7	8 15 29	1 8 22	7	22 29 8	1 8 24	8	0 8 22	1 9 23	7	15 22 0	1 8 23
(0,16,26,11,21)	10	16 26 0	1 11 22	5	11 16 26	1 6 16	5	21 26 11	1 6 28	10	11 21 0	1 11 27	5	16 21 0	1 6 22
(0,17,23,14,20)	6	17 23 0	1 7 21	3	14 17 23	1 4 10	3	20 23 14	1 4 32	6	14 20 0	1 7 24	3	17 20 0	1 4 21
(0,18,20,17,19)	2	18 20 0	1 3 20	1	17 18 20	1 2 4	1	19 20 17	1 2 36	2	17 19 0	1 3 21	1	18 19 0	1 2 20
(b) First half of the reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,5,8,27)	1	0 1 5	1 2 6	3	5 8 1	1 4 34	3	5 8 27	1 4 23	8	0 8 27	1 9 28	1	0 1 27	1 2 28
(0,1,6,3,14)	1	0 1 6	1 2 7	2	1 3 6	1 3 6	3	3 6 14	1 4 12	3	0 3 14	1 4 15	1	0 1 14	1 2 15
(0,1,7,11,29)	1	0 1 7	1 2 8	4	7 11 1	1 5 32	4	7 11 29	1 5 23	8	29 0 11	1 9 20	1	0 1 29	1 2 30
(0,1,8,3,15)	1	0 1 8	1 2 9	2	1 3 8	1 3 8	5	3 8 15	1 6 13	3	0 3 15	1 4 16	1	0 1 15	1 2 16
(0,17,25,22,1)	8	17 25 0	1 9 21	3	22 25 17	1 4 33	3	22 25 1	1 4 17	1	0 1 22	1 2 23	1	0 1 17	1 2 18
(0,2,9,15,25)	2	0 2 9	1 3 10	6	9 15 2	1 7 31	6	9 15 25	1 7 17	10	15 25 0	1 11 23	2	0 2 25	1 3 26
(0,2,17,30,12)	2	0 2 17	1 3 18	9	30 2 17	1 10 25	5	12 17 30	1 6 19	7	30 0 12	1 8 20	2	0 2 12	1 3 13
(0,18,14,28,2)	4	14 18 0	1 5 24	4	14 18 28	1 5 15	11	28 2 14	1 12 24	2	0 2 28	1 3 29	2	0 2 18	1 3 19
(0,4,13,29,9)	4	0 4 13	1 5 14	9	4 13 29	1 10 26	4	9 13 29	1 5 21	8	29 0 9	1 9 18	4	0 4 9	1 5 10
(0,5,11,26,17)	5	0 5 11	1 6 12	6	5 11 26	1 7 22	6	11 17 26	1 7 16	9	17 26 0	1 10 21	5	0 5 17	1 6 18
(0,6,14,4,17)	6	0 6 14	1 7 15	2	4 6 14	1 3 11	3	14 17 4	1 4 28	4	0 4 17	1 5 18	6	0 6 17	1 7 18
(0,7,20,34,18)	7	0 7 20	1 8 21	10	34 7 20	1 11 24	2	18 20 34	1 3 17	3	34 0 18	1 4 22	7	0 7 18	1 8 19
(c) Second half of the reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,19,22,26,27)	3	19 22 0	1 4 19	3	19 22 26	1 4 8	1	26 27 22	1 2 34	1	26 27 0	1 2 12	8	19 27 0	1 9 19
(0,11,8,13,14)	3	8 11 0	1 4 30	2	11 13 8	1 3 35	1	13 14 8	1 2 33	1	13 14 0	1 2 25	3	11 14 0	1 4 27
(0,18,22,28,29)	4	18 22 0	1 5 20	4	18 22 28	1 5 11	1	28 29 22	1 2 32	1	28 29 0	1 2 10	8	29 0 18	1 9 27
(0,12,7,14,15)	5	7 12 0	1 6 31	2	12 14 7	1 3 33	1	14 15 7	1 2 31	1	14 15 0	1 2 24	3	12 15 0	1 4 26
(0,16,13,21,1)	3	13 16 0	1 4 25	3	13 16 21	1 4 9	8	13 21 1	1 9 26	1	0 1 21	1 2 22	1	0 1 16	1 2 17
(0,10,16,23,25)	6	10 16 0	1 7 28	6	10 16 23	1 7 14	2	23 25 16	1 3 31	2	23 25 0	1 3 15	10	0 10 25	1 11 26
(0,19,32,10,12)	5	32 0 19	1 6 25	9	10 19 32	1 10 23	2	10 12 32	1 3 23	2	10 12 0	1 3 28	7	12 19 0	1 8 26
(0,11,25,21,2)	11	0 11 25	1 12 26	4	21 25 11	1 5 28	4	21 25 2	1 5 19	2	0 2 21	1 3 22	2	0 2 11	1 3 12
(0,17,33,5,9)	4	33 0 17	1 5 22	9	33 5 17	1 10 22	4	5 9 33	1 5 29	4	5 9 0	1 5 33	8	9 17 0	1 9 29
(0,28,6,12,17)	6	0 6 28	1 7 29	6	6 12 28	1 7 23	5	12 17 6	1 6 32	5	12 17 0	1 6 26	9	28 0 17	1 10 27
(0,13,3,11,17)	3	0 3 13	1 4 14	2	11 13 3	1 3 30	6	11 17 3	1 7 30	6	11 17 0	1 7 27	4	13 17 0	1 5 25
(0,21,35,11,18)	2	35 0 21	1 3 24	10	11 21 35	1 11 25	7	11 18 35	1 8 25	7	11 18 0	1 8 27	3	18 21 0	1 4 20

Table A14. Cyclic split $\mathcal{C}(3, 5, 40)$ —vertices and distances in base cycles. With red, we highlighted the special base cycles that only have $v/5$ different positions, as $i \mapsto i + v/5$ results in an automorphism. For the rest of the base cycles corresponding to symmetric difference triplets (17 triplets), the meaning of the colors is the following. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. Within each cycle, the pale green and blue colors mark the distances of one half of the reflected triplet pairs. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example. The columns of base cycles (0,2,12,28,10) and (0,30,14,32,2) are highlighted with 5 different colors to mark the connected distances of the reflected triplet pairs. Please note that, in this special case, although the differences and their orders look identical, they belong to different triple types, as it is shown in the corresponding rows of Table A15.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
2 base cycles of a cyclic $\mathcal{C}(3,5,5)$ multiplied by 8																			
0	8	16	24	32	8	8	16	8	8	16	8	8	16	8	8	16	8	8	16
0	16	32	8	24	16	16	8	16	16	8	16	16	8	16	16	8	16	16	8
2 base cycles of a cyclic $\mathcal{C}(3,5,10)$ multiplied by 4																			
0	4	8	20	24	4	4	8	4	12	16	12	4	16	4	16	20	16	4	20
0	8	20	16	28	8	12	20	12	4	8	4	12	8	12	12	16	12	8	20
9 base cycles of a cyclic $\mathcal{C}(3,5,20)$ multiplied by 2 (5 symmetric base cycles and 2 reflected pairs)																			
0	2	36	4	38	2	6	4	6	8	2	8	6	2	6	2	4	2	2	4
0	6	24	16	34	6	18	16	18	8	10	8	18	10	18	6	16	6	6	12
0	6	14	20	10	6	8	14	8	6	14	6	10	4	10	10	20	10	6	4
0	2	16	12	26	2	14	16	14	4	10	4	14	10	14	14	12	14	2	16
0	34	10	4	22	6	16	10	16	6	10	6	18	12	18	18	4	18	6	12
0	2	12	28	10	2	10	12	10	16	14	16	18	2	18	10	12	10	2	8
0	30	14	32	2	10	16	14	16	18	2	18	10	12	10	2	8	2	10	12
0	18	32	12	14	18	14	8	14	20	6	20	2	18	2	14	12	14	18	4
0	20	18	32	14	20	2	18	2	14	12	14	18	4	18	14	8	14	20	6
Symmetric cycles for odd values																			
0	1	6	34	39	1	5	6	5	12	7	12	5	7	5	1	6	1	1	2
0	3	18	22	37	3	15	18	15	4	19	4	15	19	15	3	18	3	3	6
0	5	2	38	35	5	3	2	3	4	7	4	3	7	3	5	2	5	5	10
0	7	2	38	33	7	5	2	5	4	9	4	5	9	5	7	2	7	7	14
0	9	14	26	31	9	5	14	5	12	17	12	5	17	5	9	14	9	9	18
0	11	26	14	29	11	15	14	15	12	3	12	15	3	15	11	14	11	11	18
0	13	38	2	27	13	15	2	15	4	11	4	15	11	15	13	2	13	13	14
0	15	10	30	25	15	5	10	5	20	15	20	5	15	5	15	10	15	15	10
0	17	22	18	23	17	5	18	5	4	1	4	5	1	5	17	18	17	17	6
0	19	34	6	21	19	15	6	15	12	13	12	15	13	15	19	6	19	19	2
Reflected cycles 1st half																			
0	1	3	10	9	1	2	3	2	7	9	7	1	6	1	9	10	9	1	8
0	1	4	9	15	1	3	4	3	5	8	5	6	11	6	15	9	15	1	14
0	1	8	11	17	1	7	8	7	3	10	3	6	9	6	17	11	17	1	16
0	1	11	14	23	1	10	11	10	3	13	3	9	12	9	17	14	17	1	18
0	1	12	3	16	1	11	12	11	9	2	9	13	4	13	16	3	16	1	15
0	1	13	24	21	1	12	13	12	11	17	11	3	8	3	19	16	19	1	20
0	1	14	3	22	1	13	14	13	11	2	11	19	8	19	18	3	18	1	19
0	3	14	7	26	3	11	14	11	7	4	7	19	12	19	14	7	14	3	17
0	5	16	36	19	5	11	16	11	20	9	20	17	3	17	19	4	19	5	14
0	5	21	36	13	5	16	19	16	15	9	15	17	8	17	13	4	13	5	8
0	7	16	35	22	7	9	16	9	19	12	19	13	6	13	18	5	18	7	15
0	7	20	31	13	7	13	20	13	11	16	11	18	7	18	13	9	13	7	6
0	7	24	9	17	7	17	16	17	15	2	15	8	7	8	17	9	17	7	10
0	10	29	2	19	10	19	11	19	13	8	13	17	10	17	19	2	19	10	9

Table A14. Cont.

Reflected cycles 2nd half																			
0	39	6	8	9	1	7	6	7	2	9	2	1	3	1	9	8	9	1	10
0	6	11	14	15	6	5	11	5	3	8	3	1	4	1	15	14	15	6	9
0	6	9	16	17	6	3	9	3	7	10	7	1	8	1	17	16	17	6	11
0	9	12	22	23	9	3	12	3	10	13	10	1	11	1	17	18	17	9	14
0	13	4	15	16	13	9	4	9	11	2	11	1	12	1	16	15	16	13	3
0	37	8	20	21	3	11	8	11	12	17	12	1	13	1	19	20	19	3	16
0	19	8	21	22	19	11	8	11	13	2	13	1	14	1	18	19	18	19	3
0	19	12	23	26	19	7	12	7	11	4	11	3	14	3	14	17	14	19	7
0	23	3	14	19	17	20	3	20	11	9	11	5	16	5	19	14	19	17	4
0	17	32	8	13	17	15	8	15	16	9	16	5	19	5	13	8	13	17	4
0	27	6	15	22	13	19	6	19	9	12	9	7	16	7	18	15	18	13	5
0	22	33	6	13	18	11	7	11	13	16	13	7	20	7	13	6	13	18	9
0	8	33	10	17	8	15	7	15	17	2	17	7	16	7	17	10	17	8	9
0	17	30	9	19	17	13	10	13	19	8	19	10	11	10	19	9	19	17	2

Table A15. $C(3, 5, 40)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) 2 base cycles of a cyclic $C(3, 5, 5)$ multiplied by 8.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
$(0, 8, 16, 24, 32)$	8	0 8 16	1 9 17	8	8 16 24	1 9 17	8	16 24 32	1 9 17	8	24 32 0	1 9 17	8	32 0 8	1 9 17	
$(0, 16, 32, 8, 24)$	8	32 0 16	1 9 25	8	8 16 32	1 9 25	8	24 32 8	1 9 25	8	0 8 24	1 9 25	8	16 24 0	1 9 25	
(b) 2 base cycles of a cyclic $C(3, 5, 10)$ multiplied by 4.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
$(0, 4, 8, 20, 24)$	4	0 4 8	1 5 9	4	4 8 20	1 5 17	4	20 24 8	1 5 29	4	20 24 0	1 5 21	4	0 4 24	1 5 25	
$(0, 8, 20, 16, 28)$	8	0 8 20	1 9 21	4	16 20 8	1 5 33	4	16 20 28	1 5 13	12	16 28 0	1 13 25	8	0 8 28	1 9 29	
(c) 9 base cycles of a cyclic $C(3, 5, 20)$ multiplied by 2 (5 symmetric base cycles and 2 reflected pairs).																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
$(0, 2, 36, 4, 38)$	2	0 2 36	1 3 37	2	2 4 36	1 3 35	2	36 38 4	1 3 9	2	38 0 4	1 3 7	2	38 0 2	1 3 5	
$(0, 6, 24, 16, 34)$	6	0 6 24	1 7 25	8	16 24 6	1 9 31	8	16 24 34	1 9 19	6	34 0 16	1 7 23	6	34 0 6	1 7 13	
$(0, 6, 14, 20, 10)$	6	0 6 14	1 7 15	6	14 20 6	1 7 33	4	10 14 20	1 5 11	10	0 10 20	1 11 21	4	6 10 0	1 5 35	
$(0, 2, 16, 12, 26)$	2	0 2 16	1 3 17	4	12 16 2	1 5 31	4	12 16 26	1 5 15	12	0 12 26	1 13 27	2	0 2 26	1 3 27	
$(0, 34, 10, 4, 22)$	6	34 0 10	1 7 17	6	4 10 34	1 7 31	6	4 10 22	1 7 19	4	0 4 22	1 5 23	6	34 0 22	1 7 29	
$(0, 2, 12, 28, 10)$	2	0 2 12	1 3 13	10	2 12 28	1 11 27	2	10 12 28	1 3 19	10	0 10 28	1 11 29	2	0 2 10	1 3 11	
$(0, 30, 14, 32, 2)$	10	30 0 14	1 11 25	2	30 32 14	1 3 25	10	32 2 14	1 11 23	2	0 2 32	1 3 33	2	0 2 30	1 3 31	
$(0, 18, 32, 12, 14)$	8	32 0 18	1 9 27	6	12 18 32	1 7 21	2	12 14 32	1 3 21	2	12 14 0	1 3 29	4	14 18 0	1 5 27	
$(0, 20, 18, 32, 14)$	2	18 20 0	1 3 23	2	18 20 32	1 3 15	4	14 18 32	1 5 19	8	32 0 14	1 9 23	6	14 20 0	1 7 27	
(d) Symmetric cycles for odd values.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
$(0, 1, 6, 34, 39)$	1	0 1 6	1 2 7	5	1 6 34	1 6 34	5	34 39 6	1 6 13	1	39 0 34	1 2 36	1	39 0 1	1 2 3	
$(0, 3, 18, 22, 37)$	3	0 3 18	1 4 19	4	18 22 3	1 5 26	4	18 22 37	1 5 20	3	37 0 22	1 4 26	3	37 0 3	1 4 7	
$(0, 5, 2, 38, 35)$	2	0 2 5	1 3 6	3	2 5 38	1 4 37	3	35 38 2	1 4 8	2	38 0 35	1 3 38	5	35 0 5	1 6 11	
$(0, 7, 2, 38, 33)$	2	0 2 7	1 3 8	4	38 2 7	1 5 10	4	38 2 33	1 5 36	2	38 0 33	1 3 36	7	33 0 7	1 8 15	
$(0, 9, 14, 26, 31)$	5	9 14 0	1 6 32	5	9 14 26	1 6 18	5	26 31 14	1 6 29	5	26 31 0	1 6 15	9	31 0 9	1 10 19	
$(0, 11, 26, 14, 29)$	11	0 11 26	1 12 27	3	11 14 26	1 4 16	3	26 29 14	1 4 29	11	29 0 14	1 12 26	11	29 0 11	1 12 23	
$(0, 13, 38, 2, 27)$	2	38 0 13	1 3 16	4	38 2 13	1 5 16	4	38 2 27	1 5 30	2	0 2 27	1 3 28	13	27 0 13	1 14 27	
$(0, 15, 10, 30, 25)$	5	10 15 0	1 6 31	5	10 15 30	1 6 21	5	25 30 10	1 6 26	5	25 30 0	1 6 16	10	15 25 0	1 11 26	
$(0, 17, 22, 18, 23)$	5	17 22 0	1 6 24	1	17 18 22	1 2 6	1	22 23 18	1 2 37	5	18 23 0	1 6 23	6	17 23 0	1 7 24	
$(0, 19, 34, 6, 21)$	6	34 0 19	1 7 26	12	34 6 19	1 13 26	12	34 6 21	1 13 28	6	0 6 21	1 7 22	2	19 21 0	1 3 22	
(e) First half of the reflected base cycle pairs.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
$(0, 1, 3, 10, 9)$	1	0 1 3	1 2 4	2	1 3 10	1 3 10	1	9 10 3	1 2 35	1	9 10 0	1 2 32	1	0 1 9	1 2 10	
$(0, 1, 4, 9, 15)$	1	0 1 4	1 2 5	3	1 4 9	1 4 9	5	4 9 15	1 6 12	6	9 15 0	1 7 32	1	0 1 15	1 2 16	
$(0, 1, 8, 11, 17)$	1	0 1 8	1 2 9	3	8 11 1	1 4 34	3	8 11 17	1 4 10	6	11 17 0	1 7 30	1	0 1 17	1 2 18	
$(0, 1, 11, 14, 23)$	1	0 1 11	1 2 12	3	11 14 1	1 4 31	3	11 14 23	1 4 13	9	14 23 0	1 10 27	1	0 1 23	1 2 24	
$(0, 1, 12, 3, 16)$	1	0 1 12	1 2 13	2	1 12 3	1 3 12	4	12 16 3	1 5 32	3	0 3 16	1 4 17	1	0 1 16	1 2 17	
$(0, 1, 13, 24, 21)$	1	0 1 13	1 2 14	11	13 24 1	1 12 29	3	21 24 13	1 4 33	3	21 24 0	1 4 20	1	0 1 21	1 2 22	
$(0, 1, 14, 3, 22)$	1	0 1 14	1 2 15	2	1 14 3	1 3 14	8	14 22 3	1 9 30	3	0 3 22	1 4 23	1	0 1 22	1 2 23	
$(0, 3, 14, 7, 26)$	3	0 3 14	1 4 15	4	3 7 14	1 5 12	7	7 14 26	1 8 20	7	0 7 26	1 8 27	3	0 3 26	1 4 27	
$(0, 5, 16, 36, 19)$	5	0 5 16	1 6 17	9	36 5 16	1 10 21	3	16 19 36	1 4 21	4	36 0 19	1 5 24	5	0 5 19	1 6 20	
$(0, 5, 21, 36, 13)$	5	0 5 21	1 6 22	9	36 5 21	1 10 26	8	13 21 36	1 9 24	4	36 0 13	1 5 18	5	0 5 13	1 6 14	
$(0, 7, 16, 35, 22)$	7	0 7 16	1 8 17	9	7 16 35	1 10 29	6	16 22 35	1 7 20	5	35 0 22	1 6 28	7	0 7 22	1 8 23	
$(0, 7, 20, 31, 13)$	7	0 7 20	1 8 21	11	20 31 7	1 12 28	7	13 20 31	1 8 19	9	31 0 13	1 10 23	6	7 13 0	1 7 34	
$(0, 7, 24, 9, 17)$	7	0 7 24	1 8 25	2	7 24 9	1 3 18	7	17 24 9	1 8 33	8	9 17 0	1 9 32	7	0 7 17	1 8 18	
$(0, 10, 29, 2, 19)$	10	0 10 29	1 11 30	8	2 10 29	1 9 28	10	19 29 2	1 11 24	2	0 2 19	1 3 20	9	10 19 0	1 10 31	

Table A15. Cont.

(f) Second half of the reflected base cycle pairs.															
Cycles		(x ₁ ,x ₂ ,x ₃)		(x ₂ ,x ₃ ,x ₄)		(x ₃ ,x ₄ ,x ₅)		(x ₄ ,x ₅ ,x ₁)		(x ₅ ,x ₁ ,x ₂)					
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triplet	type	d	triplet	type	d	triplet	type	d	triplet	type	d	triplet	type
(0,39,6,8,9)	1	39 0 6	1 2 8	2	6 8 39	1 3 34	1	8 9 6	1 2 39	1	8 9 0	1 2 33	1	39 0 9	1 2 11
(0,6,11,14,15)	5	6 11 0	1 6 35	3	11 14 6	1 4 36	1	14 15 11	1 2 38	1	14 15 0	1 2 27	6	0 6 15	1 7 16
(0,6,9,16,17)	3	6 9 0	1 4 35	3	6 9 16	1 4 11	1	16 17 9	1 2 34	1	16 17 0	1 2 25	6	0 6 17	1 7 18
(0,9,12,22,23)	3	9 12 0	1 4 32	3	9 12 22	1 4 14	1	22 23 12	1 2 31	1	22 23 0	1 2 19	9	0 9 23	1 10 24
(0,13,4,15,16)	4	0 4 13	1 5 14	2	13 15 4	1 3 32	1	15 16 4	1 2 30	1	15 16 0	1 2 26	3	13 16 0	1 4 28
(0,37,8,20,21)	3	37 0 8	1 4 12	11	37 8 20	1 12 24	1	20 21 8	1 2 29	1	20 21 0	1 2 21	3	37 0 21	1 4 25
(0,19,8,21,22)	8	0 8 19	1 9 20	2	19 21 8	1 3 30	1	21 22 8	1 2 28	1	21 22 0	1 2 20	3	19 22 0	1 4 22
(0,19,12,23,26)	7	12 19 0	1 8 29	4	19 23 12	1 5 34	3	23 26 12	1 4 30	3	23 26 0	1 4 18	7	19 26 0	1 8 22
(0,23,3,14,19)	3	0 3 23	1 4 24	9	14 23 3	1 10 30	5	14 19 3	1 6 30	5	14 19 0	1 6 27	4	19 23 0	1 5 22
(0,17,32,8,13)	8	32 0 17	1 9 26	9	8 17 32	1 10 25	5	8 13 32	1 6 25	5	8 13 0	1 6 33	4	13 17 0	1 5 28
(0,27,6,15,22)	6	0 6 27	1 7 28	9	6 15 27	1 10 22	7	15 22 6	1 8 32	7	15 22 0	1 8 26	5	22 27 0	1 6 19
(0,22,33,6,13)	7	33 0 22	1 8 30	11	22 33 6	1 12 25	7	6 13 33	1 8 28	6	0 6 13	1 7 14	9	13 22 0	1 10 28
(0,8,33,10,17)	7	33 0 8	1 8 16	2	8 10 33	1 3 26	7	10 17 33	1 8 24	7	10 17 0	1 8 31	8	0 8 17	1 9 18
(0,17,30,9,19)	10	30 0 17	1 11 28	8	9 17 30	1 9 22	10	9 19 30	1 11 22	9	0 9 19	1 10 20	2	17 19 0	1 3 24

Table A16. Cyclic $\mathcal{C}(3, 5, 41)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

Base Cycles (5 Vertices)					Distances in the Difference Triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
Symmetric base cycles																			
0	1	38	3	40	1	4	3	4	6	2	6	4	2	4	1	3	1	1	2
0	2	35	6	39	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	3	32	9	38	3	12	9	12	18	6	18	12	6	12	3	9	3	3	6
0	4	29	12	37	4	16	12	16	17	8	17	16	8	16	4	12	4	4	8
0	5	26	15	36	5	20	15	20	11	10	11	20	10	20	5	15	5	5	10
0	6	23	18	35	6	17	18	17	5	12	5	17	12	17	6	18	6	6	12
0	7	20	21	34	7	13	20	13	1	14	1	13	14	13	7	20	7	7	14
0	8	17	24	33	8	9	17	9	7	16	7	9	16	9	8	17	8	8	16
0	9	14	27	32	9	5	14	5	13	18	13	5	18	5	9	14	9	9	18
0	10	11	30	31	10	1	11	1	19	20	19	1	20	1	10	11	10	10	20
0	11	8	33	30	11	3	8	3	16	19	16	3	19	3	11	8	11	11	19
0	12	5	36	29	12	7	5	7	10	17	10	7	17	7	12	5	12	12	17
0	13	2	39	28	13	11	2	11	4	15	4	11	15	11	13	2	13	13	15
0	14	40	1	27	14	15	1	15	2	13	2	15	13	15	14	1	14	14	13
0	15	37	4	26	15	19	4	19	8	11	8	19	11	19	15	4	15	15	11
0	16	34	7	25	16	18	7	18	14	9	14	18	9	18	16	7	16	16	9
0	17	31	10	24	17	14	10	14	20	7	20	14	7	14	17	10	17	17	7
0	18	28	13	23	18	10	13	10	15	5	15	10	5	10	18	13	18	18	5
0	19	25	16	22	19	6	16	6	9	3	9	6	3	6	19	16	19	19	3
0	20	22	19	21	20	2	19	2	3	1	3	2	1	2	20	19	20	20	1

Table A16. Cont.

