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Parameter Elicitation for Consumer Models in Demand Response Management

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***Abstract*—Game theoretic models to demand response management typically assume that the electricity retailer has a perfect knowledge about the decision model that its consumers apply for scheduling their consumption, together with the exact parameter values. It is clearly impossible to satisfy this assumption in practice, which is a major barrier to the practical application of those approaches. At the same time, historic consumption data contains precious information about consumer behavior; in case of a variable, time-of-use electricity tariff, this also includes information about load flexibility at the consumer. This paper looks for ways to reconstruct the consumer's decision model from historic data accessible for the retailer. Assuming that consumer behavior can be captured by some formal mathematical model with a reasonable accuracy, we propose computational methods for eliciting parameter values for that consumer model by using inverse optimization and successive linear programming techniques. While the proposed approach is applicable to arbitrary consumer models that can be formulated as a linear programs, this paper investigates a special case with multiple types of controllable loads at the consumer, under a single smart metering device. Initial experimental results are presented and directions for future research are suggested.**

***Index Terms*—Smart grids, demand response management, parameter elicitation, inverse optimization.**

I. Introduction

Understanding and predicting the behavior of electricity consumers is critical for the stability and for the efficient operation of power grids. Accordingly, the profiling of consumers has been a focus of both researchers and practitioners [1]. However, in the context of *demand response management* (DRM), consumption is not a fixed characteristic of the consumer, but it is dynamically and intentionally modified by the service provider via some control signals, typically, price signals. The characterization of consumer behavior by formal models directly applicable in mathematical approaches to DRM remains a major challenge. This paper takes a step towards responding this challenge by proposing computational methods for eliciting consumer model parameters from historic data.

A. Requirements of Mathematical Models of DRM

A plethora of game theoretic models has been proposed for DRM [2]–[6], with a common and critical assumption that the retailer (typically, leader in a Stackelberg game) has a perfect knowledge about the decision model that the consumers (followers in the game) use for scheduling their consumption in response to the electricity tariff offered by the retailer. For instance, in case of a consumer with deferrable loads, a formal model would typically assume that the total load over a finite (e.g., daily) horizon is given and fixed, as well as the utility incurred by scheduling the load in a given time period.

However, this assumption can be hardly satisfied in reality, since the retailer can only make imperfect predictions about the behavior of its consumers. Furthermore, few consumers decide on their consumption according to a well-defined optimization model, and hence, it is also doubtful how precisely the models applied in the literature can capture real consumer behavior. It is typically agreed that for residential consumers, aspects of human behavior cannot be omitted, why economic rationale prevails for industrial consumers. Furthermore, load responsiveness depends on the type and controllability of the equipment, though, the spreading use of intelligent control devices will also facilitate active and conscious participation in DRM programs [7].

B. Approaches to Characterizing Consumer Behavior

There are various approaches investigated in the literature to characterize consumers from the point of view of DRM potential. A recent study [1] classifies these approaches into *technological engineering models* and *econometric empirical studies*. The former approach constructs detailed models of the individual load components and calculates the cumulated consumption from these components. These allow a formal modeling of power systems and evaluating candidate solutions even before the physical implementation of the system. At the same time, the accuracy of these approaches in practical applications is often disputed. In contrast, econometric studies do not define technical models and do not make strict assumptions on the composition of the load, but apply statistical analysis on measured data to correlate consumption to a set of external variables. These approaches estimate a so-called customer baseline load (CBL), i.e., the consumption that would arise without any DR incentives, and then correlate the deviation from CBL with the DR signal and a set of external variables.

The same paper [1] introduces a probabilistic characterization of consumer's responsiveness to occasional DR signals in the form of price-and-volume signals, which request individual

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consumers to decrease their consumption to a given level for a given payment. Another study [8] addresses quantifying the DRM potential in electric vehicle (EV) charging based on two real-world data sets. In contrast to most earlier studies that simply calculate the ratio of the connection time that the EV spent charging, this study uses a more elaborate model of the charging process to assess flexibility. A framework for the comprehensive assessment of consumer responses to electricity tariffs, and for deriving indications for tariff design is presented [9]. While the above approaches serve with a wide-ranging and well-established characterization of consumer behavior, they do not lead to formal decision models directly applicable in mathematical or game theoretical models of DRM.

C. Contribution of the Paper

This paper takes a step towards bridging the gap between the requirements of mathematical DRM models and the available methods for characterizing consumer behavior, by proposing computational techniques for eliciting parameters of formal consumer models from historic data. The proposed approach relies on inverse optimization techniques [10], [11] and *successive linear programming* (SLP) [12], [13]. While the approach is applicable to consumer models encoded in the form of linear programs (LP), this paper illustrates the approach on a specific consumer model that is often applied in the literature. Initial experimental results are presented and analyzed to evaluate the effectiveness of the approach and to point out relevant directions for future research.