Reflected cycles 1st half																			
0	1	5	8	10	1	4	5	4	3	7	3	2	5	2	10	8	10	1	9
0	1	6	8	18	1	5	6	5	2	7	2	10	12	10	18	8	18	1	17
0	1	7	11	16	1	6	7	6	4	10	4	5	9	5	16	11	16	1	15
0	1	8	10	19	1	7	8	7	2	9	2	9	11	9	19	10	19	1	18
0	1	9	4	17	1	8	9	8	5	3	5	13	8	13	17	4	17	1	16
0	1	12	7	29	1	11	12	11	5	6	5	19	17	19	12	7	12	1	13
0	2	14	23	27	2	12	14	12	9	20	9	4	13	4	14	18	14	2	16
0	2	17	5	25	2	15	17	15	12	3	12	20	8	20	16	5	16	2	18
0	3	16	33	27	3	13	16	13	17	11	17	6	11	6	14	8	14	3	17
0	3	30	14	10	3	14	11	14	16	11	16	4	20	4	10	14	10	3	7
0	6	13	2	20	6	7	13	7	11	4	11	18	7	18	20	2	20	6	14
0	6	19	38	21	6	13	19	13	19	9	19	17	2	17	20	3	20	6	15
0	7	15	35	26	7	8	15	8	20	13	20	9	11	9	15	6	15	7	19
0	8	22	3	26	8	14	19	14	19	5	19	18	4	18	15	3	15	8	18
0	10	22	35	25	10	12	19	12	13	16	13	10	3	10	16	6	16	10	15
0	26	9	30	12	15	17	9	17	20	4	20	18	3	18	12	11	12	15	14
Reflected cycles 2nd half																			
0	2	5	9	10	2	3	5	3	4	7	4	1	5	1	10	9	10	2	8
0	10	12	17	18	10	2	12	2	5	7	5	1	6	1	18	17	18	10	8
0	5	9	15	16	5	4	9	4	6	10	6	1	7	1	16	15	16	5	11
0	9	11	18	19	9	2	11	2	7	9	7	1	8	1	19	18	19	9	10
0	13	8	16	17	13	5	8	5	8	3	8	1	9	1	17	16	17	13	4
0	22	17	28	29	19	5	17	5	11	6	11	1	12	1	12	13	12	19	7
0	4	13	25	27	4	9	13	9	12	20	12	2	14	2	14	16	14	4	18
0	20	8	23	25	20	12	8	12	15	3	15	2	17	2	16	18	16	20	5
0	35	11	24	27	6	17	11	17	13	11	13	3	16	3	14	17	14	6	8
0	37	21	7	10	4	16	20	16	14	11	14	3	11	3	10	7	10	4	14
0	18	7	14	20	18	11	7	11	7	4	7	6	13	6	20	14	20	18	2
0	24	2	15	21	17	19	2	19	13	9	13	6	19	6	20	15	20	17	3
0	32	11	19	26	9	20	11	20	8	13	8	7	15	7	15	19	15	9	6
0	23	4	18	26	18	19	4	19	14	5	14	8	19	8	15	18	15	18	3
0	31	3	15	25	10	13	3	13	12	16	12	10	19	10	16	15	16	10	6
0	23	3	27	12	18	20	3	20	17	4	17	15	9	15	12	14	12	18	11

Table A17. $\mathcal{C}(3, 5, 41)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.																
Cycles		(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,38,3,40)	1	0 1 38	1 2 39	2	1 3 38	1 3 38	2	38 40 3	1 3 7	1	40 0 3	1 2 5	1	40 0 1	1 2 3	
(0,2,35,6,39)	2	0 2 35	1 3 36	4	2 6 35	1 5 34	4	35 39 6	1 5 13	2	39 0 6	1 3 9	2	39 0 2	1 3 5	
(0,3,32,9,38)	3	0 3 32	1 4 33	6	3 9 32	1 7 30	6	32 38 9	1 7 19	3	38 0 9	1 4 13	3	38 0 3	1 4 7	
(0,4,29,12,37)	4	0 4 29	1 5 30	8	4 12 29	1 9 26	8	29 37 12	1 9 25	4	37 0 12	1 5 17	4	37 0 4	1 5 9	
(0,5,26,15,36)	5	0 5 26	1 6 27	10	5 15 26	1 11 22	10	26 36 15	1 11 31	5	36 0 15	1 6 21	5	36 0 5	1 6 11	
(0,6,23,18,35)	6	0 6 23	1 7 24	5	18 23 6	1 6 30	5	18 23 35	1 6 18	6	35 0 18	1 7 25	6	35 0 6	1 7 13	
(0,7,20,21,34)	7	0 7 20	1 8 21	1	20 21 7	1 2 29	1	20 21 34	1 2 15	7	34 0 21	1 8 29	7	34 0 7	1 8 15	
(0,8,17,24,33)	8	0 8 17	1 9 18	7	17 24 8	1 8 33	7	17 24 33	1 8 17	8	33 0 24	1 9 33	8	33 0 8	1 9 17	
(0,9,14,27,32)	5	9 14 0	1 6 33	5	9 14 27	1 6 19	5	27 32 14	1 6 29	5	27 32 0	1 6 15	9	32 0 9	1 10 19	
(0,10,11,30,31)	1	10 11 0	1 2 32	1	10 11 30	1 2 21	1	30 31 11	1 2 23	1	30 31 0	1 2 12	10	31 0 10	1 11 21	
(0,11,8,33,30)	3	8 11 0	1 4 34	3	8 11 33	1 4 26	3	30 33 8	1 4 20	3	30 33 0	1 4 12	11	30 0 11	1 12 23	
(0,12,5,36,29)	5	0 5 12	1 6 13	7	5 12 36	1 8 32	7	29 36 5	1 8 18	5	36 0 29	1 6 35	12	29 0 12	1 13 25	
(0,13,2,39,28)	2	0 2 13	1 3 14	4	39 2 13	1 5 16	4	39 2 28	1 5 31	2	39 0 28	1 3 31	13	28 0 13	1 14 27	
(0,14,0,1,27)	1	40 0 14	1 2 16	2	40 1 14	1 3 16	2	40 1 27	1 3 29	1	0 1 27	1 2 28	13	14 27 0	1 14 28	
(0,15,37,4,26)	4	37 0 15	1 5 20	8	37 4 15	1 9 20	8	37 4 26	1 9 31	4	0 4 26	1 5 27	11	15 26 0	1 12 27	
(0,16,34,7,25)	7	34 0 16	1 8 24	9	7 16 34	1 10 28	9	25 34 7	1 10 24	7	0 7 25	1 8 26	9	16 25 0	1 10 26	
(0,17,31,10,24)	10	31 0 17	1 11 28	7	10 17 31	1 8 22	7	24 31 10	1 8 28	10	0 10 24	1 11 25	7	17 24 0	1 8 25	
(0,18,28,13,23)	10	18 28 0	1 11 24	5	13 18 28	1 6 16	5	23 28 13	1 6 32	10	13 23 0	1 11 29	5	18 23 0	1 6 24	
(0,19,25,16,22)	6	19 25 0	1 7 23	3	16 19 25	1 4 10	3	22 25 16	1 4 36	6	16 22 0	1 7 26	3	19 22 0	1 4 23	
(0,20,22,19,21)	2	20 22 0	1 3 22	1	19 20 22	1 2 4	1	21 22 19	1 2 40	2	19 21 0	1 3 23	1	20 21 0	1 2 22	
(b) First half of the reflected base cycle pairs.																
Cycles		(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,5,8,10)	1	0 1 5	1 2 6	3	5 8 1	1 4 38	2	8 10 5	1 3 39	2	8 10 0	1 3 34	1	0 1 10	1 2 11	
(0,1,6,8,18)	1	0 1 6	1 2 7	2	6 8 1	1 3 37	2	6 8 18	1 3 13	8	0 8 18	1 9 19	1	0 1 18	1 2 19	
(0,1,7,11,16)	1	0 1 7	1 2 8	4	7 11 1	1 5 36	4	7 11 16	1 5 10	5	11 16 0	1 6 31	1	0 1 16	1 2 17	
(0,1,8,10,19)	1	0 1 8	1 2 9	2	8 10 1	1 3 35	2	8 10 19	1 3 12	9	10 19 0	1 10 32	1	0 1 19	1 2 20	
(0,1,9,4,17)	1	0 1 9	1 2 10	3	1 4 9	1 4 9	5	4 9 17	1 6 14	4	0 4 17	1 5 18	1	0 1 17	1 2 18	
(0,1,12,7,29)	1	0 1 12	1 2 13	5	7 12 1	1 6 36	5	7 12 29	1 6 23	7	0 7 29	1 8 30	1	0 1 29	1 2 30	
(0,2,14,23,27)	2	0 2 14	1 3 15	9	14 23 2	1 10 30	4	23 27 14	1 5 33	4	23 27 0	1 5 19	2	0 2 27	1 3 28	
(0,2,17,5,25)	2	0 2 17	1 3 18	3	2 5 17	1 4 16	8	17 25 5	1 9 30	5	0 5 25	1 6 26	2	0 2 25	1 3 26	
(0,3,16,33,27)	3	0 3 16	1 4 17	11	33 16 3	1 12 25	6	27 33 16	1 7 31	6	27 33 0	1 7 15	3	0 3 27	1 4 28	
(0,3,30,14,10)	3	0 3 30	1 4 31	11	3 14 30	1 12 28	4	10 14 30	1 5 21	4	10 14 0	1 5 32	3	0 3 10	1 4 11	
(0,6,13,2,20)	6	0 6 13	1 7 14	4	2 6 13	1 5 12	7	13 20 2	1 8 31	2	0 2 20	1 3 21	6	0 6 20	1 7 21	
(0,6,19,38,21)	6	0 6 19	1 7 20	9	38 6 19	1 10 23	2	19 21 38	1 3 20	3	38 0 21	1 4 25	6	0 6 21	1 7 22	
(0,7,15,35,26)	7	0 7 15	1 8 16	8	7 15 35	1 9 29	9	26 35 15	1 10 31	6	35 0 26	1 7 33	7	0 7 26	1 8 27	
(0,8,22,3,26)	8	0 8 22	1 9 23	5	3 8 22	1 6 20	4	22 26 3	1 5 23	3	0 3 26	1 4 27	8	0 8 26	1 9 27	
(0,10,22,35,25)	10	0 10 22	1 11 23	12	10 22 35	1 13 26	3	22 25 35	1 4 14	6	35 0 25	1 7 32	10	0 10 25	1 11 26	
(0,26,9,30,12)	9	0 9 26	1 10 27	4	26 30 9	1 5 25	3	9 12 30	1 4 22	11	30 0 12	1 12 24	12	0 12 26	1 13 27	
(c) Second half of the reflected base cycle pairs.																
Cycles		(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,2,5,9,10)	2	0 2 5	1 3 6	3	2 5 9	1 4 8	1	9 10 5	1 2 38	1	9 10 0	1 2 33	2	0 2 10	1 3 11	
(0,10,12,17,18)	2	10 12 0	1 3 32	2	10 12 17	1 3 8	1	17 18 12	1 2 37	1	17 18 0	1 2 25	8	10 18 0	1 9 32	
(0,5,9,15,16)	4	5 9 0	1 5 37	4	5 9 15	1 5 11	1	15 16 9	1 2 36	1	15 16 0	1 2 27	5	0 5 16	1 6 17	
(0,9,11,18,19)	2	9 11 0	1 3 33	2	9 11 18	1 3 10	1	18 19 11	1 2 35	1	18 19 0	1 2 24	9	0 9 19	1 10 20	
(0,13,8,16,17)	5	8 13 0	1 6 34	3	13 16 8	1 4 37	1	16 17 8	1 2 34	1	16 17 0	1 2 26	4	13 17 0	1 5 29	
(0,22,17,28,29)	5	17 22 0	1 6 25	5	17 22 28	1 6 12	1	28 29 17	1 2 31	1	28 29 0	1 2 14	7	22 29 0	1 8 20	
(0,4,13,25,27)	4	0 4 13	1 5 14	9	4 13 25	1 10 22	2	25 27 13	1 3 30	2	25 27 0	1 3 17	4	0 4 27	1 5 28	
(0,20,8,23,25)	8	0 8 20	1 9 21	3	20 23 8	1 4 30	2	23 25 8	1 3 27	2	23 25 0	1 3 19	5	20 25 0	1 6 22	
(0,35,11,24,27)	6	35 0 11	1 7 18	11	24 35 11	1 12 29	3	24 27 11	1 4 29	3	24 27 0	1 4 18	6	35 0 27	1 7 34	
(0,37,21,7,10)	4	37 0 21	1 5 26	11	37 7 21	1 12 26	3	7 10 21	1 4 15	3	7 10 0	1 4 35	4	37 0 10	1 5 15	
(0,18,7,14,20)	7	0 7 18	1 8 19	4	14 18 7	1 5 35	6	14 20 7	1 7 35	6	14 20 0	1 7 28	2	18 20 0	1 3 24	
(0,24,2,15,21)	2	0 2 24	1 3 25	9	15 24 2	1 10 29	6	15 21 2	1 7 29	6	15 21 0	1 7 27	3	21 24 0	1 4 21	
(0,32,11,19,26)	9	32 0 11	1 10 21	8	11 19 32	1 9 22	7	19 26 11	1 8 34	7	19 26 0	1 8 23	6	26 32 0	1 7 16	
(0,23,4,18,26)	4	0 4 23	1 5 24	5	18 23 4	1 6 28	8	18 26 4	1 9 28	8	18 26 0	1 9 24	3	23 26 0	1 4 19	
(0,31,3,15,25)	3	0 3 31	1 4 32	12	3 15 31	1 13 29	10	15 25 3	1 11 30	10	15 25 0	1 11 27	6	25 31 0	1 7 17	
(0,23,3,27,12)	3	0 3 23	1 4 24	4	23 27 3	1 5 22	9	3 12 27	1 10 25	12	0 12 27	1 13 28	11	12 23 0	1 12 30	

Table A18. Cyclic $\mathcal{C}(3, 5, 46)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

(a) Cyclic $\mathcal{C}(3, 5, 46)$ —symmetric base cycles.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	44	2	45	1	3	2	3	4	1	4	3	1	3	1	2	1	1	2
0	2	42	4	44	2	6	4	6	8	2	8	6	2	6	2	4	2	2	4
0	3	40	6	43	3	9	6	9	12	3	12	9	3	9	3	6	3	3	6
0	4	38	8	42	4	12	8	12	16	4	16	12	4	12	4	8	4	4	8
0	5	36	10	41	5	15	10	15	20	5	20	15	5	15	5	10	5	5	10
0	6	34	12	40	6	18	12	18	22	6	22	18	6	18	6	12	6	6	12
0	7	32	14	39	7	21	14	21	18	7	18	21	7	21	7	14	7	7	14
0	8	30	16	38	8	22	16	22	14	8	14	22	8	22	8	16	8	8	16
0	9	28	18	37	9	19	18	19	10	9	10	19	9	19	9	18	9	9	18
0	10	26	20	36	10	16	20	16	6	10	6	16	10	16	10	20	10	10	20
0	11	24	22	35	11	13	22	13	2	11	2	13	11	13	11	22	11	11	22
0	12	22	24	34	12	10	22	10	2	12	2	10	12	10	12	22	12	12	22
0	13	20	26	33	13	7	20	7	6	13	6	7	13	7	13	20	13	13	20
0	14	18	28	32	14	4	18	4	10	14	10	4	14	4	14	18	14	14	18
0	15	16	30	31	15	1	16	1	14	15	14	1	15	1	15	16	15	15	16
0	16	14	32	30	16	2	14	2	18	16	18	2	16	2	16	14	16	16	14
0	17	12	34	29	17	5	12	5	22	17	22	5	17	5	17	12	17	17	12
0	18	10	36	28	18	8	10	8	20	18	20	8	18	8	18	10	18	18	10
0	19	8	38	27	19	11	8	11	16	19	16	11	19	11	19	8	19	19	8
0	20	6	40	26	20	14	6	14	12	20	12	14	20	14	20	6	20	20	6
0	21	4	42	25	21	17	4	17	8	21	8	17	21	17	21	4	21	21	4
0	22	2	44	24	22	20	2	20	4	22	4	20	22	20	22	2	22	22	2
(b) Cyclic $\mathcal{C}(3, 5, 46)$ —first half of the reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	5	8	10	1	4	5	4	3	7	3	2	5	2	10	8	10	1	9
0	1	6	8	17	1	5	6	5	2	7	2	9	11	9	17	8	17	1	16
0	1	7	11	18	1	6	7	6	4	10	4	7	11	7	18	11	18	1	17
0	1	8	10	23	1	7	8	7	2	9	2	13	15	13	23	10	23	1	22
0	1	9	4	13	1	8	9	8	5	3	5	9	4	9	13	4	13	1	12
0	1	11	4	19	1	10	11	10	7	3	7	15	8	15	19	4	19	1	18
0	1	12	4	27	1	11	12	11	8	3	8	23	15	23	19	4	19	1	20
0	1	14	9	21	1	13	14	13	5	8	5	12	7	12	21	9	21	1	20
0	2	14	17	27	2	12	14	12	3	15	3	10	13	10	19	17	19	2	21
0	2	17	6	23	2	15	17	15	11	4	11	17	6	17	23	6	23	2	21
0	2	19	5	28	2	17	19	17	14	3	14	23	9	23	18	5	18	2	20
0	3	14	9	25	3	11	14	11	5	6	5	16	11	16	21	9	21	3	22
0	3	16	25	30	3	13	16	13	9	22	9	5	14	5	16	21	16	3	19
0	3	18	40	21	3	15	18	15	22	9	22	19	3	19	21	6	21	3	18
0	3	20	35	26	3	17	20	17	15	14	15	9	6	9	20	11	20	3	23
0	4	22	35	17	4	18	22	18	13	15	13	18	5	18	17	11	17	4	13
0	5	19	40	27	5	14	19	14	21	11	21	13	8	13	19	6	19	5	22
0	7	19	42	26	7	12	19	12	23	11	23	16	7	16	20	4	20	7	19
0	7	24	33	17	7	17	22	17	9	20	9	16	7	16	17	13	17	7	10
0	8	20	36	14	8	12	20	12	16	18	16	22	6	22	14	10	14	8	6
0	26	5	36	11	20	21	5	21	15	10	15	21	6	21	11	10	11	20	15
0	27	12	34	13	19	15	12	15	22	7	22	21	1	21	13	12	13	19	14

Table A18. Cont.

(c) Cyclic $\mathcal{C}(3, 5, 46)$ —second half of the reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
0	2	5	9	10	2	3	5	3	4	7	4	1	5	1	10	9	10	2	8
0	9	11	16	17	9	2	11	2	5	7	5	1	6	1	17	16	17	9	8
0	7	11	17	18	7	4	11	4	6	10	6	1	7	1	18	17	18	7	11
0	13	15	22	23	13	2	15	2	7	9	7	1	8	1	23	22	23	13	10
0	9	4	12	13	9	5	4	5	8	3	8	1	9	1	13	12	13	9	4
0	15	8	18	19	15	7	8	7	10	3	10	1	11	1	19	18	19	15	4
0	23	15	26	27	23	8	15	8	11	3	11	1	12	1	19	20	19	23	4
0	12	7	20	21	12	5	7	5	13	8	13	1	14	1	21	20	21	12	9
0	10	13	25	27	10	3	13	3	12	15	12	2	14	2	19	21	19	10	17
0	17	6	21	23	17	11	6	11	15	4	15	2	17	2	23	21	23	17	6
0	23	9	26	28	23	14	9	14	17	3	17	2	19	2	18	20	18	23	5
0	16	11	22	25	16	5	11	5	11	6	11	3	14	3	21	22	21	16	9
0	5	14	27	30	5	9	14	9	13	22	13	3	16	3	16	19	16	5	21
0	27	3	18	21	19	22	3	22	15	9	15	3	18	3	21	18	21	19	6
0	37	6	23	26	9	15	6	15	17	14	17	3	20	3	20	23	20	9	11
0	28	41	13	17	18	13	5	13	18	15	18	4	22	4	17	13	17	18	11
0	33	8	22	27	13	21	8	21	14	11	14	5	19	5	19	22	19	13	6
0	30	7	19	26	16	23	7	23	12	11	12	7	19	7	20	19	20	16	4
0	30	39	10	17	16	9	7	9	17	20	17	7	22	7	17	10	17	16	13
0	24	40	6	14	22	16	6	16	12	18	12	8	20	8	14	6	14	22	10
0	21	6	31	11	21	15	6	15	21	10	21	20	5	20	11	15	11	21	10
0	25	1	32	13	21	22	1	22	15	7	15	19	12	19	13	14	13	21	12

Table A19. $\mathcal{C}(3, 5, 46)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,44,2,45)	1	0 1 44	1 2 45	1	1 2 44	1 2 44	1	44 45 2	1 2 5	1	45 0 2	1 2 4	1	45 0 1	1 2 3	
(0,2,42,4,44)	2	0 2 42	1 3 43	2	2 4 42	1 3 41	2	42 44 4	1 3 9	2	44 0 4	1 3 7	2	44 0 2	1 3 5	
(0,3,40,6,43)	3	0 3 40	1 4 41	3	3 6 40	1 4 38	3	40 43 6	1 4 13	3	43 0 6	1 4 10	3	43 0 3	1 4 7	
(0,4,38,8,42)	4	0 4 38	1 5 39	4	4 8 38	1 5 35	4	38 42 8	1 5 17	4	42 0 8	1 5 13	4	42 0 4	1 5 9	
(0,5,36,10,41)	5	0 5 36	1 6 37	5	5 10 36	1 6 32	5	36 41 10	1 6 21	5	41 0 10	1 6 16	5	41 0 5	1 6 11	
(0,6,34,12,40)	6	0 6 34	1 7 35	6	6 12 34	1 7 29	6	34 40 12	1 7 25	6	40 0 12	1 7 19	6	40 0 6	1 7 13	
(0,7,32,14,39)	7	0 7 32	1 8 33	7	7 14 32	1 8 26	7	32 39 14	1 8 29	7	39 0 14	1 8 22	7	39 0 7	1 8 15	
(0,8,30,16,38)	8	0 8 30	1 9 31	8	8 16 30	1 9 23	8	30 38 16	1 9 33	8	38 0 16	1 9 25	8	38 0 8	1 9 17	
(0,9,28,18,37)	9	0 9 28	1 10 29	9	9 18 28	1 10 20	9	28 37 18	1 10 37	9	37 0 18	1 10 28	9	37 0 9	1 10 19	
(0,10,26,20,36)	10	0 10 26	1 11 27	6	20 26 10	1 7 37	6	20 26 36	1 7 17	10	36 0 20	1 11 31	10	36 0 10	1 11 21	
(0,11,24,22,35)	11	0 11 24	1 12 25	2	22 24 11	1 3 36	2	22 24 35	1 3 14	11	35 0 22	1 12 34	11	35 0 11	1 12 23	
(0,12,22,24,34)	10	12 22 0	1 11 35	2	22 24 12	1 3 37	2	22 24 34	1 3 13	10	24 34 0	1 11 23	12	34 0 12	1 13 25	
(0,13,20,26,33)	7	13 20 0	1 8 34	6	20 26 13	1 7 40	6	20 26 33	1 7 14	7	26 33 0	1 8 21	13	33 0 13	1 14 27	
(0,14,18,28,32)	4	14 18 0	1 5 33	4	14 18 28	1 5 15	4	28 32 18	1 5 37	4	28 32 0	1 5 19	14	32 0 14	1 15 29	
(0,15,16,30,31)	1	15 16 0	1 2 32	1	15 16 30	1 2 16	1	30 31 16	1 2 33	1	30 31 0	1 2 17	15	31 0 15	1 16 31	
(0,16,14,32,30)	2	14 16 0	1 3 33	2	14 16 32	1 3 19	2	30 32 14	1 3 31	2	30 32 0	1 3 17	14	16 30 0	1 15 31	
(0,17,12,34,29)	5	12 17 0	1 6 35	5	12 17 34	1 6 23	5	29 34 12	1 6 30	5	29 34 0	1 6 18	12	17 29 0	1 13 30	
(0,18,10,36,28)	8	10 18 0	1 9 37	8	10 18 36	1 9 27	8	28 36 10	1 9 29	8	28 36 0	1 9 19	10	18 28 0	1 11 29	
(0,19,8,38,27)	8	0 8 19	1 9 20	11	8 19 38	1 12 31	11	27 38 8	1 12 28	8	38 0 27	1 9 36	8	19 27 0	1 9 28	
(0,20,6,40,26)	6	0 6 20	1 7 21	12	40 6 20	1 13 27	12	40 6 26	1 13 33	6	40 0 26	1 7 33	6	20 26 0	1 7 27	
(0,21,4,42,25)	4	0 4 21	1 5 22	8	42 4 21	1 9 26	8	42 4 25	1 9 30	4	42 0 25	1 5 30	4	21 25 0	1 5 26	
(0,22,2,44,24)	2	0 2 22	1 3 23	4	44 2 22	1 5 25	4	44 2 24	1 5 27	2	44 0 24	1 3 27	2	22 24 0	1 3 25	

Table A19. Cont.

(b) First half of the reflected base cycle pairs.															
Cycles	(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,5,8,10)	1	0 1 5	1 2 6	3	5 8 1	1 4 43	2	8 10 5	1 3 44	2	8 10 0	1 3 39	1	0 1 10	1 2 11
(0,1,6,8,17)	1	0 1 6	1 2 7	2	6 8 1	1 3 42	2	6 8 17	1 3 12	8	0 8 17	1 9 18	1	0 1 17	1 2 18
(0,1,7,11,18)	1	0 1 7	1 2 8	4	7 11 1	1 5 41	4	7 11 18	1 5 12	7	11 18 0	1 8 36	1	0 1 18	1 2 19
(0,1,8,10,23)	1	0 1 8	1 2 9	2	8 10 1	1 3 40	2	8 10 23	1 3 16	10	0 10 23	1 11 24	1	0 1 23	1 2 24
(0,1,9,4,13)	1	0 1 9	1 2 10	3	1 4 9	1 4 9	4	9 13 4	1 5 42	4	0 4 13	1 5 14	1	0 1 13	1 2 14
(0,1,11,4,19)	1	0 1 11	1 2 12	3	1 4 11	1 4 11	7	4 11 19	1 8 16	4	0 4 19	1 5 20	1	0 1 19	1 2 20
(0,1,12,4,27)	1	0 1 12	1 2 13	3	1 4 12	1 4 12	8	4 12 27	1 9 24	4	0 4 27	1 5 28	1	0 1 27	1 2 28
(0,1,14,9,21)	1	0 1 14	1 2 15	5	9 14 1	1 6 39	5	9 14 21	1 6 13	9	0 9 21	1 10 22	1	0 1 21	1 2 22
(0,2,14,17,27)	2	0 2 14	1 3 15	3	14 17 2	1 4 35	3	14 17 27	1 4 14	10	17 27 0	1 11 30	2	0 2 27	1 3 28
(0,2,17,6,23)	2	0 2 17	1 3 18	4	2 6 17	1 5 16	6	17 23 6	1 7 36	6	0 6 23	1 7 24	2	0 2 23	1 3 24
(0,2,19,5,28)	2	0 2 19	1 3 20	3	2 5 19	1 4 18	9	19 28 5	1 10 33	5	0 5 28	1 6 29	2	0 2 28	1 3 29
(0,3,14,9,25)	3	0 3 14	1 4 15	5	9 14 3	1 6 41	5	9 14 25	1 6 17	9	0 9 25	1 10 26	3	0 3 25	1 4 26
(0,3,16,25,30)	3	0 3 16	1 4 17	9	16 25 3	1 10 34	5	25 30 16	1 6 38	5	25 30 0	1 6 22	3	0 3 30	1 4 31
(0,3,18,40,21)	3	0 3 18	1 4 19	9	40 3 18	1 10 25	3	18 21 40	1 4 23	6	40 0 21	1 7 28	3	0 3 21	1 4 22
(0,3,20,35,26)	3	0 3 20	1 4 21	14	35 3 20	1 15 32	6	20 26 35	1 7 16	9	26 35 0	1 10 21	3	0 3 26	1 4 27
(0,4,22,35,17)	4	0 4 22	1 5 23	13	22 35 4	1 14 29	5	17 22 35	1 6 19	11	35 0 17	1 12 29	4	0 4 17	1 5 18
(0,5,19,40,27)	5	0 5 19	1 6 20	11	40 5 19	1 12 26	8	19 27 40	1 9 22	6	40 0 27	1 7 34	5	0 5 27	1 6 28
(0,7,19,42,26)	7	0 7 19	1 8 20	11	42 7 19	1 12 24	7	19 26 42	1 8 24	4	42 0 26	1 5 31	7	0 7 26	1 8 27
(0,7,24,33,17)	7	0 7 24	1 8 25	9	24 33 7	1 10 30	7	17 24 33	1 8 17	13	33 0 17	1 14 31	7	0 7 17	1 8 18
(0,8,20,36,14)	8	0 8 20	1 9 21	12	8 20 36	1 13 29	6	14 20 36	1 7 23	10	36 0 14	1 11 25	6	8 14 0	1 7 39
(0,26,5,36,11)	5	0 5 26	1 6 27	10	26 36 5	1 11 26	6	5 11 36	1 7 32	10	36 0 11	1 11 22	11	0 11 26	1 12 27
(0,27,12,34,13)	12	0 12 27	1 13 28	7	27 34 12	1 8 32	1	12 13 34	1 2 23	12	34 0 13	1 13 26	13	0 13 27	1 14 28
(c) Second half of the reflected base cycle pairs.															
Cycles	(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,2,5,9,10)	2	0 2 5	1 3 6	3	2 5 9	1 4 8	1	9 10 5	1 2 43	1	9 10 0	1 2 38	2	0 2 10	1 3 11
(0,9,11,16,17)	2	9 11 0	1 3 38	2	9 11 16	1 3 8	1	16 17 11	1 2 42	1	16 17 0	1 2 31	8	9 17 0	1 9 38
(0,7,11,17,18)	4	7 11 0	1 5 40	4	7 11 17	1 5 11	1	17 18 11	1 2 41	1	17 18 0	1 2 30	7	0 7 18	1 8 19
(0,13,15,22,23)	2	13 15 0	1 3 34	2	13 15 22	1 3 10	1	22 23 15	1 2 40	1	22 23 0	1 2 25	10	13 23 0	1 11 34
(0,9,4,12,13)	4	0 4 9	1 5 10	3	9 12 4	1 4 42	1	12 13 4	1 2 39	1	12 13 0	1 2 35	4	9 13 0	1 5 38
(0,15,8,18,19)	7	8 15 0	1 8 39	3	15 18 8	1 4 40	1	18 19 8	1 2 37	1	18 19 0	1 2 29	4	15 19 0	1 5 32
(0,23,15,26,27)	8	15 23 0	1 9 32	3	23 26 15	1 4 39	1	26 27 15	1 2 36	1	26 27 0	1 2 21	4	23 27 0	1 5 24
(0,12,7,20,21)	5	7 12 0	1 6 40	5	7 12 20	1 6 14	1	20 21 7	1 2 34	1	20 21 0	1 2 27	9	12 21 0	1 10 35
(0,10,13,25,27)	3	10 13 0	1 4 37	3	10 13 25	1 4 16	2	25 27 13	1 3 35	2	25 27 0	1 3 22	10	0 10 27	1 11 28
(0,17,6,21,23)	6	0 6 17	1 7 18	4	17 21 6	1 5 36	2	21 23 6	1 3 32	2	21 23 0	1 3 26	6	17 23 0	1 7 30
(0,23,9,26,28)	9	0 9 23	1 10 24	3	23 26 9	1 4 33	2	26 28 9	1 3 30	2	26 28 0	1 3 21	5	23 28 0	1 6 24
(0,16,11,22,25)	5	11 16 0	1 6 36	5	11 16 22	1 6 12	3	22 25 11	1 4 36	3	22 25 0	1 4 25	9	16 25 0	1 10 31
(0,5,14,27,30)	5	0 5 14	1 6 15	9	5 14 27	1 10 23	3	27 30 14	1 4 34	3	27 30 0	1 4 20	5	0 5 30	1 6 31
(0,27,3,18,21)	3	0 3 27	1 4 28	9	18 27 3	1 10 32	3	18 21 3	1 4 32	3	18 21 0	1 4 29	6	21 27 0	1 7 26
(0,37,6,23,26)	6	0 6 37	1 7 38	14	23 37 6	1 15 30	3	23 26 6	1 4 30	3	23 26 0	1 4 24	9	37 0 26	1 10 36
(0,28,41,13,17)	5	41 0 28	1 6 34	13	28 41 13	1 14 32	4	13 17 41	1 5 29	4	13 17 0	1 5 34	11	17 28 0	1 12 30
(0,33,8,22,27)	8	0 8 33	1 9 34	11	22 33 8	1 12 33	5	22 27 8	1 6 33	5	22 27 0	1 6 25	6	27 33 0	1 7 20
(0,30,7,19,26)	7	0 7 30	1 8 31	11	19 30 7	1 12 35	7	19 26 7	1 8 35	7	19 26 0	1 8 28	4	26 30 0	1 5 21
(0,30,39,10,17)	7	39 0 30	1 8 38	9	30 39 10	1 10 27	7	10 17 39	1 8 30	7	10 17 0	1 8 37	13	17 30 0	1 14 30
(0,24,40,6,14)	6	40 0 24	1 7 31	12	40 6 24	1 13 31	8	6 14 40	1 9 35	6	0 6 14	1 7 15	10	14 24 0	1 11 33
(0,21,6,31,11)	6	0 6 21	1 7 22	10	21 31 6	1 11 32	5	6 11 31	1 6 26	11	0 11 31	1 12 32	10	11 21 0	1 11 36
(0,25,1,32,13)	1	0 1 25	1 2 26	7	25 32 1	1 8 23	12	1 13 32	1 13 32	13	0 13 32	1 14 33	12	13 25 0	1 13 34

Table A20. Cyclic $C(3, 5, 47)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

(a) Cyclic $C(3, 5, 47)$ —symmetric base cycles.																		
Base cycles (5 vertices)					Distances in the difference triplets													
v1	v2	v3	v4	v5	1-2-3		2-3-4		3-4-5		4-5-1		5-1-2					
0	1	44	3	46	1	4	3	4	6	2	6	4	2	4	1	1	2	2
0	2	41	6	45	2	8	6	8	12	4	12	8	4	8	2	2	4	4
0	3	38	9	44	3	12	9	12	18	6	18	12	6	12	3	3	6	6
0	4	35	12	43	4	16	12	16	23	8	23	16	8	16	4	4	8	8
0	5	32	15	42	5	20	15	20	17	10	17	20	10	20	5	5	10	10
0	6	29	18	41	6	23	18	23	11	12	11	23	12	23	6	6	12	12
0	7	26	21	40	7	19	21	19	5	14	5	19	14	19	7	7	14	14
0	8	23	24	39	8	15	23	15	1	16	1	15	16	15	8	8	16	16
0	9	20	27	38	9	11	20	11	7	18	7	11	18	11	9	9	18	18
0	10	17	30	37	10	7	17	7	13	20	13	7	20	7	10	10	20	20
0	11	14	33	36	11	3	14	3	19	22	19	3	22	3	11	11	22	22
0	12	11	36	35	12	1	11	1	22	23	22	1	23	1	12	12	23	23
0	13	8	39	34	13	5	8	5	16	21	16	5	21	5	13	13	21	21
0	14	5	42	33	14	9	5	9	10	19	10	9	19	9	14	14	19	19
0	15	2	45	32	15	13	2	13	4	17	4	13	17	13	15	15	17	17
0	16	46	1	31	16	17	1	17	2	15	2	17	15	17	16	16	15	15
0	17	43	4	30	17	21	4	21	8	13	8	21	13	21	17	17	13	13
0	18	40	7	29	18	22	7	22	14	11	14	22	11	22	18	18	11	11
0	19	37	10	28	19	18	10	18	20	9	20	18	9	18	19	19	9	9
0	20	34	13	27	20	14	13	14	21	7	21	14	7	14	20	20	7	7
0	21	31	16	26	21	10	16	10	15	5	15	10	5	10	21	21	5	5
0	22	28	19	25	22	6	19	6	9	3	9	6	3	6	22	22	3	3
0	23	25	22	24	23	2	22	2	3	1	3	2	1	2	23	23	1	1
(b) Cyclic $C(3, 5, 47)$ —first half of the reflected base cycle pairs.																		
Base cycles (5 vertices)					Distances in the difference triplets													
v1	v2	v3	v4	v5	1-2-3		2-3-4		3-4-5		4-5-1		5-1-2					
0	1	5	8	10	1	4	5	4	3	7	3	2	5	2	10	8	10	9
0	1	6	8	18	1	5	6	5	2	7	2	10	12	10	18	8	18	17
0	1	7	11	34	1	6	7	6	4	10	4	23	20	23	13	11	13	14
0	1	8	10	21	1	7	8	7	2	9	2	11	13	11	21	10	21	20
0	1	9	4	15	1	8	9	8	5	3	5	11	6	11	15	4	15	14
0	1	11	8	35	1	10	11	10	3	7	3	20	23	20	12	8	12	13
0	1	19	3	26	1	18	19	18	16	2	16	23	7	23	21	3	21	22
0	2	11	6	22	2	9	11	9	5	4	5	16	11	16	22	6	22	20
0	2	14	9	26	2	12	14	12	5	7	5	17	12	17	21	9	21	23
0	2	16	6	19	2	14	16	14	10	4	10	13	3	13	19	6	19	17
0	2	20	17	28	2	18	20	18	3	15	3	11	8	11	19	17	19	21
0	3	15	22	30	3	12	15	12	7	19	7	8	15	8	17	22	17	20
0	3	16	9	31	3	13	16	13	7	6	7	22	15	22	16	9	16	19
0	3	17	35	21	3	14	17	14	18	15	18	14	4	14	21	12	21	18
0	23	32	40	4	23	9	15	9	8	17	8	11	19	11	4	7	4	19
0	6	17	32	26	6	11	17	11	15	21	15	6	9	6	21	15	21	20
0	6	20	36	16	6	14	20	14	16	17	16	20	4	20	16	11	16	10
0	23	30	39	6	23	7	17	7	9	16	9	14	23	14	6	8	6	17
0	8	26	13	22	8	18	21	18	13	5	13	9	4	9	22	13	22	14
0	25	5	33	10	22	20	5	20	19	8	19	23	5	23	10	14	10	15
0	23	5	35	10	23	18	5	18	17	12	17	22	5	22	10	12	10	13
0	12	25	43	21	12	13	22	13	18	16	18	22	4	22	21	4	21	9
0	28	13	32	12	19	15	13	15	19	4	19	20	1	20	12	15	12	16
(c) Cyclic $C(3, 5, 47)$ —second half of the reflected base cycle pairs.																		
Base cycles (5 vertices)					Distances in the difference triplets													
v1	v2	v3	v4	v5	1-2-3		2-3-4		3-4-5		4-5-1		5-1-2					
0	2	5	9	10	2	3	5	3	4	7	4	1	5	1	10	9	10	8
0	10	12	17	18	10	2	12	2	5	7	5	1	6	1	18	17	18	11
0	23	27	33	34	23	4	20	4	6	10	6	1	7	1	13	14	13	23
0	11	13	20	21	11	2	13	2	7	9	7	1	8	1	21	20	21	10
0	11	6	14	15	11	5	6	5	8	3	8	1	9	1	15	14	15	4
0	27	24	34	35	20	3	23	3	10	7	10	1	11	1	12	13	12	8
0	23	7	25	26	23	16	7	16	18	2	18	1	19	1	21	22	21	3
0	16	11	20	22	16	5	11	5	9	4	9	2	11	2	22	20	22	6
0	17	12	24	26	17	5	12	5	12	7	12	2	14	2	21	23	21	9
0	13	3	17	19	13	10	3	10	14	4	14	2	16	2	19	17	19	6
0	11	8	26	28	11	3	8	3	18	15	18	2	20	2	19	21	19	17
0	8	15	27	30	8	7	15	7	12	19	12	3	15	3	17	20	17	22
0	22	15	28	31	22	7	15	7	13	6	13	3	16	3	16	19	16	9
0	33	4	18	21	14	18	4	18	14	15	14	3	17	3	21	18	21	12
0	11	19	28	4	11	8	19	8	9	17	9	23	15	23	4	19	4	7
0	41	9	20	26	6	15	9	15	11	21	11	6	17	6	21	20	21	15
0	27	43	10	16	20	16	4	16	14	17	14	6	20	6	16	10	16	11
0	14	23	30	6	14	9	23	9	7	16	7	23	17	23	6	17	6	8
0	9	43	14	22	9	13	4	13	18	5	18	8	21	8	22	14	22	13
0	24	5	32	10	23	19	5	19	20	8	20	22	5	22	10	15	10	14
0	22	5	34	10	22	17	5	17	18	12	18	23	5	23	10	13	10	12
0	25	43	9	21	22	18	4	18	13	16	13	12	22	12	21	9	21	4
0	27	46	31	12	20	19	1	19	15	4	15	19	13	19	12	16	12	15

Table A21. $\mathcal{C}(3,5,47)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric base cycles.															
Cycles		(x ₁ ,x ₂ ,x ₃)		(x ₂ ,x ₃ ,x ₄)		(x ₃ ,x ₄ ,x ₅)		(x ₄ ,x ₅ ,x ₁)		(x ₅ ,x ₁ ,x ₂)					
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,44,3,46)	1	0 1 44	1 2 45	2	1 3 44	1 3 44	2	44 46 3	1 3 7	1	46 0 3	1 2 5	1	46 0 1	1 2 3
(0,2,41,6,45)	2	0 2 41	1 3 42	4	2 6 41	1 5 40	4	41 45 6	1 5 13	2	45 0 6	1 3 9	2	45 0 2	1 3 5
(0,3,38,9,44)	3	0 3 38	1 4 39	6	3 9 38	1 7 36	6	38 44 9	1 7 19	3	44 0 9	1 4 13	3	44 0 3	1 4 7
(0,4,35,12,43)	4	0 4 35	1 5 36	8	4 12 35	1 9 32	8	35 43 12	1 9 25	4	43 0 12	1 5 17	4	43 0 4	1 5 9
(0,5,32,15,42)	5	0 5 32	1 6 33	10	5 15 32	1 11 28	10	32 42 15	1 11 31	5	42 0 15	1 6 21	5	42 0 5	1 6 11
(0,6,29,18,41)	6	0 6 29	1 7 30	11	18 29 6	1 12 36	11	18 29 41	1 12 24	6	41 0 18	1 7 25	6	41 0 6	1 7 13
(0,7,26,21,40)	7	0 7 26	1 8 27	5	21 26 7	1 6 34	5	21 26 40	1 6 20	7	40 0 21	1 8 29	7	40 0 7	1 8 15
(0,8,23,24,39)	8	0 8 23	1 9 24	1	23 24 8	1 2 33	1	23 24 39	1 2 17	8	39 0 24	1 9 33	8	39 0 8	1 9 17
(0,9,20,27,38)	9	0 9 20	1 10 21	7	20 27 9	1 8 37	7	20 27 38	1 8 19	9	38 0 27	1 10 37	9	38 0 9	1 10 19
(0,10,17,30,37)	7	10 17 0	1 8 38	7	10 17 30	1 8 21	7	30 37 17	1 8 35	7	30 37 0	1 8 18	10	37 0 10	1 11 21
(0,11,14,33,36)	3	11 14 0	1 4 37	3	11 14 33	1 4 23	3	33 36 14	1 4 29	3	33 36 0	1 4 15	11	36 0 11	1 12 23
(0,12,11,36,35)	1	11 12 0	1 2 37	1	11 12 36	1 2 26	1	35 36 11	1 2 24	1	35 36 0	1 2 13	12	35 0 12	1 13 25
(0,13,8,39,34)	5	8 13 0	1 6 40	5	8 13 39	1 6 32	5	34 39 8	1 6 22	5	34 39 0	1 6 14	13	34 0 13	1 14 27
(0,14,5,42,33)	5	0 5 14	1 6 15	9	5 14 42	1 10 38	9	33 42 5	1 10 20	5	42 0 33	1 6 39	14	33 0 14	1 15 29
(0,15,2,45,32)	2	0 2 15	1 3 16	4	45 2 15	1 5 18	4	45 2 32	1 5 35	2	45 0 32	1 3 35	15	32 0 15	1 16 31
(0,16,46,1,31)	1	46 0 16	1 2 18	2	46 1 16	1 3 18	2	46 1 31	1 3 33	1	0 1 31	1 2 32	16	46 1 0	1 16 32
(0,17,43,4,30)	4	43 0 17	1 5 22	8	43 4 17	1 9 22	8	43 4 30	1 9 35	4	0 4 30	1 5 31	13	43 0 4	1 13 31
(0,18,40,7,29)	7	40 0 18	1 8 26	11	7 18 40	1 12 34	11	29 40 7	1 12 26	7	0 7 29	1 8 30	11	40 0 7	1 12 30
(0,19,37,10,28)	10	37 0 19	1 11 30	9	10 19 37	1 10 28	9	28 37 10	1 10 30	10	0 10 28	1 11 29	9	19 28 0	1 10 29
(0,20,34,13,27)	13	34 0 20	1 14 34	7	13 20 34	1 8 22	7	27 34 13	1 8 34	13	0 13 27	1 14 28	7	20 27 0	1 8 28
(0,21,31,16,26)	10	21 31 0	1 11 27	5	16 21 31	1 6 16	5	26 31 16	1 6 38	10	16 26 0	1 11 32	5	21 26 0	1 6 27
(0,22,28,19,25)	6	22 28 0	1 7 26	3	19 22 28	1 4 10	3	25 28 19	1 4 42	6	19 25 0	1 7 29	3	22 25 0	1 4 26
(0,23,25,22,24)	2	23 25 0	1 3 25	1	22 23 25	1 2 4	1	24 25 22	1 2 46	2	22 24 0	1 3 26	1	23 24 0	1 2 25

(b) First half of the reflected base cycle pairs.															
Cycles		(x ₁ ,x ₂ ,x ₃)		(x ₂ ,x ₃ ,x ₄)		(x ₃ ,x ₄ ,x ₅)		(x ₄ ,x ₅ ,x ₁)		(x ₅ ,x ₁ ,x ₂)					
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,5,8,10)	1	0 1 5	1 2 6	3	5 8 1	1 4 44	2	8 10 5	1 3 45	2	8 10 0	1 3 40	1	0 1 10	1 2 11
(0,1,6,8,18)	1	0 1 6	1 2 7	2	6 8 1	1 3 43	2	6 8 18	1 3 13	8	0 8 18	1 9 19	1	0 1 18	1 2 19
(0,1,7,11,34)	1	0 1 7	1 2 8	4	7 11 1	1 5 42	4	7 11 34	1 5 28	11	0 11 34	1 12 35	1	0 1 34	1 2 35
(0,1,8,10,21)	1	0 1 8	1 2 9	2	8 10 1	1 3 41	2	8 10 21	1 3 14	10	0 10 21	1 11 22	1	0 1 21	1 2 22
(0,1,9,4,15)	1	0 1 9	1 2 10	3	1 4 9	1 4 9	5	4 9 15	1 6 12	4	0 4 15	1 5 16	1	0 1 15	1 2 16
(0,1,11,8,35)	1	0 1 11	1 2 12	3	8 11 1	1 4 41	3	8 11 35	1 4 28	8	0 8 35	1 9 36	1	0 1 35	1 2 36
(0,1,19,3,26)	1	0 1 19	1 2 20	2	1 3 19	1 3 19	7	19 26 3	1 8 32	3	0 3 26	1 4 27	1	0 1 26	1 2 27
(0,2,11,6,22)	2	0 2 11	1 3 12	4	2 6 11	1 5 10	5	6 11 22	1 6 17	6	0 6 22	1 7 23	2	0 2 22	1 3 23
(0,2,14,9,26)	2	0 2 14	1 3 15	5	9 14 2	1 6 41	5	9 14 26	1 6 18	9	0 9 26	1 10 27	2	0 2 26	1 3 27
(0,2,16,6,19)	2	0 2 16	1 3 17	4	2 6 16	1 5 15	3	16 19 6	1 4 38	6	0 6 19	1 7 20	2	0 2 19	1 3 20
(0,2,20,17,28)	2	0 2 20	1 3 21	3	17 20 2	1 4 33	3	17 20 28	1 4 12	11	17 28 0	1 12 31	2	0 2 28	1 3 29
(0,3,15,22,30)	3	0 3 15	1 4 16	7	15 22 3	1 8 36	7	15 22 30	1 8 16	8	22 30 0	1 9 26	3	0 3 30	1 4 31
(0,3,16,9,31)	3	0 3 16	1 4 17	6	3 9 16	1 7 14	7	9 16 31	1 8 23	9	0 9 31	1 10 32	3	0 3 31	1 4 32
(0,3,17,35,21)	3	0 3 17	1 4 18	14	3 17 35	1 15 33	4	17 21 35	1 5 19	12	35 0 21	1 13 34	3	0 3 21	1 4 22
(0,23,32,40,4)	9	23 32 0	1 10 25	8	32 40 23	1 9 39	8	32 40 4	1 9 20	4	0 4 40	1 5 41	4	0 4 23	1 5 24
(0,6,17,32,26)	6	0 6 17	1 7 18	11	6 17 32	1 12 27	6	26 32 17	1 7 39	6	26 32 0	1 7 22	6	0 6 26	1 7 27
(0,6,20,36,16)	6	0 6 20	1 7 21	14	6 20 36	1 15 31	4	16 20 36	1 5 21	11	36 0 16	1 12 28	6	0 6 16	1 7 17
(0,23,30,39,6)	7	23 30 0	1 8 25	7	23 30 39	1 8 17	9	30 39 6	1 10 24	6	0 6 39	1 7 40	6	0 6 23	1 7 24
(0,8,26,13,22)	8	0 8 26	1 9 27	5	8 13 26	1 6 19	4	22 26 13	1 5 39	9	13 22 0	1 10 35	8	0 8 22	1 9 23
(0,25,5,33,10)	5	0 5 25	1 6 26	8	25 33 5	1 9 28	5	5 10 33	1 6 29	10	0 10 33	1 11 34	10	0 10 25	1 11 26
(0,23,5,35,10)	5	0 5 23	1 6 24	12	23 35 5	1 13 30	5	5 10 35	1 6 31	10	0 10 35	1 11 36	10	0 10 23	1 11 24
(0,12,25,43,21)	12	0 12 25	1 13 26	13	12 25 43	1 14 32	4	21 25 43	1 5 23	4	43 0 21	1 5 26	9	12 21 0	1 10 36
(0,28,13,32,12)	13	0 13 28	1 14 29	4	28 32 13	1 5 33	1	12 13 32	1 2 21	12	0 12 32	1 13 33	12	0 12 28	1 13 29

(c) Second half of the reflected base cycle pairs.															
Cycles		(x ₁ ,x ₂ ,x ₃)		(x ₂ ,x ₃ ,x ₄)		(x ₃ ,x ₄ ,x ₅)		(x ₄ ,x ₅ ,x ₁)		(x ₅ ,x ₁ ,x ₂)					
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,2,5,9,10)	2	0 2 5	1 3 6	3	2 5 9	1 4 8	1	9 10 5	1 2 44	1	9 10 0	1 2 39	2	0 2 10	1 3 11
(0,10,12,17,18)	2	10 12 0	1 3 38	2	10 12 17	1 3 8	1	17 18 12	1 2 43	1	17 18 0	1 2 31	8	10 18 0	1 9 38
(0,23,27,33,34)	4	23 27 0	1 5 25	4	23 27 33	1 5 11	1	33 34 27	1 2 42	1	33 34 0	1 2 15	11	23 34 0	1 12 25
(0,11,13,20,21)	2	11 13 0	1 3 37	2	11 13 20	1 3 10	1	20 21 13	1 2 41	1	20 21 0	1 2 28	10	11 21 0	1 11 37
(0,11,6,14,15)	5	6 11 0	1 6 42	3	11 14 6	1 4 43	1	14 15 6	1 2 40	1	14 15 0	1 2 34	4	11 15 0	1 5 37
(0,27,24,34,35)	3	24 27 0	1 4 24	3	24 27 34	1 4 11	1	34 35 24	1 2 38	1	34 35 0	1 2 14	8	27 35 0	1 9 21
(0,23,7,25,26)	7	0 7 23	1 8 24	2	23 25 7	1 3 32	1	25 26 7	1 2 30	1	25 26 0	1 2 23	3	23 26 0	1 4 25
(0,16,11,20,22)	5	11 16 0	1 6 37	4	16 20 11	1 5 43	2	20 22 11	1 3 39	2	20 22 0	1 3 28	6	16 22 0	1 7 32
(0,17,12,24,26)	5	12 17 0	1 6 36	5	12 17 24	1 6 13	2	24 26 12	1 3 36	2	24 26 0	1 3 24	9	17 26 0	1 10 31
(0,13,3,17,19)	3	0 3 13	1 4 14	4	13 17 3	1 5 38	2	17 19 3	1 3 34	2	17 19 0	1 3 31	6	13 19 0	1 7 35
(0,11,8,26,28)	3	8 11 0	1 4 40	3	8 11 26	1 4 19	2	26 28 8	1 3 30	2	26 28 0	1 3 22	11	0 11 28	1 12 29
(0,8,15,27,30)	7	8 15 0	1 8 40	7	8 15 27	1 8 20	3	27 30 15	1 4 36	3	27 30 0	1 4 21	8	0 8 30	1 9 31
(0,22,15,28,31)	7	15 22 0	1 8 33	6	22 28 15	1 7 41	3	28 31 15							

Table A22. Cyclic split $\mathcal{C}(3, 5, 50)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. The bright green cells mark the base cycle vertices where the original +3 rule had to be modified for v4 (which means a modification for v3 as well.) With red, we highlighted the special base cycles, that only have $v/5$ different positions, as $i \mapsto i + v/5$ results in an automorphism of these cycles. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

(a) Cyclic $\mathcal{C}(3, 5, 50)$ —symmetric cycles for odd values.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	48	2	49	1	3	2	3	4	1	4	3	1	3	1	2	1	1	2
0	3	44	6	47	3	9	6	9	12	3	12	9	3	9	3	6	3	3	6
0	5	38	12	45	5	17	12	17	24	7	24	17	7	17	5	12	5	5	10
0	7	36	14	43	7	21	14	21	22	7	22	21	7	21	7	14	7	7	14
0	9	32	18	41	9	23	18	23	14	9	14	23	9	23	9	18	9	9	18
0	11	28	22	39	11	17	22	17	6	11	6	17	11	17	11	22	11	11	22
0	13	24	26	37	13	11	24	11	2	13	2	11	13	11	13	24	13	13	24
0	15	20	30	35	15	5	20	5	10	15	10	5	15	5	15	20	15	15	20
0	17	16	34	33	17	1	16	1	18	17	18	1	17	1	17	16	17	17	16
0	19	12	38	31	19	7	12	7	24	19	24	7	19	7	19	12	19	19	12
0	21	8	42	29	21	13	8	13	16	21	16	13	21	13	21	8	21	21	8
0	23	4	46	27	23	19	4	19	8	23	8	19	23	19	23	4	23	23	4

(b) Cyclic $\mathcal{C}(3, 5, 50)$ —symmetric base cycles for even values with 4 pairs of reflected base cycles: the base cycles of $\mathcal{C}(3, 5, 25)$ with vertex indices multiplied by two.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	2	44	6	48	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	4	38	12	46	4	16	12	16	24	8	24	16	8	16	4	12	4	4	8
0	6	32	18	44	6	24	18	24	14	12	14	24	12	24	6	18	6	6	12
0	8	26	24	42	8	18	24	18	2	16	2	18	16	18	8	24	8	8	16
0	10	20	30	40	10	10	20	10	10	20	10	10	20	10	10	20	10	10	20
0	12	14	36	38	12	2	14	2	22	24	22	2	24	2	12	14	12	12	24
0	14	8	42	36	14	6	8	6	16	22	16	6	22	6	14	8	14	14	22
0	16	2	48	34	16	14	2	14	4	18	4	14	18	14	16	2	16	16	18
0	18	46	4	32	18	22	4	22	8	14	8	22	14	22	18	4	18	18	14
0	20	40	10	30	20	20	10	20	20	10	20	20	10	20	20	10	20	20	10
0	22	34	16	28	22	12	16	12	18	6	18	12	6	12	22	16	22	22	6
0	24	28	22	26	24	4	22	4	6	2	6	4	2	4	24	22	24	24	2
0	2	10	22	20	2	8	10	8	12	20	12	2	10	2	20	22	20	2	18
0	4	10	24	20	4	6	10	6	14	20	14	4	10	4	20	24	20	4	16
0	6	26	40	16	6	20	24	20	14	16	14	24	10	24	16	10	16	6	10
0	8	28	40	18	8	20	22	20	12	18	12	22	10	22	18	10	18	8	10
0	48	10	18	20	2	12	10	12	8	20	8	2	10	2	20	18	20	2	22
0	46	10	16	20	4	14	10	14	6	20	6	4	10	4	20	16	20	4	24
0	26	40	10	16	24	14	10	14	20	16	20	6	24	6	16	10	16	24	10
0	28	40	10	18	22	12	10	12	20	18	20	8	22	8	18	10	18	22	10

Table A22. Cont.

(c) Cyclic $C(3, 5, 50)$ —first half of the remaining reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	5	8	13	1	4	5	4	3	7	3	5	8	5	13	8	13	1	12
0	1	6	3	14	1	5	6	5	3	2	3	11	8	11	14	3	14	1	13
0	1	7	12	21	1	6	7	6	5	11	5	9	14	9	21	12	21	1	20
0	1	8	3	15	1	7	8	7	5	2	5	12	7	12	15	3	15	1	14
0	1	9	16	19	1	8	9	8	7	15	7	3	10	3	19	16	19	1	18
0	1	10	3	16	1	9	10	9	7	2	7	13	6	13	16	3	16	1	15
0	1	11	14	29	1	10	11	10	3	13	3	15	18	15	21	14	21	1	22
0	1	12	3	20	1	11	12	11	9	2	9	17	8	17	20	3	20	1	19
0	1	23	4	25	1	22	23	22	19	3	19	21	2	21	25	4	25	1	24
0	2	15	6	25	2	13	15	13	9	4	9	19	10	19	25	6	25	2	23
0	2	17	6	29	2	15	17	15	11	4	11	23	12	23	21	6	21	2	23
0	3	17	34	27	3	14	17	14	17	19	17	7	10	7	23	16	23	3	24
0	3	21	8	25	3	18	21	18	13	5	13	17	4	17	25	8	25	3	22
0	3	23	10	29	3	20	23	20	13	7	13	19	6	19	21	10	21	3	24
0	4	33	16	11	4	21	17	21	17	12	17	5	22	5	11	16	11	4	7
0	5	21	40	29	5	16	21	16	19	15	19	11	8	11	21	10	21	5	24
0	5	23	34	9	5	18	23	18	11	21	11	25	14	25	9	16	9	5	4
0	5	31	4	19	5	24	19	24	23	1	23	15	12	15	19	4	19	5	14
0	6	35	26	15	6	21	15	21	9	20	9	11	20	11	15	24	15	6	9
0	9	22	39	16	9	13	22	13	17	20	17	23	6	23	16	11	16	9	7
0	9	24	2	33	9	15	24	15	22	7	22	19	9	19	17	2	17	9	24
0	11	30	43	18	11	19	20	19	13	18	13	25	12	25	18	7	18	11	7
0	13	36	8	23	13	23	14	23	22	5	22	15	13	15	23	8	23	13	10
0	15	32	5	25	15	17	18	17	23	10	23	20	7	20	25	5	25	15	10
(d) Cyclic $C(3, 5, 50)$ —second half of the remaining reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	5	8	12	13	5	3	8	3	4	7	4	1	5	1	13	12	13	5	8
0	11	8	13	14	11	3	8	3	5	2	5	1	6	1	14	13	14	11	3
0	9	14	20	21	9	5	14	5	6	11	6	1	7	1	21	20	21	9	12
0	12	7	14	15	12	5	7	5	7	2	7	1	8	1	15	14	15	12	3
0	3	10	18	19	3	7	10	7	8	15	8	1	9	1	19	18	19	3	16
0	13	6	15	16	13	7	6	7	9	2	9	1	10	1	16	15	16	13	3
0	15	18	28	29	15	3	18	3	10	13	10	1	11	1	21	22	21	15	14
0	17	8	19	20	17	9	8	9	11	2	11	1	12	1	20	19	20	17	3
0	21	2	24	25	21	19	2	19	22	3	22	1	23	1	25	24	25	21	4
0	19	10	23	25	19	9	10	9	13	4	13	2	15	2	25	23	25	19	6
0	23	12	27	29	23	11	12	11	15	4	15	2	17	2	21	23	21	23	6
0	43	10	24	27	7	17	10	17	14	19	14	3	17	3	23	24	23	7	16
0	17	4	22	25	17	13	4	13	18	5	18	3	21	3	25	22	25	17	8
0	19	6	26	29	19	13	6	13	20	7	20	3	23	3	21	24	21	19	10
0	45	28	7	11	5	17	22	17	21	12	21	4	17	4	11	7	11	5	16
0	39	8	24	29	11	19	8	19	16	15	16	5	21	5	21	24	21	11	10
0	25	36	4	9	25	11	14	11	18	21	18	5	23	5	9	4	9	25	16
0	15	38	14	19	15	23	12	23	24	1	24	5	19	5	19	14	19	15	4
0	39	30	9	15	11	9	20	9	21	20	21	6	15	6	15	9	15	11	24
0	27	44	7	16	23	17	6	17	13	20	13	9	22	9	16	7	16	23	11
0	31	9	24	33	19	22	9	22	15	7	15	9	24	9	17	24	17	19	2
0	25	38	7	18	25	13	12	13	19	18	19	11	20	11	18	7	18	25	7
0	15	37	10	23	15	22	13	22	23	5	23	13	14	13	23	10	23	15	8
0	20	43	10	25	20	23	7	23	17	10	17	15	18	15	25	10	25	20	5

Table A23. $\mathcal{C}(3, 5, 50)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) $\mathcal{C}(3, 5, 50)$ —symmetric base cycles.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,48,2,49)	1	0 1 48	1 2 49	1	1 2 48	1 2 48	1	48 49 2	1 2 5	1	49 0 2	1 2 4	1	49 0 1	1 2 3
(0,3,44,6,47)	3	0 3 44	1 4 45	3	3 6 44	1 4 42	3	44 47 6	1 4 13	3	47 0 6	1 4 10	3	47 0 3	1 4 7
(0,5,38,12,45)	5	0 5 38	1 6 39	7	5 12 38	1 8 34	7	38 45 12	1 8 25	5	45 0 12	1 6 18	5	45 0 5	1 6 11
(0,7,36,14,43)	7	0 7 36	1 8 37	7	7 14 36	1 8 30	7	36 43 14	1 8 29	7	43 0 14	1 8 22	7	43 0 7	1 8 15
(0,9,32,18,41)	9	0 9 32	1 10 33	9	9 18 32	1 10 24	9	32 41 18	1 10 37	9	41 0 18	1 10 28	9	41 0 9	1 10 19
(0,11,28,22,39)	11	0 11 28	1 12 29	6	22 28 11	1 7 40	6	22 28 39	1 7 18	11	39 0 22	1 12 34	11	39 0 11	1 12 23
(0,13,24,26,37)	11	13 24 0	1 12 38	2	24 26 13	1 3 40	2	24 26 37	1 3 14	11	26 37 0	1 12 25	13	37 0 13	1 14 27
(0,15,20,30,35)	5	15 20 0	1 6 36	5	15 20 30	1 6 16	5	30 35 20	1 6 41	5	30 35 0	1 6 21	15	35 0 15	1 16 31
(0,17,16,34,33)	1	16 17 0	1 2 35	1	16 17 34	1 2 19	1	33 34 16	1 2 34	1	33 34 0	1 2 18	16	17 33 0	1 17 34
(0,19,12,38,31)	7	12 19 0	1 8 39	7	12 19 38	1 8 27	7	31 38 12	1 8 32	7	31 38 0	1 8 20	12	19 31 0	1 13 32
(0,21,8,42,29)	8	0 8 21	1 9 22	13	8 21 42	1 14 35	13	29 42 8	1 14 30	8	42 0 29	1 9 38	8	21 29 0	1 9 30
(0,23,4,46,27)	4	0 4 23	1 5 24	8	46 4 23	1 9 28	8	46 4 27	1 9 32	4	46 0 27	1 5 32	4	23 27 0	1 5 28

(b) $\mathcal{C}(3, 5, 50)$ —symmetric cycles for even values with 4 pairs of reflected base cycles: the base cycles of $\mathcal{C}(3, 5, 50)$ with vertex indices multiplied by two.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,2,44,6,48)	2	0 2 44	1 3 45	4	2 6 44	1 5 43	4	44 48 6	1 5 13	2	48 0 6	1 3 9	2	48 0 2	1 3 5
(0,4,38,12,46)	4	0 4 38	1 5 39	8	4 12 38	1 9 35	8	38 46 12	1 9 25	4	46 0 12	1 5 17	4	46 0 4	1 5 9
(0,6,32,18,44)	6	0 6 32	1 7 33	12	6 18 32	1 13 27	12	32 44 18	1 13 37	6	44 0 18	1 7 25	6	44 0 6	1 7 13
(0,8,26,24,42)	8	0 8 26	1 9 27	2	24 26 8	1 3 35	2	24 26 42	1 3 19	8	42 0 24	1 9 33	8	42 0 8	1 9 17
(0,10,20,30,40)	10	0 10 20	1 11 21	10	10 20 30	1 11 21	10	20 30 40	1 11 21	10	30 40 0	1 11 21	10	40 0 10	1 11 21
(0,12,14,36,38)	2	12 14 0	1 3 39	2	12 14 36	1 3 25	2	36 38 14	1 3 29	2	36 38 0	1 3 15	12	38 0 12	1 13 25
(0,14,8,42,36)	6	8 14 0	1 7 43	6	8 14 42	1 7 35	6	36 42 8	1 7 23	6	36 42 0	1 7 15	14	36 0 14	1 15 29
(0,16,2,48,34)	2	0 2 16	1 3 17	4	48 2 16	1 5 19	4	48 2 34	1 5 37	2	48 0 34	1 3 37	16	34 0 16	1 17 33
(0,18,46,4,32)	4	46 0 18	1 5 23	8	46 4 18	1 9 23	8	46 4 32	1 9 37	4	0 4 32	1 5 33	14	18 32 0	1 15 33
(0,20,40,10,30)	10	40 0 20	1 11 31	10	10 20 40	1 11 31	10	30 40 10	1 11 31	10	0 10 30	1 11 31	10	20 30 0	1 11 31
(0,22,34,16,28)	12	22 34 0	1 13 29	6	16 22 34	1 7 19	6	28 34 16	1 7 39	12	16 28 0	1 13 35	6	22 28 0	1 7 29
(0,24,28,22,26)	4	24 28 0	1 5 27	2	22 24 28	1 3 7	2	26 28 22	1 3 47	4	22 26 0	1 5 29	2	24 26 0	1 3 27
(0,2,10,22,20)	2	0 2 10	1 3 11	8	2 10 22	1 9 21	2	20 22 10	1 3 41	2	20 22 0	1 3 31	2	0 2 20	1 3 21
(0,4,10,24,20)	4	0 4 10	1 5 11	6	4 10 24	1 7 21	4	20 24 10	1 5 41	4	20 24 0	1 5 31	4	0 4 20	1 5 21
(0,6,26,40,16)	6	0 6 26	1 7 27	14	26 40 6	1 15 31	10	16 26 40	1 11 25	10	40 0 16	1 11 27	6	0 6 16	1 7 17
(0,8,28,40,18)	8	0 8 28	1 9 29	12	28 40 8	1 13 31	10	18 28 40	1 11 23	10	40 0 18	1 11 29	8	0 8 18	1 9 19
(0,48,10,18,20)	2	48 0 10	1 3 13	8	10 18 48	1 9 39	2	18 20 10	1 3 43	2	18 20 0	1 3 33	2	48 0 20	1 3 23
(0,46,10,16,20)	4	46 0 10	1 5 15	6	10 16 46	1 7 37	4	16 20 10	1 5 45	4	16 20 0	1 5 35	4	46 0 20	1 5 25
(0,26,40,10,16)	10	40 0 26	1 11 37	14	26 40 10	1 15 35	6	10 16 40	1 7 31	6	10 16 0	1 7 41	10	16 26 0	1 11 35
(0,28,40,10,18)	10	40 0 28	1 11 39	12	28 40 10	1 13 33	8	10 18 40	1 9 31	8	10 18 0	1 9 41	10	18 28 0	1 11 33

(c) $\mathcal{C}(3, 5, 50)$ —first half of the remaining reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,5,8,13)	1	0 1 5	1 2 6	3	5 8 1	1 4 47	3	5 8 13	1 4 9	5	8 13 0	1 6 43	1	0 1 13	1 2 14
(0,1,6,3,14)	1	0 1 6	1 2 7	2	1 3 6	1 3 6	3	3 6 14	1 4 12	3	0 3 14	1 4 15	1	0 1 14	1 2 15
(0,1,7,12,21)	1	0 1 7	1 2 8	5	7 12 1	1 6 45	5	7 12 21	1 6 15	9	12 21 0	1 10 39	1	0 1 21	1 2 22
(0,1,8,3,15)	1	0 1 8	1 2 9	2	1 3 8	1 3 8	5	3 8 15	1 6 13	3	0 3 15	1 4 16	1	0 1 15	1 2 16
(0,1,9,16,19)	1	0 1 9	1 2 10	7	9 16 1	1 8 43	3	16 19 9	1 4 44	3	16 19 0	1 4 35	1	0 1 19	1 2 20
(0,1,10,3,16)	1	0 1 10	1 2 11	2	1 3 10	1 3 10	6	10 16 3	1 7 44	3	0 3 16	1 4 17	1	0 1 16	1 2 17
(0,1,11,14,29)	1	0 1 11	1 2 12	3	11 14 1	1 4 41	3	11 14 29	1 4 19	14	0 14 29	1 15 30	1	0 1 29	1 2 30
(0,1,12,3,20)	1	0 1 12	1 2 13	2	1 3 12	1 3 12	8	12 20 3	1 9 42	3	0 3 20	1 4 21	1	0 1 20	1 2 21
(0,1,23,4,25)	1	0 1 23	1 2 24	3	1 4 23	1 4 23	2	23 25 4	1 3 32	4	0 4 25	1 5 26	1	0 1 25	1 2 26
(0,2,15,6,25)	2	0 2 15	1 3 16	4	2 6 15	1 5 14	9	6 15 25	1 10 20	6	0 6 25	1 7 26	2	0 2 25	1 3 26
(0,2,17,6,29)	2	0 2 17	1 3 18	4	2 6 17	1 5 16	11	6 17 29	1 12 24	6	0 6 29	1 7 30	2	0 2 29	1 3 30
(0,3,17,34,27)	3	0 3 17	1 4 18	14	3 17 34	1 15 32	7	27 34 17	1 8 41	7	27 34 0	1 8 24	3	0 3 27	1 4 28
(0,3,21,8,25)	3	0 3 21	1 4 22	5	3 8 21	1 6 19	4	21 25 8	1 5 38	8	0 8 25	1 9 26	3	0 3 25	1 4 26
(0,3,23,10,29)	3	0 3 23	1 4 24	7	3 10 23	1 8 21	6	23 29 10	1 7 38	10	0 10 29	1 11 30	3	0 3 29	1 4 30
(0,4,33,16,11)	4	0 4 33	1 5 34	12	4 16 33	1 13 30	5	11 16 33	1 6 23	5	11 16 0	1 6 40	4	0 4 11	1 5 12
(0,5,21,40,29)	5	0 5 21	1 6 22	15	40 5 21	1 16 32	8	21 29 40	1 9 20	10	40 0 29	1 11 40	5	0 5 29	1 6 30
(0,5,23,34,9)	5	0 5 23	1 6 24	11	23 34 5	1 12 33	11	23 34 9	1 12 37	9	0 9 34	1 10 35	4	5 9 0	1 5 46
(0,5,31,4,19)	5	0 5 31	1 6 32	1	4 5 31	1 2 28	12	19 31 4	1 13 36	4	0 4 19	1 5 20	5	0 5 19	1 6 20
(0,6,35,26,15)	6	0 6 35	1 7 36	9	26 35 6	1 10 31	9	26 35 15	1 10 40	11	15 26 0	1 12 36	6	0 6 15	1 7 16
(0,9,22,39,16)	9	0 9 22	1 10 23	13	9 22 39	1 14 31	6	16 22 39	1 7 24	11	39 0 16	1 12 28	7	9 16 0	1 8 42
(0,9,24,2,33)	9	0 9 24	1 10 25	7	2 9 24	1 8 23	9	24 33 2	1 10 29	2	0 2 33	1 3 34	9	0 9 33	1 10 34
(0,11,30,43,18)	11	0 11 30	1 12 31	13	30 43 11	1 14 32	12	18 30 43	1 13 26	7	43 0 18	1 8 26	7	11 18 0	1 8 40
(0,13,36,8,23)	13	0 13 36	1 14 37	5	8 13 36	1 6 29	13	23 36 8	1 14 36	8	0 8 23	1 9 24	10	13 23 0	1 11 38
(0,15,32,5,25)	15	0 15 32	1 16 33	10	5 15 32	1 11 28	7	25 32 5	1 8 31	5	0 5 25	1 6 26	10	15 25 0	1 11 36

Table A23. Cont.

(d) $\mathcal{C}(3, 5, 50)$ —second half of the remaining reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,5,8,12,13)	3	5 8 0	1 4 46	3	5 8 12	1 4 8	1	12 13 8	1 2 47	1	12 13 0	1 2 39	5	0 5 13	1 6 14
(0,11,8,13,14)	3	8 11 0	1 4 43	2	11 13 8	1 3 48	1	13 14 8	1 2 46	1	13 14 0	1 2 38	3	11 14 0	1 4 40
(0,9,14,20,21)	5	9 14 0	1 6 42	5	9 14 20	1 6 12	1	20 21 14	1 2 45	1	20 21 0	1 2 31	9	0 9 21	1 10 22
(0,12,7,14,15)	5	7 12 0	1 6 44	2	12 14 7	1 3 46	1	14 15 7	1 2 44	1	14 15 0	1 2 37	3	12 15 0	1 4 39
(0,3,10,18,19)	3	0 3 10	1 4 11	7	3 10 18	1 8 16	1	18 19 10	1 2 43	1	18 19 0	1 2 33	3	0 3 19	1 4 20
(0,13,6,15,16)	6	0 6 13	1 7 14	2	13 15 6	1 3 44	1	15 16 6	1 2 42	1	15 16 0	1 2 36	3	13 16 0	1 4 38
(0,15,18,28,29)	3	15 18 0	1 4 36	3	15 18 28	1 4 14	1	28 29 18	1 2 41	1	28 29 0	1 2 23	14	15 29 0	1 15 36
(0,17,8,19,20)	8	0 8 17	1 9 18	2	17 19 8	1 3 42	1	19 20 8	1 2 40	1	19 20 0	1 2 32	3	17 20 0	1 4 34
(0,21,2,24,25)	2	0 2 21	1 3 22	3	21 24 2	1 4 32	1	24 25 2	1 2 29	1	24 25 0	1 2 27	4	21 25 0	1 5 30
(0,19,10,23,25)	9	10 19 0	1 10 41	4	19 23 10	1 5 42	2	23 25 10	1 3 38	2	23 25 0	1 3 28	6	19 25 0	1 7 32
(0,23,12,27,29)	11	12 23 0	1 12 39	4	23 27 12	1 5 40	2	27 29 12	1 3 36	2	27 29 0	1 3 24	6	23 29 0	1 7 28
(0,43,10,24,27)	7	43 0 10	1 8 18	14	10 24 43	1 15 34	3	24 27 10	1 4 37	3	24 27 0	1 4 27	7	43 0 27	1 8 35
(0,17,4,22,25)	4	0 4 17	1 5 18	5	17 22 4	1 6 38	3	22 25 4	1 4 33	3	22 25 0	1 4 29	8	17 25 0	1 9 34
(0,19,6,26,29)	6	0 6 19	1 7 20	7	19 26 6	1 8 38	3	26 29 6	1 4 31	3	26 29 0	1 4 25	10	19 29 0	1 11 32
(0,45,28,7,11)	5	45 0 28	1 6 34	12	45 7 28	1 13 34	4	7 11 28	1 5 22	4	7 11 0	1 5 44	5	45 0 11	1 6 17
(0,39,8,24,29)	8	0 8 39	1 9 40	15	24 39 8	1 16 35	5	24 29 8	1 6 35	5	24 29 0	1 6 27	10	29 39 0	1 11 22
(0,25,36,4,9)	11	25 36 0	1 12 26	11	25 36 4	1 12 30	5	4 9 36	1 6 33	4	0 4 9	1 5 10	9	0 9 25	1 10 26
(0,15,38,14,19)	12	38 0 15	1 13 28	1	14 15 38	1 2 25	5	14 19 38	1 6 25	5	14 19 0	1 6 37	4	15 19 0	1 5 36
(0,39,30,9,15)	9	30 39 0	1 10 21	9	30 39 9	1 10 30	6	9 15 30	1 7 22	6	9 15 0	1 7 42	11	39 0 15	1 12 27
(0,27,44,7,16)	6	44 0 27	1 7 34	13	44 7 27	1 14 34	9	7 16 44	1 10 38	7	0 7 16	1 8 17	11	16 27 0	1 12 35
(0,31,9,24,33)	9	0 9 31	1 10 32	7	24 31 9	1 8 36	9	24 33 9	1 10 36	9	24 33 0	1 10 27	2	31 33 0	1 3 20
(0,25,38,7,18)	12	38 0 25	1 13 38	13	25 38 7	1 14 33	11	7 18 38	1 12 32	7	0 7 18	1 8 19	7	18 25 0	1 8 33
(0,15,37,10,23)	13	37 0 15	1 14 29	5	10 15 37	1 6 28	13	10 23 37	1 14 28	10	0 10 23	1 11 24	8	15 23 0	1 9 36
(0,20,43,10,25)	7	43 0 20	1 8 28	10	10 20 43	1 11 34	15	10 25 43	1 16 34	10	0 10 25	1 11 26	5	20 25 0	1 6 31

Table A24. Cyclic split $\mathcal{C}(3, 5, 52)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. The bright green cells mark the base cycle vertices where the original +3 rule had to be modified for v4 (which means a modification for v3 as well.) In subtable b, the 4 pairs of reflected base cycles derived from $\mathcal{C}(3, 5, 26)$ are highlighted with 2 shades of grey. The rest of the reflected base cycle pairs are listed in subtables c and d, where the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

(a) Cyclic $\mathcal{C}(3, 5, 52)$ —symmetric cycles for odd values.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	50	2	51	1	3	2	3	4	1	4	3	1	3	1	2	1	1	2
0	3	46	6	49	3	9	6	9	12	3	12	9	3	9	3	6	3	3	6
0	5	42	10	47	5	15	10	15	20	5	20	15	5	15	5	10	5	5	10
0	7	38	14	45	7	21	14	21	24	7	24	21	7	21	7	14	7	7	14
0	9	34	18	43	9	25	18	25	16	9	16	25	9	25	9	18	9	9	18
0	11	30	22	41	11	19	22	19	8	11	8	19	11	19	11	22	11	11	22
0	13	28	24	39	13	15	24	15	4	11	4	15	11	15	13	24	13	13	26
0	15	22	30	37	15	7	22	7	8	15	8	7	15	7	15	22	15	15	22
0	17	18	34	35	17	1	18	1	16	17	16	1	17	1	17	18	17	17	18
0	19	14	38	33	19	5	14	5	24	19	24	5	19	5	19	14	19	19	14
0	21	10	42	31	21	11	10	11	20	21	20	11	21	11	21	10	21	21	10
0	23	6	46	29	23	17	6	17	12	23	12	17	23	17	23	6	23	23	6
0	25	2	50	27	25	23	2	23	4	25	4	23	25	23	25	2	25	25	2

Table A24. Cont.

(b) Cyclic $\mathcal{C}(3,5,52)$ —symmetric base cycles for even values with 4 pairs of reflected base cycles: the base cycles of $\mathcal{C}(3,5,26)$ with vertex indices multiplied by two.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	2	48	4	50	2	6	4	6	8	2	8	6	2	6	2	4	2	2	4
0	6	40	12	46	6	18	12	18	24	6	24	18	6	18	6	12	6	6	12
0	10	32	20	42	10	22	20	22	12	10	12	22	10	22	10	20	10	10	20
0	14	24	28	38	14	10	24	10	4	14	4	10	14	10	14	24	14	14	24
0	18	16	36	34	18	2	16	2	20	18	20	2	18	2	18	16	18	18	16
0	22	8	44	30	22	14	8	14	16	22	16	14	22	14	22	8	22	22	8
0	4	44	8	48	4	12	8	12	16	4	16	12	4	12	4	8	4	4	8
0	8	36	16	44	8	24	16	24	20	8	20	24	8	24	8	16	8	8	16
0	12	44	8	40	12	20	8	20	16	4	16	20	4	20	12	8	12	12	24
0	16	42	10	36	16	26	10	26	20	6	20	26	6	26	16	10	16	16	20
0	20	50	2	32	20	22	2	22	4	18	4	22	18	22	20	2	20	20	12
0	24	46	6	28	24	22	6	22	12	18	12	22	18	22	24	6	24	24	4
0	2	16	34	26	2	14	16	14	18	20	18	8	10	8	26	18	26	2	24
0	2	24	50	12	2	22	24	22	26	4	26	14	12	14	12	2	12	2	10
0	24	40	46	4	24	16	12	16	6	22	6	10	16	10	4	6	4	24	20
0	6	14	4	38	6	8	14	8	10	2	10	18	24	18	14	4	14	6	20
0	44	10	24	26	8	18	10	18	14	20	14	2	16	2	26	24	26	8	18
0	14	40	10	12	14	26	12	26	22	4	22	2	24	2	12	10	12	14	2
0	10	16	32	4	10	6	16	6	16	22	16	24	12	24	4	20	4	10	6
0	34	24	32	38	18	10	24	10	8	2	8	6	14	6	20	14	18	18	4
(c) Cyclic $\mathcal{C}(3,5,52)$ —first half of the remaining reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	5	8	13	1	4	5	4	3	7	3	5	8	5	13	8	13	1	12
0	1	6	3	14	1	5	6	5	3	2	3	11	8	11	14	3	14	1	13
0	1	7	12	19	1	6	7	6	5	11	5	7	12	7	19	12	19	1	18
0	1	8	3	20	1	7	8	7	5	2	5	17	12	17	20	3	20	1	19
0	1	9	18	11	1	8	9	8	9	17	9	7	2	7	11	18	11	1	10
0	1	10	5	21	1	9	10	9	5	4	5	16	11	16	21	5	21	1	20
0	1	12	3	16	1	11	12	11	9	2	9	13	4	13	16	3	16	1	15
0	1	15	6	25	1	14	15	14	9	5	9	19	10	19	25	6	25	1	24
0	1	22	3	26	1	21	22	21	19	2	19	23	4	23	26	3	26	1	25
0	1	23	10	29	1	22	23	22	13	9	13	19	6	19	23	10	23	1	24
0	2	13	6	31	2	11	13	11	7	4	7	25	18	25	21	6	21	2	23
0	2	15	8	35	2	13	15	13	7	6	7	25	20	25	17	8	17	2	19
0	3	10	19	31	3	7	10	7	9	16	9	12	21	12	21	19	21	3	24
0	3	13	20	33	3	10	13	10	7	17	7	13	20	13	19	20	19	3	22
0	3	15	30	21	3	12	15	12	15	25	15	9	6	9	21	22	21	3	18
0	3	18	38	27	3	15	18	15	20	17	20	11	9	11	25	14	25	3	24
0	25	12	36	3	25	13	12	13	24	11	24	19	9	19	3	16	3	25	22
0	23	12	38	3	23	11	12	11	26	15	26	17	9	17	3	14	3	23	20
0	4	21	46	31	4	17	21	17	25	10	25	15	10	15	21	6	21	4	25
0	5	18	41	25	5	13	18	13	23	16	23	16	7	16	25	11	25	5	20
0	5	27	10	23	5	22	25	22	17	5	17	13	4	13	23	10	23	5	18
0	7	30	41	24	7	23	22	23	11	18	11	17	6	17	24	11	24	7	17
0	9	32	47	24	9	23	20	23	15	14	15	23	8	23	24	5	24	9	15
0	13	30	44	21	13	17	22	17	14	21	14	23	9	23	21	8	21	13	8
0	13	34	8	27	13	21	18	21	26	5	26	19	7	19	25	8	25	13	14
0	15	31	48	33	15	16	21	16	17	19	17	15	2	15	19	4	19	15	18

Table A24. Cont.

(d) Cyclic $\mathcal{C}(3, 5, 52)$ —second half of the remaining 26 reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	5	8	12	13	5	3	8	3	4	7	4	1	5	1	13	12	13	5	8
0	11	8	13	14	11	3	8	3	5	2	5	1	6	1	14	13	14	11	3
0	7	12	18	19	7	5	12	5	6	11	6	1	7	1	19	18	19	7	12
0	17	12	19	20	17	5	12	5	7	2	7	1	8	1	20	19	20	17	3
0	45	2	10	11	7	9	2	9	8	17	8	1	9	1	11	10	11	7	18
0	16	11	20	21	16	5	11	5	9	4	9	1	10	1	21	20	21	16	5
0	13	4	15	16	13	9	4	9	11	2	11	1	12	1	16	15	16	13	3
0	19	10	24	25	19	9	10	9	14	5	14	1	15	1	25	24	25	19	6
0	23	4	25	26	23	19	4	19	21	2	21	1	22	1	26	25	26	23	3
0	19	6	28	29	19	13	6	13	22	9	22	1	23	1	23	24	23	19	10
0	25	18	29	31	25	7	18	7	11	4	11	2	13	2	21	23	21	25	6
0	27	20	33	35	25	7	20	7	13	6	13	2	15	2	17	19	17	25	8
0	12	21	28	31	12	9	21	9	7	16	7	3	10	3	21	24	21	12	19
0	13	20	30	33	13	7	20	7	10	17	10	3	13	3	19	22	19	13	20
0	43	6	18	21	9	15	6	15	12	25	12	3	15	3	21	18	21	9	22
0	41	9	24	27	11	20	9	20	15	17	15	3	18	3	25	24	25	11	14
0	19	43	30	3	19	24	9	24	13	11	13	25	12	25	3	22	3	19	16
0	17	43	32	3	17	26	9	26	11	15	11	23	12	23	3	20	3	17	14
0	37	10	27	31	15	25	10	25	17	10	17	4	21	4	21	25	21	15	6
0	36	7	20	25	16	23	7	23	13	16	13	5	18	5	25	20	25	16	11
0	13	48	18	23	13	17	4	17	22	5	22	5	25	5	23	18	23	13	10
0	35	46	17	24	17	11	6	11	23	18	23	7	22	7	24	17	24	17	11
0	29	44	15	24	23	15	8	15	23	14	23	9	20	9	24	15	24	23	5
0	29	43	8	21	23	14	9	14	17	21	17	13	22	13	21	8	21	23	8
0	19	45	14	27	19	26	7	26	21	5	21	13	18	13	25	14	25	19	8
0	37	2	18	33	15	17	2	17	16	19	16	15	21	15	19	18	19	15	4

Table A25. $\mathcal{C}(3, 5, 52)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) $\mathcal{C}(3, 5, 52)$ —symmetric cycles for odd values.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,50,2,51)	1	0 1 50	1 2 51	1	1 2 50	1 2 50	1	50 51 2	1 2 5	1	51 0 2	1 2 4	1	51 0 1	1 2 3
(0,3,46,6,49)	3	0 3 46	1 4 47	3	3 6 46	1 4 44	3	46 49 6	1 4 13	3	49 0 6	1 4 10	3	49 0 3	1 4 7
(0,5,42,10,47)	5	0 5 42	1 6 43	5	5 10 42	1 6 38	5	42 47 10	1 6 21	5	47 0 10	1 6 16	5	47 0 5	1 6 11
(0,7,38,14,45)	7	0 7 38	1 8 39	7	7 14 38	1 8 32	7	38 45 14	1 8 29	7	45 0 14	1 8 22	7	45 0 7	1 8 15
(0,9,34,18,43)	9	0 9 34	1 10 35	9	9 18 34	1 10 26	9	34 43 18	1 10 37	9	43 0 18	1 10 28	9	43 0 9	1 10 19
(0,11,30,22,41)	11	0 11 30	1 12 31	8	22 30 11	1 9 42	8	22 30 41	1 9 20	11	41 0 22	1 12 34	11	41 0 11	1 12 23
(0,13,28,24,39)	13	0 13 28	1 14 29	4	24 28 13	1 5 42	4	24 28 39	1 5 16	13	39 0 24	1 14 38	13	39 0 13	1 14 27
(0,15,22,30,37)	7	15 22 0	1 8 38	7	15 22 30	1 8 16	7	30 37 22	1 8 45	7	30 37 0	1 8 23	15	37 0 15	1 16 31
(0,17,18,34,35)	1	17 18 0	1 2 36	1	17 18 34	1 2 18	1	34 35 18	1 2 37	1	34 35 0	1 2 19	17	35 0 17	1 18 35
(0,19,14,38,33)	5	14 19 0	1 6 39	5	14 19 38	1 6 25	5	33 38 14	1 6 34	5	33 38 0	1 6 20	14	19 33 0	1 15 34
(0,21,10,42,31)	10	0 10 21	1 11 22	11	10 21 42	1 12 33	11	31 42 10	1 12 32	10	42 0 31	1 11 42	10	21 31 0	1 11 32
(0,23,6,46,29)	6	0 6 23	1 7 24	12	46 6 23	1 13 30	12	46 6 29	1 13 36	6	46 0 29	1 7 36	6	23 29 0	1 7 30
(0,25,2,50,27)	2	0 2 25	1 3 26	4	50 2 25	1 5 28	4	50 2 27	1 5 30	2	50 0 27	1 3 30	2	25 27 0	1 3 28

Table A25. Cont.

(b) $\mathcal{C}(3, 5, 52)$ —symmetric base cycles for even values with 4 pairs of reflected base cycles: the base cycles of $\mathcal{C}(3, 5, 26)$ with vertex indices multiplied by two.																
Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,2,48,4,50)	2	0 2 48	1 3 49	2	2 4 48	1 3 47	2	48 50 4	1 3 9	2	50 0 4	1 3 7	2	50 0 2	1 3 5	
(0,6,40,12,46)	6	0 6 40	1 7 41	6	6 12 40	1 7 35	6	40 46 12	1 7 25	6	46 0 12	1 7 19	6	46 0 6	1 7 13	
(0,10,32,20,42)	10	0 10 32	1 11 33	10	10 20 32	1 11 23	10	32 42 20	1 11 41	10	42 0 20	1 11 31	10	42 0 10	1 11 21	
(0,14,24,28,38)	10	14 24 0	1 11 39	4	24 28 14	1 5 43	4	24 28 38	1 5 15	10	28 38 0	1 11 25	14	38 0 14	1 15 29	
(0,18,16,36,34)	2	16 18 0	1 3 37	2	16 18 36	1 3 21	2	34 36 16	1 3 35	2	34 36 0	1 3 19	16	18 34 0	1 17 35	
(0,22,8,44,30)	8	0 8 22	1 9 23	14	8 22 44	1 15 37	14	30 44 8	1 15 31	8	44 0 30	1 9 39	8	22 30 0	1 9 31	
(0,4,44,8,48)	4	0 4 44	1 5 45	4	4 8 44	1 5 41	4	44 48 8	1 5 17	4	48 0 8	1 5 13	4	48 0 4	1 5 9	
(0,8,36,16,44)	8	0 8 36	1 9 37	8	8 16 36	1 9 29	8	36 44 16	1 9 33	8	44 0 16	1 9 25	8	44 0 8	1 9 17	
(0,12,44,8,40)	8	44 0 12	1 9 21	4	8 12 44	1 5 37	4	40 44 8	1 5 21	8	0 8 40	1 9 41	12	40 0 12	1 13 25	
(0,16,42,10,36)	10	42 0 16	1 11 27	6	10 16 42	1 7 33	6	36 42 10	1 7 27	10	0 10 36	1 11 37	16	36 0 16	1 17 33	
(0,20,50,2,32)	2	50 0 20	1 3 23	4	50 2 20	1 5 23	4	50 2 32	1 5 35	2	0 2 32	1 3 33	12	20 32 0	1 13 33	
(0,24,46,6,28)	6	46 0 24	1 7 31	12	46 6 24	1 13 31	12	46 6 28	1 13 35	6	0 6 28	1 7 29	4	24 28 0	1 5 29	
(0,2,16,34,26)	2	0 2 16	1 3 17	14	2 16 34	1 15 33	8	26 34 16	1 9 43	8	26 34 0	1 9 27	2	0 2 26	1 3 27	
(0,2,24,50,12)	2	0 2 24	1 3 25	4	50 2 24	1 5 27	12	12 24 50	1 13 39	2	50 0 12	1 3 15	2	0 2 12	1 3 13	
(0,24,40,46,4)	12	40 0 24	1 13 37	6	40 46 24	1 7 37	6	40 46 4	1 7 17	4	0 4 46	1 5 47	4	0 4 24	1 5 25	
(0,6,14,4,38)	6	0 6 14	1 7 15	2	4 6 14	1 3 11	10	4 14 38	1 11 35	4	0 4 38	1 5 39	6	0 6 38	1 7 39	
(0,44,10,24,26)	8	44 0 10	1 9 19	14	10 24 44	1 15 35	2	24 26 10	1 3 39	2	24 26 0	1 3 29	8	44 0 26	1 9 35	
(0,14,40,10,12)	12	40 0 14	1 13 27	4	10 14 40	1 5 31	2	10 12 40	1 3 31	2	10 12 0	1 3 43	2	12 14 0	1 3 41	
(0,10,16,32,4)	6	10 16 0	1 7 43	6	10 16 32	1 7 23	12	4 16 32	1 13 29	4	0 4 32	1 5 33	4	0 4 10	1 5 11	
(0,34,24,32,38)	10	24 34 0	1 11 29	2	32 34 24	1 3 45	6	32 38 24	1 7 45	6	32 38 0	1 7 21	4	34 38 0	1 5 19	

(c) First half of the remaining reflected base cycle pairs.

Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,1,5,8,13)	1	0 1 5	1 2 6	3	5 8 1	1 4 49	3	5 8 13	1 4 9	5	8 13 0	1 6 45	1	0 1 13	1 2 14	
(0,1,6,3,14)	1	0 1 6	1 2 7	2	1 3 6	1 3 6	3	3 6 14	1 4 12	3	0 3 14	1 4 15	1	0 1 14	1 2 15	
(0,1,7,12,19)	1	0 1 7	1 2 8	5	7 12 1	1 6 47	5	7 12 19	1 6 13	7	12 19 0	1 8 41	1	0 1 19	1 2 20	
(0,1,8,3,20)	1	0 1 8	1 2 9	2	1 3 8	1 3 8	5	3 8 20	1 6 18	3	0 3 20	1 4 21	1	0 1 20	1 2 21	
(0,1,9,18,11)	1	0 1 9	1 2 10	8	1 9 18	1 9 18	2	9 11 18	1 3 10	7	11 18 0	1 8 42	1	0 1 11	1 2 12	
(0,1,10,5,21)	1	0 1 10	1 2 11	4	1 5 10	1 5 10	5	5 10 21	1 6 17	5	0 5 21	1 6 22	1	0 1 21	1 2 22	
(0,1,12,3,16)	1	0 1 12	1 2 13	2	1 3 12	1 3 12	4	12 16 3	1 5 44	3	0 3 16	1 4 17	1	0 1 16	1 2 17	
(0,1,15,6,25)	1	0 1 15	1 2 16	5	1 6 15	1 6 15	9	6 15 25	1 10 20	6	0 6 25	1 7 26	1	0 1 25	1 2 26	
(0,1,22,3,26)	1	0 1 22	1 2 23	2	1 3 22	1 3 22	4	22 26 3	1 5 34	3	0 3 26	1 4 27	1	0 1 26	1 2 27	
(0,1,23,10,29)	1	0 1 23	1 2 24	9	1 10 23	1 10 23	6	23 29 10	1 7 40	10	0 10 29	1 11 30	1	0 1 29	1 2 30	
(0,2,13,6,31)	2	0 2 13	1 3 14	4	2 6 13	1 5 12	7	6 13 31	1 8 26	6	0 6 31	1 7 32	2	0 2 31	1 3 32	
(0,2,15,8,35)	2	0 2 15	1 3 16	6	2 8 15	1 7 14	7	8 15 35	1 8 28	8	0 8 35	1 9 36	2	0 2 35	1 3 36	
(0,3,10,19,31)	3	0 3 10	1 4 11	7	3 10 19	1 8 17	9	10 19 31	1 10 22	12	19 31 0	1 13 34	3	0 3 31	1 4 32	
(0,3,13,20,33)	3	0 3 13	1 4 14	7	13 20 3	1 8 43	7	13 20 33	1 8 21	13	20 33 0	1 14 33	3	0 3 33	1 4 34	
(0,3,15,30,21)	3	0 3 15	1 4 16	12	3 15 30	1 13 28	6	15 21 30	1 7 16	9	21 30 0	1 10 32	3	0 3 21	1 4 22	
(0,3,18,38,27)	3	0 3 18	1 4 19	15	3 18 38	1 16 36	9	18 27 38	1 10 21	11	27 38 0	1 12 26	3	0 3 27	1 4 28	
(0,25,12,36,3)	12	0 12 25	1 13 26	11	25 36 12	1 12 40	9	3 12 36	1 10 34	3	0 3 36	1 4 37	3	0 3 25	1 4 26	
(0,23,12,38,3)	11	12 23 0	1 12 41	11	12 23 38	1 12 27	9	3 12 38	1 10 36	3	0 3 38	1 4 39	3	0 3 23	1 4 24	
(0,4,21,46,31)	4	0 4 21	1 5 22	10	46 4 21	1 11 28	10	21 31 46	1 11 26	6	46 0 31	1 7 38	4	0 4 31	1 5 32	
(0,5,18,41,25)	5	0 5 18	1 6 19	13	5 18 41	1 14 37	7	18 25 41	1 8 24	11	41 0 25	1 12 37	5	0 5 25	1 6 26	
(0,5,27,10,23)	5	0 5 27	1 6 28	5	5 10 27	1 6 23	4	23 27 10	1 5 40	10	0 10 23	1 11 24	5	0 5 23	1 6 24	
(0,7,30,41,24)	7	0 7 30	1 8 31	11	30 41 7	1 12 30	6	24 30 41	1 7 18	11	41 0 24	1 12 36	7	0 7 24	1 8 25	
(0,9,32,47,24)	9	0 9 32	1 10 33	14	47 9 32	1 15 38	8	24 32 47	1 9 24	5	47 0 24	1 6 30	9	0 9 24	1 10 25	
(0,13,30,44,21)	13	0 13 30	1 14 31	14	30 44 13	1 15 36	9	21 30 44	1 10 24	8	44 0 21	1 9 30	8	13 21 0	1 9 40	
(0,13,34,8,27)	13	0 13 34	1 14 35	5	8 13 34	1 6 27	7	27 34 8	1 8 34	8	0 8 27	1 9 28	13	0 13 27	1 14 28	
(0,15,31,48,33)	15	0 15 31	1 16 32	16	15 31 48	1 17 34	2	31 33 48	1 3 18	4	48 0 33	1 5 38	15	0 15 33	1 16 34	

(d) Second half of the remaining reflected base cycle pairs.

Cycles		(x_1, x_2, x_3)			(x_2, x_3, x_4)			(x_3, x_4, x_5)			(x_4, x_5, x_1)			(x_5, x_1, x_2)		
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,5,8,12,13)	3	5 8 0	1 4 48	3	5 8 12	1 4 8	1	12 13 8	1 2 49	1	12 13 0	1 2 41	5	0 5 13	1 6 14	
(0,11,8,13,14)	3	8 11 0	1 4 45	2	11 13 8	1 3 50	1	13 14 8	1 2 48	1	13 14 0	1 2 40	3	11 14 0	1 4 42	
(0,7,12,18,19)	5	7 12 0	1 6 46	5	7 12 18	1 6 12	1	18 19 12	1 2 47	1	18 19 0	1 2 35	7	0 7 19	1 8 20	
(0,17,12,19,20)	5	12 17 0	1 6 41	2	17 19 12	1 3 48	1	19 20 12	1 2 46	1	19 20 0	1 2 34	3	17 20 0	1 4 36	
(0,45,2,10,11)	2	0 2 45	1 3 46	8	2 10 45	1 9 44	1	10 11 2	1 2 45	1	10 11 0	1 2 43	7	45 0 11	1 8 19	
(0,16,11,20,21)	5	11 16 0	1 6 42	4	16 20 11	1 5 48	1	20 21 11	1 2 44	1	20 21 0	1 2 33	5	16 21 0	1 6 37	
(0,13,4,15,16)	4	0 4 13	1 5 14	2	13 15 4	1 3 44	1	15 16 4	1 2 42	1	15 16 0	1 2 38	3	13 16 0	1 4 40	
(0,19,10,24,25)	9	10 19 0	1 10 43	5	19 24 10	1 6 44	1	24 25 10	1 2 39	1	24 25 0	1 2 29	6	19 25 0	1 7 34	
(0,23,4,25,26)	4	0 4 23	1 5 24	2	23 25 4	1 3 34	1	25 26 4	1 2 32	1	25 26 0	1 2 28	3	23 26 0	1 4 30	
(0,19,6,28,29)	6	0 6 19	1 7 20	9	19 28 6	1 10 40	1	28 29 6	1 2 31	1	28 29 0	1 2 25	10	19 29 0	1 11 34	
(0,25,18,29,31)	7	18 25 0	1 8 35	4	25 29 18	1 5 46	2	29 31 18	1 3 42	2	29 31 0	1 3 24	6	25 31 0	1 7 28	
(0,27,20,33,35)	7	20 27 0	1 8 33	6	27 33 20	1 7 46	2	33 35 20	1 3 40	2	33 35 0	1 3 20	8	27 35 0	1 9 26	
(0,12,21,28,31)	9	12 21 0	1 10 41</													

Table A26. Cyclic $\mathcal{C}(3, 5, 55)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. $v = 55$ is a special case as 55 can be divided by 5 and 11 as well. The symmetric cycles corresponding to the difference triplets (5,5,10), (10,10,20), (15,15,25), (20,20,15), and (25,25,5) are shown in a separate subtable. Among the rest of the symmetric cycles, we highlighted with red the ones that correspond to (11,11,22) and (22,22,11). The detailed explanation for why these cases have to be handled differently can be found in Section 4. In the case of reflected base cycles (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

(a) Cyclic $\mathcal{C}(3, 5, 55)$ —symmetric cycles for the symmetric triplets (5,5,10), (10,10,20), (15,15,25), (20,20,15), and (25,25,5).																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	5	53	2	50	5	7	2	7	4	3	4	7	3	7	5	2	5	5	10
0	10	51	4	45	10	14	4	14	8	6	8	14	6	14	10	4	10	10	20
0	15	49	6	40	15	21	6	21	12	9	12	21	9	21	15	6	15	15	25
0	20	47	8	35	20	27	8	27	16	12	16	27	12	27	20	8	20	20	15
0	25	43	12	30	25	18	12	18	24	13	24	18	13	18	25	12	25	25	5
(b) Cyclic $\mathcal{C}(3, 5, 55)$ —the remaining symmetric cycles.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	52	3	54	1	4	3	4	6	2	6	4	2	4	1	3	1	1	2
0	2	49	6	53	2	8	6	8	12	4	12	8	4	8	2	6	2	2	4
0	3	46	9	52	3	12	9	12	18	6	18	12	6	12	3	9	3	3	6
0	4	43	12	51	4	16	12	16	24	8	24	16	8	16	4	12	4	4	8
0	6	37	18	49	6	24	18	24	19	12	19	24	12	24	6	18	6	6	12
0	7	34	21	48	7	27	21	27	13	14	13	27	14	27	7	21	7	7	14
0	8	31	24	47	8	23	24	23	7	16	7	23	16	23	8	24	8	8	16
0	9	28	27	46	9	19	27	19	1	18	1	19	18	19	9	27	9	9	18
0	11	22	33	44	11	11	22	11	11	22	11	11	22	11	11	22	11	11	22
0	12	19	36	43	12	7	19	7	17	24	17	7	24	7	12	19	12	12	24
0	13	16	39	42	13	3	16	3	23	26	23	3	26	3	13	16	13	13	26
0	14	13	42	41	14	1	13	1	26	27	26	1	27	1	14	13	14	14	27
0	16	7	48	39	16	9	7	9	14	23	14	9	23	9	16	7	16	16	23
0	17	4	51	38	17	13	4	13	8	21	8	13	21	13	17	4	17	17	21
0	18	1	54	37	18	17	1	17	2	19	2	17	19	17	18	1	18	18	19
0	19	53	2	36	19	21	2	21	4	17	4	21	17	21	19	2	19	19	17
0	21	47	8	34	21	26	8	26	16	13	16	26	13	26	21	8	21	21	13
0	22	44	11	33	22	22	11	22	22	11	22	22	11	22	22	11	22	22	11
0	23	41	14	32	23	18	14	18	27	9	27	18	9	18	23	14	23	23	9
0	24	38	17	31	24	14	17	14	21	7	21	14	7	14	24	17	24	24	7
0	26	32	23	29	26	6	23	6	9	3	9	6	3	6	26	23	26	26	3
0	27	29	26	28	27	2	26	2	3	1	3	2	1	2	27	26	27	27	1

Table A26. Cont.

(c) Cyclic $\mathcal{C}(3,5,55)$ —first half of the reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	5	10	7	1	4	5	4	5	9	5	3	2	3	7	10	7	1	6
0	1	6	9	17	1	5	6	5	3	8	3	8	11	8	17	9	17	1	16
0	1	8	10	21	1	7	8	7	2	9	2	11	13	11	21	10	21	1	20
0	1	9	11	20	1	8	9	8	2	10	2	9	11	9	20	11	20	1	19
0	1	10	14	25	1	9	10	9	4	13	4	11	15	11	25	14	25	1	24
0	1	11	5	16	1	10	11	10	6	4	6	11	5	11	16	5	16	1	15
0	1	12	5	22	1	11	12	11	7	4	7	17	10	17	22	5	22	1	21
0	1	13	3	30	1	12	13	12	10	2	10	27	17	27	25	3	25	1	26
0	1	15	3	23	1	14	15	14	12	2	12	20	8	20	23	3	23	1	22
0	2	15	5	20	2	13	15	13	10	3	10	15	5	15	20	5	20	2	18
0	2	16	5	23	2	14	16	14	11	3	11	18	7	18	23	5	23	2	21
0	2	17	5	27	2	15	17	15	12	3	12	22	10	22	27	5	27	2	25
0	2	18	8	26	2	16	18	16	10	6	10	18	8	18	26	8	26	2	24
0	2	22	6	25	2	20	22	20	16	4	16	19	3	19	25	6	25	2	23
0	3	17	8	27	3	14	17	14	9	5	9	19	10	19	27	8	27	3	24
0	3	18	9	33	3	15	18	15	9	6	9	24	15	24	22	9	22	3	25
0	3	20	14	34	3	17	20	17	6	11	6	20	14	20	21	14	21	3	24
0	3	21	7	36	3	18	21	18	14	4	14	26	15	26	19	7	19	3	22
0	4	19	11	24	4	15	19	15	8	7	8	13	5	13	24	11	24	4	20
0	4	22	9	29	4	18	22	18	13	5	13	20	7	20	26	9	26	4	25
0	4	23	10	32	4	19	23	19	13	6	13	22	9	22	23	10	23	4	27
0	4	25	48	33	4	21	25	21	23	11	23	15	8	15	22	7	22	4	26
0	5	12	18	34	5	7	12	7	6	13	6	16	22	16	21	18	21	5	26
0	5	21	49	25	5	16	21	16	27	11	27	24	4	24	25	6	25	5	20
0	9	34	45	26	9	25	21	25	11	19	11	19	8	19	26	10	26	9	17
0	10	24	53	31	10	14	24	14	26	12	26	22	7	22	24	2	24	10	21
0	10	35	48	25	10	25	20	25	13	17	13	23	10	23	25	7	25	10	15
0	11	37	6	26	11	26	18	26	24	5	24	20	11	20	26	6	26	11	15
0	12	27	43	17	12	15	27	15	16	24	16	26	10	26	17	12	17	12	5
0	12	34	6	23	12	22	21	22	27	6	27	17	11	17	23	6	23	12	11
0	12	35	7	25	12	23	20	23	27	5	27	18	10	18	25	7	25	12	13
0	13	28	50	36	13	15	27	15	22	18	22	14	8	14	19	5	19	13	23
0	16	41	8	25	16	25	14	25	22	8	22	17	16	17	25	8	25	16	9
0	17	37	1	32	17	20	18	20	19	16	19	24	5	24	23	1	23	17	15
0	19	40	13	33	19	21	15	21	27	6	27	20	7	20	22	13	22	19	14

Table A26. Cont.

(d) Cyclic $\mathcal{C}(3, 5, 55)$ —second half of the reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	52	2	6	7	3	5	2	5	4	9	4	1	5	1	7	6	7	3	10
0	8	11	16	17	8	3	11	3	5	8	5	1	6	1	17	16	17	8	9
0	11	13	20	21	11	2	13	2	7	9	7	1	8	1	21	20	21	11	10
0	9	11	19	20	9	2	11	2	8	10	8	1	9	1	20	19	20	9	11
0	11	15	24	25	11	4	15	4	9	13	9	1	10	1	25	24	25	11	14
0	11	5	15	16	11	6	5	6	10	4	10	1	11	1	16	15	16	11	5
0	17	10	21	22	17	7	10	7	11	4	11	1	12	1	22	21	22	17	5
0	27	17	29	30	27	10	17	10	12	2	12	1	13	1	25	26	25	27	3
0	20	8	22	23	20	12	8	12	14	2	14	1	15	1	23	22	23	20	3
0	15	5	18	20	15	10	5	10	13	3	13	2	15	2	20	18	20	15	5
0	18	7	21	23	18	11	7	11	14	3	14	2	16	2	23	21	23	18	5
0	22	10	25	27	22	12	10	12	15	3	15	2	17	2	27	25	27	22	5
0	18	8	24	26	18	10	8	10	16	6	16	2	18	2	26	24	26	18	8
0	19	3	23	25	19	16	3	16	20	4	20	2	22	2	25	23	25	19	6
0	19	10	24	27	19	9	10	9	14	5	14	3	17	3	27	24	27	19	8
0	24	15	30	33	24	9	15	9	15	6	15	3	18	3	22	25	22	24	9
0	20	14	31	34	20	6	14	6	17	11	17	3	20	3	21	24	21	20	14
0	29	15	33	36	26	14	15	14	18	4	18	3	21	3	19	22	19	26	7
0	13	5	20	24	13	8	5	8	15	7	15	4	19	4	24	20	24	13	11
0	20	7	25	29	20	13	7	13	18	5	18	4	22	4	26	25	26	20	9
0	22	9	28	32	22	13	9	13	19	6	19	4	23	4	23	27	23	22	10
0	40	8	29	33	15	23	8	23	21	11	21	4	25	4	22	26	22	15	7
0	16	22	29	34	16	6	22	6	7	13	7	5	12	5	21	26	21	16	18
0	31	4	20	25	24	27	4	27	16	11	16	5	21	5	25	20	25	24	6
0	36	47	17	26	19	11	8	11	25	19	25	9	21	9	26	17	26	19	10
0	33	7	21	31	22	26	7	26	14	12	14	10	24	10	24	21	24	22	2
0	32	45	15	25	23	13	10	13	25	17	25	10	20	10	25	15	25	23	7
0	20	44	15	26	20	24	11	24	26	5	26	11	18	11	26	15	26	20	6
0	29	45	5	17	26	16	10	16	15	24	15	12	27	12	17	5	17	26	12
0	17	44	11	23	17	27	11	27	22	6	22	12	21	12	23	11	23	17	6
0	18	45	13	25	18	27	10	27	23	5	23	12	20	12	25	13	25	18	7
0	41	8	23	36	14	22	8	22	15	18	15	13	27	13	19	23	19	14	5
0	17	39	9	25	17	22	16	22	25	8	25	16	14	16	25	9	25	17	8
0	31	50	15	32	24	19	5	19	20	16	20	17	18	17	23	15	23	24	1
0	20	48	14	33	20	27	7	27	21	6	21	19	15	19	22	14	22	20	13

Table A27. $\mathcal{C}(3, 5, 55)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) $\mathcal{C}(3, 5, 55)$ —symmetric cycles for the symmetric triplets (5,5,10), (10,10,20), (15,15,25), (20,20,15), and (25,25,5).																
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)						
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	
(0,5,53,2,50)	2	53 0 5	1 3 8	3	2 5 53	1 4 52	3	50 53 2	1 4 8	2	0 2 50	1 3 51	5	50 0 5	1 6 11	
(0,10,51,4,45)	4	51 0 10	1 5 15	6	4 10 51	1 7 48	6	45 51 4	1 7 15	4	0 4 45	1 5 46	10	45 0 10	1 11 21	
(0,15,49,6,40)	6	49 0 15	1 7 22	9	6 15 49	1 10 44	9	40 49 6	1 10 22	6	0 6 40	1 7 41	15	40 0 15	1 16 31	
(0,20,47,8,35)	8	47 0 20	1 9 29	12	8 20 47	1 13 40	12	35 47 8	1 13 29	8	0 8 35	1 9 36	15	20 35 0	1 16 36	
(0,25,43,12,30)	12	43 0 25	1 13 38	13	12 25 43	1 14 32	13	30 43 12	1 14 38	12	0 12 30	1 13 31	5	25 30 0	1 6 31	

Table A27. Cont.

(b) C(3, 5, 55)—the remaining symmetric cycles.																		
Cycles		(x ₁ , x ₂ , x ₃)		(x ₂ , x ₃ , x ₄)		(x ₃ , x ₄ , x ₅)		(x ₄ , x ₅ , x ₁)		(x ₅ , x ₁ , x ₂)								
(x ₁ , x ₂ , x ₃ , x ₄ , x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0, 1, 52, 3, 54)	1	0 1 52	1 2 53	2	1 3 52	1 3 52	2	52 54 3	1 3 7	1	54 0 3	1 2 5	1	54 0 1	1 2 3			
(0, 2, 49, 6, 53)	2	0 2 49	1 3 50	4	2 6 49	1 5 48	4	49 53 6	1 5 13	2	53 0 6	1 3 9	2	53 0 2	1 3 5			
(0, 3, 46, 9, 52)	3	0 3 46	1 4 47	6	3 9 46	1 7 44	6	46 52 9	1 7 19	3	52 0 9	1 4 13	3	52 0 3	1 4 7			
(0, 4, 43, 12, 51)	4	0 4 43	1 5 44	8	4 12 43	1 9 40	8	43 51 12	1 9 25	4	51 0 12	1 5 17	4	51 0 4	1 5 9			
(0, 6, 37, 18, 49)	6	0 6 37	1 7 38	12	6 18 37	1 13 32	12	37 49 18	1 13 37	6	49 0 18	1 7 25	6	49 0 6	1 7 13			
(0, 7, 34, 21, 48)	7	0 7 34	1 8 35	13	21 34 7	1 14 42	13	21 34 48	1 14 28	7	48 0 21	1 8 29	7	48 0 7	1 8 15			
(0, 8, 31, 24, 47)	8	0 8 31	1 9 32	7	24 31 8	1 8 40	7	24 31 47	1 8 24	8	47 0 24	1 9 33	8	47 0 8	1 9 17			
(0, 9, 28, 27, 46)	9	0 9 28	1 10 29	1	27 28 9	1 2 38	1	27 28 46	1 2 20	9	46 0 27	1 10 37	9	46 0 9	1 10 19			
(0, 11, 22, 33, 44)	11	0 11 22	1 12 23	11	11 22 33	1 12 23	11	22 33 44	1 12 23	11	33 44 0	1 12 23	11	44 0 11	1 12 23			
(0, 12, 19, 36, 43)	7	12 19 0	1 8 44	7	12 19 36	1 8 25	7	36 43 19	1 8 39	7	36 43 0	1 8 20	12	43 0 12	1 13 25			
(0, 13, 16, 39, 42)	3	13 16 0	1 4 43	3	13 16 39	1 4 27	3	39 42 16	1 4 33	3	39 42 0	1 4 17	13	42 0 13	1 14 27			
(0, 14, 13, 42, 41)	1	13 14 0	1 2 43	1	13 14 42	1 2 30	1	41 42 13	1 2 28	1	41 42 0	1 2 15	14	41 0 14	1 15 29			
(0, 16, 7, 48, 39)	7	0 7 16	1 8 17	9	7 16 48	1 10 42	9	39 48 7	1 10 24	7	48 0 39	1 8 47	16	39 0 16	1 17 33			
(0, 17, 4, 51, 38)	4	0 4 17	1 5 18	8	51 4 17	1 9 22	8	51 4 38	1 9 43	4	51 0 38	1 5 43	17	38 0 17	1 18 35			
(0, 18, 1, 54, 37)	1	0 1 18	1 2 19	2	54 1 18	1 3 20	2	54 1 37	1 3 39	1	54 0 37	1 2 39	18	37 0 18	1 19 37			
(0, 19, 53, 2, 36)	2	53 0 19	1 3 22	4	53 2 19	1 5 22	4	53 2 36	1 5 39	2	0 2 36	1 3 37	17	19 36 0	1 18 37			
(0, 21, 47, 8, 34)	8	47 0 21	1 9 30	13	8 21 47	1 14 40	13	34 47 8	1 14 30	8	0 8 34	1 9 35	13	21 34 0	1 14 35			
(0, 22, 44, 11, 33)	11	44 0 22	1 12 34	11	11 22 44	1 12 34	11	33 44 11	1 12 34	11	0 11 33	1 12 34	11	22 33 0	1 12 34			
(0, 23, 41, 14, 32)	14	41 0 23	1 15 38	9	14 23 41	1 10 28	9	32 41 14	1 10 38	14	0 14 32	1 15 33	9	23 32 0	1 10 33			
(0, 24, 38, 17, 31)	14	24 38 0	1 15 32	7	17 24 38	1 8 22	7	31 38 17	1 8 42	14	17 31 0	1 15 39	7	24 31 0	1 8 32			
(0, 26, 32, 23, 29)	6	26 32 0	1 7 30	3	23 26 32	1 4 10	3	29 32 23	1 4 50	6	23 29 0	1 7 33	3	26 29 0	1 4 30			
(0, 27, 29, 26, 28)	2	27 29 0	1 3 29	1	26 27 29	1 2 4	1	28 29 26	1 2 54	2	26 28 0	1 3 30	1	27 28 0	1 2 29			
(c) First half of the remaining 26 reflected base cycle pairs.																		
Cycles		(x ₁ , x ₂ , x ₃)		(x ₂ , x ₃ , x ₄)		(x ₃ , x ₄ , x ₅)		(x ₄ , x ₅ , x ₁)		(x ₅ , x ₁ , x ₂)								
(x ₁ , x ₂ , x ₃ , x ₄ , x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0, 1, 5, 10, 7)	1	0 1 5	1 2 6	4	1 5 10	1 5 10	2	5 7 10	1 3 6	3	7 10 0	1 4 49	1	0 1 7	1 2 8			
(0, 1, 6, 9, 17)	1	0 1 6	1 2 7	3	6 9 1	1 4 51	3	6 9 17	1 4 12	8	9 17 0	1 9 47	1	0 1 17	1 2 18			
(0, 1, 8, 10, 21)	1	0 1 8	1 2 9	2	8 10 1	1 3 49	2	8 10 21	1 3 14	10	0 10 21	1 11 22	1	0 1 21	1 2 22			
(0, 1, 9, 11, 20)	1	0 1 9	1 2 10	2	9 11 1	1 3 48	2	9 11 20	1 3 12	9	11 20 0	1 10 45	1	0 1 20	1 2 21			
(0, 1, 10, 14, 25)	1	0 1 10	1 2 11	4	10 14 1	1 5 47	4	10 14 25	1 5 16	11	14 25 0	1 12 42	1	0 1 25	1 2 26			
(0, 1, 11, 5, 16)	1	0 1 11	1 2 12	4	1 5 11	1 5 11	5	11 16 5	1 6 50	5	0 5 16	1 6 17	1	0 1 16	1 2 17			
(0, 1, 12, 5, 22)	1	0 1 12	1 2 13	4	1 5 12	1 5 12	7	5 12 22	1 8 18	5	0 5 22	1 6 23	1	0 1 22	1 2 23			
(0, 1, 13, 3, 30)	1	0 1 13	1 2 14	2	1 3 13	1 3 13	10	3 13 30	1 11 28	3	0 3 30	1 4 31	1	0 1 30	1 2 31			
(0, 1, 15, 3, 23)	1	0 1 15	1 2 16	2	1 3 15	1 3 15	8	15 23 3	1 9 44	3	0 3 23	1 4 24	1	0 1 23	1 2 24			
(0, 2, 15, 5, 20)	2	0 2 15	1 3 16	3	2 5 15	1 4 14	5	15 20 5	1 6 46	5	0 5 20	1 6 21	2	0 2 20	1 3 21			
(0, 2, 16, 5, 23)	2	0 2 16	1 3 17	3	2 5 16	1 4 15	7	16 23 5	1 8 45	5	0 5 23	1 6 24	2	0 2 23	1 3 24			
(0, 2, 17, 5, 27)	2	0 2 17	1 3 18	3	2 5 17	1 4 16	10	17 27 5	1 11 44	5	0 5 27	1 6 28	2	0 2 27	1 3 28			
(0, 2, 18, 8, 26)	2	0 2 18	1 3 19	6	2 8 18	1 7 17	8	18 26 8	1 9 46	8	0 8 26	1 9 27	2	0 2 26	1 3 27			
(0, 2, 22, 6, 25)	2	0 2 22	1 3 23	4	2 6 22	1 5 21	3	22 25 6	1 4 40	6	0 6 25	1 7 26	2	0 2 25	1 3 26			
(0, 3, 17, 8, 27)	3	0 3 17	1 4 18	5	3 8 17	1 6 15	9	8 17 27	1 10 20	8	0 8 27	1 9 28	3	0 3 27	1 4 28			
(0, 3, 18, 9, 33)	3	0 3 18	1 4 19	6	3 9 18	1 7 16	9	9 18 33	1 10 25	9	0 9 33	1 10 34	3	0 3 33	1 4 34			
(0, 3, 20, 14, 34)	3	0 3 20	1 4 21	6	14 20 3	1 7 45	6	14 20 34	1 7 21	14	0 14 34	1 15 35	3	0 3 34	1 4 35			
(0, 3, 21, 7, 36)	3	0 3 21	1 4 22	4	3 7 21	1 5 19	14	7 21 36	1 15 30	7	0 7 36	1 8 37	3	0 3 36	1 4 37			
(0, 4, 19, 11, 24)	4	0 4 19	1 5 20	7	4 11 19	1 8 16	5	19 24 11	1 6 48	11	0 11 24	1 12 25	4	0 4 24	1 5 25			
(0, 4, 22, 9, 29)	4	0 4 22	1 5 23	5	4 9 22	1 6 19	7	22 29 9	1 8 43	9	0 9 29	1 10 30	4	0 4 29	1 5 30			
(0, 4, 23, 10, 32)	4	0 4 23	1 5 24	6	4 10 23	1 7 20	9	23 32 10	1 10 43	10	0 10 32	1 11 33	4	0 4 32	1 5 33			
(0, 4, 25, 48, 33)	4	0 4 25	1 5 26	11	48 4 25	1 12 33	8	25 33 48	1 9 24	7	48 0 33	1 8 41	4	0 4 33	1 5 34			
(0, 5, 12, 18, 34)	5	0 5 12	1 6 13	6	12 18 5	1 7 49	6	12 18 34	1 7 23	16	18 34 0	1 17 38	5	0 5 34	1 6 35			
(0, 5, 21, 49, 25)	5	0 5 21	1 6 22	11	49 5 21	1 12 28	4	21 25 49	1 5 29	6	49 0 25	1 7 32	5	0 5 25	1 6 26			
(0, 9, 34, 45, 26)	9	0 9 34	1 10 35	11	34 45 9	1 12 31	8	26 34 45	1 9 20	10	45 0 26	1 11 37	9	0 9 26	1 10 27			
(0, 10, 24, 53, 31)	10	0 10 24	1 11 25	12	53 10 24	1 13 27	7	24 31 53	1 8 30	2	53 0 31	1 3 34	10	0 10 31	1 11 32			
(0, 10, 35, 48, 25)	10	0 10 35	1 11 36	13	35 48 10	1 14 31	10	25 35 48	1 11 24	7	48 0 25	1 8 33	10	0 10 25	1 11 26			
(0, 11, 37, 6, 26)	11	0 11 37	1 12 38	5	6 11 37	1 6 32	11	26 37 6	1 12 36	6	0 6 26	1 7 27	11	0 11 26	1 12 27			
(0, 12, 27, 43, 17)	12	0 12 27	1 13 38	15	12 27 43	1 16 32	10	17 27 43	1 11 27	12	43 0 17	1 13 30	5	12 17 0	1 6 44			
(0, 12, 34, 6, 23)	12	0 12 34	1 13 35	6	6 12 34	1 7 29	11	23 34 6	1 12 39	6	0 6 23	1 7 24	11	12 23 0	1 12 44			
(0, 12, 35, 7, 25)	12	0 12 35	1 13 36	5	7 12 35	1 6 29	10	25 35 7	1 11 38	7	0 7 25	1 8 26	12	0 12 25	1 13 26			
(0, 13, 28, 50, 36)	13	0 13 28	1 14 29	15	13 28 50	1 16 38	8	28 36 50	1 9 23	5	50 0 36	1 6 42	13	0 13 36	1 14 37			
(0, 16, 41, 8, 25)	14	41 0 16	1 15 31	8	8 16 41	1 9 34	16	25 41 8	1 17 39	8	0 8 25	1 9 26	9	16 25 0	1 10 40			
(0, 17, 37, 1, 32)	17	0 17 37	1 18 38	16	1 17 37	1 17 37	5	32 37 1	1 6 25	1	0 1 32	1 2 33	15	17 32 0	1 16 39			
(0, 19, 40, 13, 33)	15	40 0 19	1 16 35	6	13 19 40	1 7 28	7	33 40 13	1 8 36	13	0 13 33	1 14 34	14	19 33 0	1 15 37			

Table A27. Cont.

(d) Second half of the remaining 26 reflected base cycle pairs.															
Cycles		(x ₁ ,x ₂ ,x ₃)		(x ₂ ,x ₃ ,x ₄)		(x ₃ ,x ₄ ,x ₅)		(x ₄ ,x ₅ ,x ₁)		(x ₅ ,x ₁ ,x ₂)					
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,52,2,6,7)	2	0 2 52	1 3 53	4	2 6 52	1 5 51	1	6 7 2	1 2 52	1	6 7 0	1 2 50	3	52 0 7	1 4 11
(0,8,11,16,17)	3	8 11 0	1 4 48	3	8 11 16	1 4 9	1	16 17 11	1 2 51	1	16 17 0	1 2 40	8	0 8 17	1 9 18
(0,11,13,20,21)	2	11 13 0	1 3 45	2	11 13 20	1 3 10	1	20 21 13	1 2 49	1	20 21 0	1 2 36	10	11 21 0	1 11 45
(0,9,11,19,20)	2	9 11 0	1 3 47	2	9 11 19	1 3 11	1	19 20 11	1 2 48	1	19 20 0	1 2 37	9	0 9 20	1 10 21
(0,11,15,24,25)	4	11 15 0	1 5 45	4	11 15 24	1 5 14	1	24 25 15	1 2 47	1	24 25 0	1 2 32	11	0 11 25	1 12 26
(0,11,5,15,16)	5	0 5 11	1 6 12	4	11 15 5	1 5 50	1	15 16 5	1 2 46	1	15 16 0	1 2 41	5	11 16 0	1 6 45
(0,17,10,21,22)	7	10 17 0	1 8 46	4	17 21 10	1 5 49	1	21 22 10	1 2 45	1	21 22 0	1 2 35	5	17 22 0	1 6 39
(0,27,17,29,30)	10	17 27 0	1 11 39	2	27 29 17	1 3 46	1	29 30 17	1 2 44	1	29 30 0	1 2 27	3	27 30 0	1 4 29
(0,20,8,22,23)	8	0 8 20	1 9 21	2	20 22 8	1 3 44	1	22 23 8	1 2 42	1	22 23 0	1 2 34	3	20 23 0	1 4 36
(0,15,5,18,20)	5	0 5 15	1 6 16	3	15 18 5	1 4 46	2	18 20 5	1 3 43	2	18 20 0	1 3 38	5	15 20 0	1 6 41
(0,18,7,21,23)	7	0 7 18	1 8 19	3	18 21 7	1 4 45	2	21 23 7	1 3 42	2	21 23 0	1 3 35	5	18 23 0	1 6 38
(0,22,10,25,27)	10	0 10 22	1 11 23	3	22 25 10	1 4 44	2	25 27 10	1 3 41	2	25 27 0	1 3 31	5	22 27 0	1 6 34
(0,18,8,24,26)	8	0 8 18	1 9 19	6	18 24 8	1 7 46	2	24 26 8	1 3 40	2	24 26 0	1 3 32	8	18 26 0	1 9 38
(0,19,3,23,25)	3	0 3 19	1 4 20	4	19 23 3	1 5 40	2	23 25 3	1 3 36	2	23 25 0	1 3 33	6	19 25 0	1 7 37
(0,19,10,24,27)	9	10 19 0	1 10 46	5	19 24 10	1 6 47	3	24 27 10	1 4 42	3	24 27 0	1 4 32	8	19 27 0	1 9 37
(0,24,15,30,33)	9	15 24 0	1 10 41	6	24 30 15	1 7 47	3	30 33 15	1 4 41	3	30 33 0	1 4 26	9	24 33 0	1 10 32
(0,20,14,31,34)	6	14 20 0	1 7 42	6	14 20 31	1 7 18	3	31 34 14	1 4 39	3	31 34 0	1 4 25	14	20 34 0	1 15 36
(0,29,15,33,36)	14	15 29 0	1 15 41	4	29 33 15	1 5 42	3	33 36 15	1 4 38	3	33 36 0	1 4 23	7	29 36 0	1 8 27
(0,13,5,20,24)	5	0 5 13	1 6 14	7	13 20 5	1 8 48	4	20 24 5	1 5 41	4	20 24 0	1 5 36	11	13 24 0	1 12 43
(0,20,7,25,29)	7	0 7 20	1 8 21	5	20 25 7	1 6 43	4	25 29 7	1 5 38	4	25 29 0	1 5 31	9	20 29 0	1 10 36
(0,22,9,28,32)	9	0 9 22	1 10 23	6	22 28 9	1 7 43	4	28 32 9	1 5 37	4	28 32 0	1 5 28	10	22 32 0	1 11 34
(0,40,8,29,33)	8	0 8 40	1 9 41	11	29 40 8	1 12 35	4	29 33 8	1 5 35	4	29 33 0	1 5 27	7	33 40 0	1 8 23
(0,16,22,29,34)	6	16 22 0	1 7 40	6	16 22 29	1 7 14	5	29 34 22	1 6 49	5	29 34 0	1 6 27	16	0 16 34	1 17 35
(0,31,4,20,25)	4	0 4 31	1 5 32	11	20 31 4	1 12 40	5	20 25 4	1 6 40	5	20 25 0	1 6 36	6	25 31 0	1 7 31
(0,36,47,17,26)	8	47 0 36	1 9 45	11	36 47 17	1 12 37	9	17 26 47	1 10 31	9	17 26 0	1 10 39	10	26 36 0	1 11 30
(0,33,7,21,31)	7	0 7 33	1 8 34	12	21 33 7	1 13 42	10	21 31 7	1 11 42	10	21 31 0	1 11 35	2	31 33 0	1 3 25
(0,32,45,15,25)	10	45 0 32	1 11 43	13	32 45 15	1 14 39	10	15 25 45	1 11 31	10	15 25 0	1 11 41	7	25 32 0	1 8 31
(0,20,44,15,26)	11	44 0 20	1 12 32	5	15 20 44	1 6 30	11	15 26 44	1 12 30	11	15 26 0	1 12 41	6	20 26 0	1 7 36
(0,29,45,5,17)	10	45 0 29	1 11 40	15	45 5 29	1 16 40	12	5 17 45	1 13 41	5	0 5 17	1 6 18	12	17 29 0	1 13 39
(0,17,44,11,23)	11	44 0 17	1 12 29	6	11 17 44	1 7 34	12	11 23 44	1 13 34	11	0 11 23	1 12 24	6	17 23 0	1 7 39
(0,18,45,13,25)	10	45 0 18	1 11 29	5	13 18 45	1 6 33	12	13 25 45	1 13 33	12	13 25 0	1 13 43	7	18 25 0	1 8 38
(0,41,8,23,36)	8	0 8 41	1 9 42	15	8 23 41	1 16 34	13	23 36 8	1 14 41	13	23 36 0	1 14 33	5	36 41 0	1 6 20
(0,17,39,9,25)	16	39 0 17	1 17 34	8	9 17 39	1 9 31	14	25 39 9	1 15 40	9	0 9 25	1 10 26	8	17 25 0	1 9 39
(0,31,50,15,32)	5	50 0 31	1 6 37	16	15 31 50	1 17 36	17	15 32 50	1 18 36	15	0 15 32	1 16 33	1	31 32 0	1 2 25
(0,20,48,14,33)	7	48 0 20	1 8 28	6	14 20 48	1 7 35	15	33 48 14	1 16 37	14	0 14 33	1 15 34	13	20 33 0	1 14 36

Table A28. Cyclic $\mathcal{C}(3,5,56)$ —vertices and distances in base cycles. The bright yellow cells within the symmetric cycles correspond to the symmetric difference triplets. The distances in the pale green and blue cells have their reflected pairs within each cycle. The bright green cells mark the base cycle vertices where the original +2 rule had to be modified for v4 (which means a modification for v3 as well.) In the case of reflected base cycle pairs (1st and 2nd halves), the cells corresponding to reflected difference triplets are marked by the same color for the first cycle pair as an example.

(a) Cyclic $\mathcal{C}(3,5,56)$ —symmetric cycles for odd values. The vertices in the 4th column were selected using a +2 rule, but a modification had to be made. The exception is marked.

Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	54	2	55	1	3	2	3	4	1	4	3	1	3	1	2	1	1	2
0	3	50	6	53	3	9	6	9	12	3	12	9	3	9	3	6	3	3	6
0	5	46	10	51	5	15	10	15	20	5	20	15	5	15	5	10	5	5	10
0	7	44	12	49	7	19	12	19	24	5	24	19	5	19	7	12	7	7	14
0	9	38	18	47	9	27	18	27	20	9	20	27	9	27	9	18	9	9	18
0	11	34	22	45	11	23	22	23	12	11	12	23	11	23	11	22	11	11	22
0	13	30	26	43	13	17	26	17	4	13	4	17	13	17	13	26	13	13	26
0	15	26	30	41	15	11	26	11	4	15	4	11	15	11	15	26	15	15	26
0	17	22	34	39	17	5	22	5	12	17	12	5	17	5	17	22	17	17	22
0	19	18	38	37	19	1	18	1	20	19	20	1	19	1	19	18	19	19	18
0	21	14	42	35	21	7	14	7	28	21	28	7	21	7	21	14	21	21	14
0	23	10	46	33	23	13	10	13	20	23	20	13	23	13	23	10	23	23	10
0	25	6	50	31	25	19	6	19	12	25	12	19	25	19	25	6	25	25	6
0	27	2	54	29	27	25	2	25	4	27	4	25	27	25	27	2	27	27	2

Table A28. Cont.

(b) $\mathcal{C}(3, 5, 56)$ —symmetric cycles for even values. The vertices in the 4th column were selected using a +2 rule, but modifications had to be made. The exceptions are marked.

Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	2	52	4	54	2	6	4	6	8	2	8	6	2	6	2	4	2	2	4
0	4	48	8	52	4	12	8	12	16	4	16	12	4	12	4	8	4	4	8
0	6	44	12	50	6	18	12	18	24	6	24	18	6	18	6	12	6	6	12
0	8	42	14	48	8	22	14	22	28	6	28	22	6	22	8	14	8	8	16
0	10	36	20	46	10	26	20	26	16	10	16	26	10	26	10	20	10	10	20
0	12	32	24	44	12	20	24	20	8	12	8	20	12	20	12	24	12	12	24
0	14	32	24	42	14	18	24	18	8	10	8	18	10	18	14	24	14	14	28
0	16	30	26	40	16	14	26	14	4	10	4	14	10	14	16	26	16	16	24
0	18	20	36	38	18	2	20	2	16	18	16	2	18	2	18	20	18	18	20
0	20	16	40	36	20	4	16	4	24	20	24	4	20	4	20	16	20	20	16
0	22	12	44	34	22	10	12	10	24	22	24	10	22	10	22	12	22	22	12
0	24	10	46	32	24	14	10	14	20	22	20	14	22	14	24	10	24	24	8
0	26	4	52	30	26	22	4	22	8	26	8	22	26	22	26	4	26	26	4

(c) $\mathcal{C}(3, 5, 56)$ —first half of the reflected base cycle pairs.

Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	1	5	8	10	1	4	5	4	3	7	3	2	5	2	10	8	10	1	9
0	1	6	8	15	1	5	6	5	2	7	2	7	9	7	15	8	15	1	14
0	1	7	11	16	1	6	7	6	4	10	4	5	9	5	16	11	16	1	15
0	1	8	11	21	1	7	8	7	3	10	3	10	13	10	21	11	21	1	20
0	1	9	4	18	1	8	9	8	5	3	5	14	9	14	18	4	18	1	17
0	1	11	13	22	1	10	11	10	2	12	2	9	11	9	22	13	22	1	21
0	1	12	4	23	1	11	12	11	8	3	8	19	11	19	23	4	23	1	22
0	1	13	6	17	1	12	13	12	7	5	7	11	4	11	17	6	17	1	16
0	1	14	3	27	1	13	14	13	11	2	11	24	13	24	27	3	27	1	26
0	1	24	3	28	1	23	24	23	21	2	21	25	4	25	28	3	28	1	27
0	1	25	8	31	1	24	25	24	17	7	17	23	6	23	25	8	25	1	26
0	2	14	17	28	2	12	14	12	3	15	3	11	14	11	28	17	28	2	26
0	2	15	6	21	2	13	15	13	9	4	9	15	6	15	21	6	21	2	19
0	2	16	8	25	2	14	16	14	8	6	8	17	9	17	25	8	25	2	23
0	2	17	20	39	2	15	17	15	3	18	3	19	22	19	17	20	17	2	19
0	2	22	5	26	2	20	22	20	17	3	17	21	4	21	26	5	26	2	24
0	3	16	8	29	3	13	16	13	8	5	8	21	13	21	27	8	27	3	26
0	3	17	22	35	3	14	17	14	5	19	5	13	18	13	21	22	21	3	24
0	3	19	9	26	3	16	19	16	10	6	10	17	7	17	26	9	26	3	23
0	3	21	43	25	3	18	21	18	22	16	22	18	4	18	25	13	25	3	22
0	4	19	26	32	4	15	19	15	7	22	7	6	13	6	24	26	24	4	28
0	5	11	24	31	5	6	11	6	13	19	13	7	20	7	25	24	25	5	26
0	5	21	12	33	5	16	21	16	9	7	9	21	12	21	23	12	23	5	28
0	6	22	49	27	6	16	22	16	27	13	27	22	5	22	27	7	27	6	21
0	7	36	48	18	7	27	20	27	12	15	12	26	18	26	18	8	18	7	11
0	9	33	50	23	9	24	23	24	17	15	17	27	10	27	23	6	23	9	14
0	10	31	48	19	10	21	25	21	17	18	17	27	12	27	19	8	19	10	9
0	11	35	48	20	11	24	21	24	13	19	13	28	15	28	20	8	20	11	9
0	11	36	51	27	11	25	20	25	15	16	15	24	9	24	27	5	27	11	16
0	12	28	5	25	12	16	28	16	23	7	23	20	3	20	25	5	25	12	13
0	14	29	50	20	14	15	27	15	21	20	21	26	9	26	20	6	20	14	6
0	14	37	7	26	14	23	19	23	26	7	26	19	11	19	26	7	26	14	12
0	14	43	10	25	14	27	13	27	23	4	23	15	18	15	25	10	25	14	11
0	15	34	5	23	15	19	22	19	27	10	27	18	11	18	23	5	23	15	8
0	16	33	2	24	16	17	23	17	25	14	25	22	9	22	24	2	24	16	8
0	16	35	7	25	16	19	21	19	28	9	28	18	10	18	25	7	25	16	9

Table A28. Cont.

(d) $\mathcal{C}(3, 5, 56)$ —second half of the reflected base cycle pairs.																			
Base cycles (5 vertices)					Distances in the difference triplets														
v1	v2	v3	v4	v5	1-2-3			2-3-4			3-4-5			4-5-1			5-1-2		
0	2	5	9	10	2	3	5	3	4	7	4	1	5	1	10	9	10	2	8
0	7	9	14	15	7	2	9	2	5	7	5	1	6	1	15	14	15	7	8
0	5	9	15	16	5	4	9	4	6	10	6	1	7	1	16	15	16	5	11
0	10	13	20	21	10	3	13	3	7	10	7	1	8	1	21	20	21	10	11
0	14	9	17	18	14	5	9	5	8	3	8	1	9	1	18	17	18	14	4
0	9	11	21	22	9	2	11	2	10	12	10	1	11	1	22	21	22	9	13
0	19	11	22	23	19	8	11	8	11	3	11	1	12	1	23	22	23	19	4
0	11	4	16	17	11	7	4	7	12	5	12	1	13	1	17	16	17	11	6
0	24	13	26	27	24	11	13	11	13	2	13	1	14	1	27	26	27	24	3
0	25	4	27	28	25	21	4	21	23	2	23	1	24	1	28	27	28	25	3
0	23	6	30	31	23	17	6	17	24	7	24	1	25	1	25	26	25	23	8
0	11	14	26	28	11	3	14	3	12	15	12	2	14	2	28	26	28	11	17
0	15	6	19	21	15	9	6	9	13	4	13	2	15	2	21	19	21	15	6
0	17	9	23	25	17	8	9	8	14	6	14	2	16	2	25	23	25	17	8
0	19	22	37	39	19	3	22	3	15	18	15	2	17	2	17	19	17	19	20
0	21	4	24	26	21	17	4	17	20	3	20	2	22	2	26	24	26	21	5
0	21	13	26	29	21	8	13	8	13	5	13	3	16	3	27	26	27	21	8
0	13	18	32	35	13	5	18	5	14	19	14	3	17	3	21	24	21	13	22
0	17	7	23	26	17	10	7	10	16	6	16	3	19	3	26	23	26	17	9
0	38	4	22	25	18	22	4	22	18	16	18	3	21	3	25	22	25	18	13
0	6	13	28	32	6	7	13	7	15	22	15	4	19	4	24	28	24	6	26
0	7	20	26	31	7	13	20	13	6	19	6	5	11	5	25	26	25	7	24
0	21	12	28	33	21	9	12	9	16	7	16	5	21	5	23	28	23	21	12
0	34	5	21	27	22	27	5	27	16	13	16	6	22	6	27	21	27	22	7
0	26	38	11	18	26	12	18	12	27	15	27	7	20	7	18	11	18	26	8
0	29	46	14	23	27	17	10	17	24	15	24	9	23	9	23	14	23	27	6
0	27	44	9	19	27	17	12	17	21	18	21	10	25	10	19	9	19	27	8
0	28	41	9	20	28	13	15	13	24	19	24	11	21	11	20	9	20	28	8
0	32	47	16	27	24	15	9	15	25	16	25	11	20	11	27	16	27	24	5
0	20	53	13	25	20	23	3	23	16	7	16	12	28	12	25	13	25	20	5
0	26	47	6	20	26	21	9	21	15	20	15	14	27	14	20	6	20	26	6
0	19	45	12	26	19	26	11	26	23	7	23	14	19	14	26	12	26	19	7
0	15	38	11	25	15	23	18	23	27	4	27	14	13	14	25	11	25	15	10
0	18	45	8	23	18	27	11	27	19	10	19	15	22	15	23	8	23	18	5
0	22	47	8	24	22	25	9	25	17	14	17	16	23	16	24	8	24	22	2
0	18	46	9	25	18	28	10	28	19	9	19	16	21	16	25	9	25	18	7

Table A29. $\mathcal{C}(3, 5, 56)$ —for each base cycle, the corresponding 5 triples are analyzed. Three columns belong to each triple: **d**—the minimum difference value, **triple**—the ordered version of the original triple, and **type**—the triple type. The meaning of these is discussed at the beginning of the Appendix A.

(a) Symmetric cycles for odd values.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,54,2,55)	1	0 1 54	1 2 55	1	1 2 54	1 2 54	1	54 55 2	1 2 5	1	55 0 2	1 2 4	1	55 0 1	1 2 3
(0,3,50,6,53)	3	0 3 50	1 4 51	3	3 6 50	1 4 48	3	50 53 6	1 4 13	3	53 0 6	1 4 10	3	53 0 3	1 4 7
(0,5,46,10,51)	5	0 5 46	1 6 47	5	5 10 46	1 6 42	5	46 51 10	1 6 21	5	51 0 10	1 6 16	5	51 0 5	1 6 11
(0,7,44,12,49)	7	0 7 44	1 8 45	5	7 12 44	1 6 38	5	44 49 12	1 6 25	7	49 0 12	1 8 20	7	49 0 7	1 8 15
(0,9,38,18,47)	9	0 9 38	1 10 39	9	9 18 38	1 10 30	9	38 47 18	1 10 37	9	47 0 18	1 10 28	9	47 0 9	1 10 19
(0,11,34,22,45)	11	0 11 34	1 12 35	11	11 22 34	1 12 24	11	34 45 22	1 12 45	11	45 0 22	1 12 34	11	45 0 11	1 12 23
(0,13,30,26,43)	13	0 13 30	1 14 31	4	26 30 13	1 5 44	4	26 30 43	1 5 18	13	43 0 26	1 14 40	13	43 0 13	1 14 27
(0,15,26,30,41)	11	15 26 0	1 12 42	4	26 30 15	1 5 46	4	26 30 41	1 5 16	11	30 41 0	1 12 27	15	41 0 15	1 16 31
(0,17,22,34,39)	5	17 22 0	1 6 40	5	17 22 34	1 6 18	5	34 39 22	1 6 45	5	34 39 0	1 6 23	17	39 0 17	1 18 35
(0,19,18,38,37)	1	18 19 0	1 2 39	1	18 19 38	1 2 21	1	37 38 18	1 2 38	1	37 38 0	1 2 20	18	19 37 0	1 19 38
(0,21,14,42,35)	7	14 21 0	1 8 43	7	14 21 42	1 8 29	7	35 42 14	1 8 36	7	35 42 0	1 8 22	14	21 35 0	1 15 36
(0,23,10,46,33)	10	0 10 23	1 11 24	13	10 23 46	1 14 37	13	33 46 10	1 14 34	10	46 0 33	1 11 44	10	23 33 0	1 11 34
(0,25,6,50,31)	6	0 6 25	1 7 26	12	50 6 25	1 13 32	12	50 6 31	1 13 38	6	50 0 31	1 7 38	6	25 31 0	1 7 32
(0,27,2,54,29)	2	0 2 27	1 3 28	4	54 2 27	1 5 30	4	54 2 29	1 5 32	2	54 0 29	1 3 32	2	27 29 0	1 3 30

(b) Symmetric cycles for even values.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,2,52,4,54)	2	0 2 52	1 3 53	2	2 4 52	1 3 51	2	52 54 4	1 3 9	2	54 0 4	1 3 7	2	54 0 2	1 3 5
(0,4,48,8,52)	4	0 4 48	1 5 49	4	4 8 48	1 5 45	4	48 52 8	1 5 17	4	52 0 8	1 5 13	4	52 0 4	1 5 9
(0,6,44,12,50)	6	0 6 44	1 7 45	6	6 12 44	1 7 39	6	44 50 12	1 7 25	6	50 0 12	1 7 19	6	50 0 6	1 7 13
(0,8,42,14,48)	8	0 8 42	1 9 43	6	8 14 42	1 7 35	6	42 48 14	1 7 29	8	48 0 14	1 9 23	8	48 0 8	1 9 17
(0,10,36,20,46)	10	0 10 36	1 11 37	10	10 20 36	1 11 27	10	36 46 20	1 11 41	10	46 0 20	1 11 31	10	46 0 10	1 11 21
(0,12,32,24,44)	12	0 12 32	1 13 33	8	24 32 12	1 9 45	8	24 32 44	1 9 21	12	44 0 24	1 13 37	12	44 0 12	1 13 25
(0,14,32,24,42)	14	0 14 32	1 15 33	8	24 32 14	1 9 47	8	24 32 42	1 9 19	14	42 0 24	1 15 39	14	42 0 14	1 15 29
(0,16,30,26,40)	14	16 30 0	1 15 41	4	26 30 16	1 5 47	4	26 30 40	1 5 15	14	26 40 0	1 15 31	16	40 0 16	1 17 33
(0,18,20,36,38)	2	18 20 0	1 3 39	2	18 20 36	1 3 19	2	36 38 20	1 3 41	2	36 38 0	1 3 21	18	38 0 18	1 19 37
(0,20,16,40,36)	4	16 20 0	1 5 41	4	16 20 40	1 5 25	4	36 40 16	1 5 37	4	36 40 0	1 5 21	16	20 36 0	1 17 37
(0,22,12,44,34)	10	12 22 0	1 11 45	10	12 22 44	1 11 33	10	34 44 12	1 11 35	10	34 44 0	1 11 23	12	22 34 0	1 13 35
(0,24,10,46,32)	10	0 10 24	1 11 25	14	10 24 46	1 15 37	14	32 46 10	1 15 35	10	46 0 32	1 11 43	8	24 32 0	1 9 33
(0,26,4,52,30)	4	0 4 26	1 5 27	8	52 4 26	1 9 31	8	52 4 30	1 9 35	4	52 0 30	1 5 35	4	26 30 0	1 5 31

(c) First half of the reflected base cycle pairs.															
Cycles		(x_1, x_2, x_3)		(x_2, x_3, x_4)		(x_3, x_4, x_5)		(x_4, x_5, x_1)		(x_5, x_1, x_2)					
$(x_1, x_2, x_3, x_4, x_5)$	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type
(0,1,5,8,10)	1	0 1 5	1 2 6	3	5 8 1	1 4 53	2	8 10 5	1 3 54	2	8 10 0	1 3 49	1	0 1 10	1 2 11
(0,1,6,8,15)	1	0 1 6	1 2 7	2	6 8 1	1 3 52	2	6 8 15	1 3 10	7	8 15 0	1 8 49	1	0 1 15	1 2 16
(0,1,7,11,16)	1	0 1 7	1 2 8	4	7 11 1	1 5 51	4	7 11 16	1 5 10	5	11 16 0	1 6 46	1	0 1 16	1 2 17
(0,1,8,11,21)	1	0 1 8	1 2 9	3	8 11 1	1 4 50	3	8 11 21	1 4 14	10	11 21 0	1 11 46	1	0 1 21	1 2 22
(0,1,9,4,18)	1	0 1 9	1 2 10	3	1 4 9	1 4 9	5	4 9 18	1 6 15	4	0 4 18	1 5 19	1	0 1 18	1 2 19
(0,1,11,13,22)	1	0 1 11	1 2 12	2	11 13 1	1 3 47	2	11 13 22	1 3 12	9	13 22 0	1 10 44	1	0 1 22	1 2 23
(0,1,12,4,23)	1	0 1 12	1 2 13	3	1 4 12	1 4 12	8	4 12 23	1 9 20	4	0 4 23	1 5 24	1	0 1 23	1 2 24
(0,1,13,6,17)	1	0 1 13	1 2 14	5	1 6 13	1 6 13	4	13 17 6	1 5 50	6	0 6 17	1 7 18	1	0 1 17	1 2 18
(0,1,14,3,27)	1	0 1 14	1 2 15	2	1 3 14	1 3 14	11	3 14 27	1 12 25	3	0 3 27	1 4 28	1	0 1 27	1 2 28
(0,1,24,3,28)	1	0 1 24	1 2 25	2	1 3 24	1 3 24	4	24 28 3	1 5 36	3	0 3 28	1 4 29	1	0 1 28	1 2 29
(0,1,25,8,31)	1	0 1 25	1 2 26	7	1 8 25	1 8 25	6	25 31 8	1 7 40	8	0 8 31	1 9 32	1	0 1 31	1 2 32
(0,2,14,17,28)	2	0 2 14	1 3 15	3	14 17 2	1 4 45	3	14 17 28	1 4 15	11	17 28 0	1 12 40	2	0 2 28	1 3 29
(0,2,15,6,21)	2	0 2 15	1 3 16	4	2 6 15	1 5 14	6	15 21 6	1 7 48	6	0 6 21	1 7 22	2	0 2 21	1 3 22
(0,2,16,8,25)	2	0 2 16	1 3 17	6	2 8 16	1 7 15	8	8 16 25	1 9 18	8	0 8 25	1 9 26	2	0 2 25	1 3 26
(0,2,17,20,39)	2	0 2 17	1 3 18	3	17 20 2	1 4 42	3	17 20 39	1 4 23	17	39 0 20	1 18 38	2	0 2 39	1 3 40
(0,2,22,5,26)	2	0 2 22	1 3 23	3	2 5 22	1 4 21	4	22 26 5	1 5 40	5	0 5 26	1 6 27	2	0 2 26	1 3 27
(0,3,16,8,29)	3	0 3 16	1 4 17	5	3 8 16	1 6 14	8	8 16 29	1 9 22	8	0 8 29	1 9 30	3	0 3 29	1 4 30
(0,3,17,22,35)	3	0 3 17	1 4 18	5	17 22 3	1 6 43	5	17 22 35	1 6 19	13	22 35 0	1 14 35	3	0 3 35	1 4 36
(0,3,19,9,26)	3	0 3 19	1 4 20	6	3 9 19	1 7 17	7	19 26 9	1 8 47	9	0 9 26	1 10 27	3	0 3 26	1 4 27
(0,3,21,43,25)	3	0 3 21	1 4 22	16	43 3 21	1 17 35	4	21 25 43	1 5 23	13	43 0 25	1 14 39	3	0 3 25	1 4 26
(0,4,19,26,32)	4	0 4 19	1 5 20	7	19 26 4	1 8 42	6	26 32 19	1 7 50	6	26 32 0	1 7 31	4	0 4 32	1 5 33
(0,5,11,24,31)	5	0 5 11	1 6 12	6	5 11 24	1 7 20	7	24 31 11	1 8 44	7	24 31 0	1 8 33	5	0 5 31	1 6 32
(0,5,21,12,33)	5	0 5 21	1 6 22	7	5 12 21	1 8 17	9	12 21 33	1 10 22	12	0 12 33	1 13 34	5	0 5 33	1 6 34
(0,6,22,49,27)	6	0 6 22	1 7 23	13	49 6 22	1 14 30	5	22 27 49	1 6 28	7	49 0 27	1 8 35	6	0 6 27	1 7 28
(0,7,36,48,18)	7	0 7 36	1 8 37	12	36 48 7	1 13 28	12	36 48 18	1 13 39	8	48 0 18	1 9 27	7	0 7 18	1 8 19
(0,9,33,50,23)	9	0 9 33	1 10 34	15	50 9 33	1 16 40	10	23 33 50	1 11 28	6	50 0 23	1 7 30	9	0 9 23	1 10 24
(0,10,31,48,19)	10	0 10 31	1 11 32	17	31 48 10	1 18 36	12	19 31 48	1 13 30	8	48 0 19	1 9 28	9	10 19 0	1 10 47
(0,11,35,48,20)	11	0 11 35	1 12 36	13	35 48 11	1 14 33	13	35 48 20	1 14 42	8	48 0 20	1 9 29	9	11 20 0	1 10 46
(0,11,36,51,27)	11	0 11 36	1 12 37	15	36 51 11	1 16 32	9	27 36 51	1 10 25	5	51 0 27	1 6 33	11	0 11 27	1 12 28
(0,12,28,5,25)	12	0 12 28	1 13 29	7	5 12 28	1 8 24	3	25 28 5	1 4 37	5	0 5 25	1 6 26	12	0 12 25	1 13 26
(0,14,29,50,20)	14	0 14 29	1 15 30	15	14 29 50	1 16 37	9	20 29 50	1 10 31	6	50 0 20	1 7 27	6	14 20 0	1 7 43
(0,14,37,7,26)	14	0 14 37	1 15 38	7	7 14 37	1 8 31	11	26 37 7	1 12 38	7	0 7 26	1 8 27	12	14 26 0	1 13 43
(0,14,43,10,25)	13	43 0 14	1 14 28	4	10 14 43	1 5 34	15	10 25 43	1 16 34	10	0 10 25	1 11 26	11	14 25 0	1 12 43
(0,15,34,5,23)	15	0 15 34	1 16 35</												

Table A29. Cont.

(d) Second half of the reflected base cycle pairs.																	
Cycles			(x ₁ ,x ₂ ,x ₃)			(x ₂ ,x ₃ ,x ₄)			(x ₃ ,x ₄ ,x ₅)			(x ₄ ,x ₅ ,x ₁)			(x ₅ ,x ₁ ,x ₂)		
(x ₁ ,x ₂ ,x ₃ ,x ₄ ,x ₅)	d	triple	type	d	triple	type	d	triple	type	d	triple	type	d	triple	type		
(0,2,5,9,10)	2	0 2 5	1 3 6	3	2 5 9	1 4 8	1	9 10 5	1 2 53	1	9 10 0	1 2 48	2	0 2 10	1 3 11		
(0,7,9,14,15)	2	7 9 0	1 3 50	2	7 9 14	1 3 8	1	14 15 9	1 2 52	1	14 15 0	1 2 43	7	0 7 15	1 8 16		
(0,5,9,15,16)	4	5 9 0	1 5 52	4	5 9 15	1 5 11	1	15 16 9	1 2 51	1	15 16 0	1 2 42	5	0 5 16	1 6 17		
(0,10,13,20,21)	3	10 13 0	1 4 47	3	10 13 20	1 4 11	1	20 21 13	1 2 50	1	20 21 0	1 2 37	10	0 10 21	1 11 22		
(0,14,9,17,18)	5	9 14 0	1 6 48	3	14 17 9	1 4 52	1	17 18 9	1 2 49	1	17 18 0	1 2 40	4	14 18 0	1 5 43		
(0,9,11,21,22)	2	9 11 0	1 3 48	2	9 11 21	1 3 13	1	21 22 11	1 2 47	1	21 22 0	1 2 36	9	0 9 22	1 10 23		
(0,19,11,22,23)	8	11 19 0	1 9 46	3	19 22 11	1 4 49	1	22 23 11	1 2 46	1	22 23 0	1 2 35	4	19 23 0	1 5 38		
(0,11,4,16,17)	4	0 4 11	1 5 12	5	11 16 4	1 6 50	1	16 17 4	1 2 45	1	16 17 0	1 2 41	6	11 17 0	1 7 46		
(0,24,13,26,27)	11	13 24 0	1 12 44	2	24 26 13	1 3 46	1	26 27 13	1 2 44	1	26 27 0	1 2 31	3	24 27 0	1 4 33		
(0,25,4,27,28)	4	0 4 25	1 5 26	2	25 27 4	1 3 36	1	27 28 4	1 2 34	1	27 28 0	1 2 30	3	25 28 0	1 4 32		
(0,23,6,30,31)	6	0 6 23	1 7 24	7	23 30 6	1 8 40	1	30 31 6	1 2 33	1	30 31 0	1 2 27	8	23 31 0	1 9 34		
(0,11,14,26,28)	3	11 14 0	1 4 46	3	11 14 26	1 4 16	2	26 28 14	1 3 45	2	26 28 0	1 3 31	11	0 11 28	1 12 29		
(0,15,6,19,21)	6	0 6 15	1 7 16	4	15 19 6	1 5 48	2	19 21 6	1 3 44	2	19 21 0	1 3 38	6	15 21 0	1 7 42		
(0,17,9,23,25)	8	9 17 0	1 9 48	6	17 23 9	1 7 49	2	23 25 9	1 3 43	2	23 25 0	1 3 34	8	17 25 0	1 9 40		
(0,19,22,37,39)	3	19 22 0	1 4 38	3	19 22 37	1 4 19	2	37 39 22	1 3 42	2	37 39 0	1 3 20	17	39 0 19	1 18 37		
(0,21,4,24,26)	4	0 4 21	1 5 22	3	21 24 4	1 4 40	2	24 26 4	1 3 37	2	24 26 0	1 3 33	5	21 26 0	1 6 36		
(0,21,13,26,29)	8	13 21 0	1 9 44	5	21 26 13	1 6 49	3	26 29 13	1 4 44	3	26 29 0	1 4 31	8	21 29 0	1 9 36		
(0,13,18,32,35)	5	13 18 0	1 6 44	5	13 18 32	1 6 20	3	32 35 18	1 4 43	3	32 35 0	1 4 25	13	0 13 35	1 14 36		
(0,17,7,23,26)	7	0 7 17	1 8 18	6	17 23 7	1 7 47	3	23 26 7	1 4 41	3	23 26 0	1 4 34	9	17 26 0	1 10 40		
(0,38,4,22,25)	4	0 4 38	1 5 39	16	22 38 4	1 17 39	3	22 25 4	1 4 39	3	22 25 0	1 4 35	13	25 38 0	1 14 32		
(0,6,13,28,32)	6	0 6 13	1 7 14	7	6 13 28	1 8 23	4	28 32 13	1 5 42	4	28 32 0	1 5 29	6	0 6 32	1 7 33		
(0,7,20,26,31)	7	0 7 20	1 8 21	6	20 26 7	1 7 44	5	26 31 20	1 6 51	5	26 31 0	1 6 31	7	0 7 31	1 8 32		
(0,21,12,28,33)	9	12 21 0	1 10 45	7	21 28 12	1 8 48	5	28 33 12	1 6 41	5	28 33 0	1 6 29	12	21 33 0	1 13 36		
(0,34,5,21,27)	5	0 5 34	1 6 35	13	21 34 5	1 14 41	6	21 27 5	1 7 41	6	21 27 0	1 7 36	7	27 34 0	1 8 30		
(0,26,38,11,18)	12	26 38 0	1 13 31	12	26 38 11	1 13 42	7	11 18 38	1 8 28	7	11 18 0	1 8 46	8	18 26 0	1 9 39		
(0,29,46,14,23)	10	46 0 29	1 11 40	15	14 29 46	1 16 33	9	14 23 46	1 10 33	9	14 23 0	1 10 43	6	23 29 0	1 7 34		
(0,27,44,9,19)	12	44 0 27	1 13 40	17	27 44 9	1 18 39	10	9 19 44	1 11 36	9	0 9 19	1 10 20	8	19 27 0	1 9 38		
(0,28,41,9,20)	13	28 41 0	1 14 29	13	28 41 9	1 14 38	11	9 20 41	1 12 33	9	0 9 20	1 10 21	8	20 28 0	1 9 37		
(0,32,47,16,27)	9	47 0 32	1 10 42	15	32 47 16	1 16 41	11	16 27 47	1 12 32	11	16 27 0	1 12 41	5	27 32 0	1 6 30		
(0,20,53,13,25)	3	53 0 20	1 4 24	7	13 20 53	1 8 41	12	13 25 53	1 13 41	12	13 25 0	1 13 44	5	20 25 0	1 6 37		
(0,26,47,6,20)	9	47 0 26	1 10 36	15	47 6 26	1 16 36	14	6 20 47	1 15 42	6	0 6 20	1 7 21	6	20 26 0	1 7 37		
(0,19,45,12,26)	11	45 0 19	1 12 31	7	12 19 45	1 8 34	14	12 26 45	1 15 34	12	0 12 26	1 13 27	7	19 26 0	1 8 38		
(0,15,38,11,25)	15	0 15 38	1 16 39	4	11 15 38	1 5 28	13	25 38 11	1 14 43	11	0 11 25	1 12 26	10	15 25 0	1 11 42		
(0,18,45,8,23)	11	45 0 18	1 12 30	10	8 18 45	1 11 38	15	8 23 45	1 16 38	8	0 8 23	1 9 24	5	18 23 0	1 6 39		
(0,22,47,8,24)	9	47 0 22	1 10 32	14	8 22 47	1 15 40	16	8 24 47	1 17 40	8	0 8 24	1 9 25	2	22 24 0	1 3 35		
(0,18,46,9,25)	10	46 0 18	1 11 29	9	9 18 46	1 10 38	16	9 25 46	1 17 38	9	0 9 25	1 10 26	7	18 25 0	1 8 39		

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