II. Problem Definition

While this paper aims at developing a generic approach to parameter elicitation for consumer models, the approach is presented on a specific model that is often investigated in the literature, with multiple controllable loads for the single consumer. According to this model, the consumer must decide on the timing of N different types of controllable load over a finite time horizon divided into T time periods of equal length. For each type of load $i = 1, ..., N$, the total demand M_i over the horizon is given, in addition to the upper bounds $L_{i,t}$ on the load in each period. Scheduling a unit of load of type i into period t incurs a monetary equivalent utility of $U_{i,t}$ for the consumer. The unit price of electricity Q_t also varies over time. Then, the problem of the consumer consists in maximizing the criterion composed of its total utility minus the cost of electricity. This problem can be formulated as an LP as follows. Symbols in brackets on the r.h.s of the constraints stand for the dual variables assigned to the constraint, whereas the applied notation is summarized in Table I.

Maximize

$$\sum_{t=1}^{T} \sum_{i=1}^{N} (U_{i,t} x_{i,t} \; - \; Q_t x_{i,t}) \tag{1}$$

subject to

$$\sum_{t=1}^{T} x_{i,t} = M_i \quad \forall i \qquad [\alpha_i] \tag{2}$$

$$x_{i,t} \leq L_{i,t} \qquad \forall i, t \quad [\beta_{i,t}] \tag{3}$$

$$x_{i,t} \geq 0 \qquad \forall i, t \tag{4}$$

In this LP formulation, the objective (1) states that the consumer maximizes its total utility minus the cost of electricity. Equality (2) declares that the total load of type i must equal M_i, whereas constraint (3) specifies the upper bound on the per period load for each type. This problem, with decision variables $x_{i,t}$, will be called the *direct* problem.

This paper assumes that the above optimization model captures the consumer's behavior with some reasonable accuracy. The electricity retailer is aware of the electricity price Q_t and the cumulated electricity consumption $z_t = \sum_{i=1}^{N} x_{i,t}$ of the consumer, but it cannot observe the detailed, per device consumption profile $x_{i,t}$ or the parameter values that the consumer used when scheduling its consumption. Then, the focus of the paper is eliciting the unknown parameter values $U_{i,t}$, M_i, and $L_{i,t}$ from historic data given in the form of tuples (Q^k, z^k), $k = 1, ..., K$ i.e., the consumer's past demand responses to the variation of the electricity tariff. It is also assumed that each sample is available with the same time resolution, i.e., $Q^k = [Q^k_1, Q^k_2, ..., Q^k_T]$ and $z^k = [z^k_1, z^k_2, ..., z^k_T]$. This problem will be referred to as the *inverse* problem, and the rest of the paper addresses the solution of this inverse problem.

III. Solution Approach

This section proposes an inverse optimization solution approach to the parameter elicitation problem. For this purpose, it looks for a combination of parameter values $U_{i,t}$, M_i, and $L_{i,t}$ (common over all samples) such that for each historic sample k, the historic electricity tariff Q^k induces an approximate consumption $\tilde{z}^k_t = \sum_{i=1}^{N} x^k_{i,t}$ that is as close to the historic

TABLE I

Notation used in the paper. Wherever necessary, the notation will be extended with upper index $\circ^k$ to denote the value of entity $\circ$ in sample k.

Notation		Observable
Dimensions		
T	Number of time periods	Yes
N	Number of controllable loads	Yes
K	Number of historic samples	Yes
Grid parameters		
Q_t	Unit price of electricity [$/kWh]	Yes
Consumer's parameters		
M_i	Total controllable load during the horizon [kWh]	No
$L_{i,t}$	Maximum controllable load scheduled [kWh]	No
$U_{i,t}$	Utility of controllable load scheduled [$/kWh]	No
Decision variables of the consumer		
$x_{i,t}$	Controllable load of type i in period t [kWh]	No
z_t	Cumulated consumption of the consumer [kWh]	Yes
Auxiliary variables		
α_i	Dual variable for constraint (2)	No
$\beta_{i,t}$	Dual variable for constraint (3)	No
ε_t	Model prediction error in period t	No

consumption z_t^k as possible. In this setting, $x_{i,t}^k$ (and the induced $\tilde{z}_t^k$) are an optimal solution of the direct problem faced by the consumer.

This optimization problem is formulated by exploiting LP duality for the direct problem, which states that a given solution is optimal if and only if the primal and dual objectives are equal. Accordingly, the proposed mathematical formulation is composed of the following main parts: the primal of the direct problem stated above; the dual of the direct problem; a constraint that the primal and the dual objectives match each other; and an objective function and constraints that ensure that the approximation error is minimized. For the consumer model at hand, this formulation can be defined as follows:

Minimize

$$\sum_{k=1}^{K}\sum_{t=1}^{T}\varepsilon_t^k \tag{5}$$

subject to

$$\varepsilon_t^k \geq \sum_{i=1}^{N} x_{i,t}^k \; - \; z_t^k \qquad \forall\, k,t \tag{6}$$

$$\varepsilon_t^k \geq z_t^k \; - \; \sum_{i=1}^{N} x_{i,t}^k \qquad \forall\, k,t \tag{7}$$

$$\sum_{t=1}^{T} x_{i,t}^k = M_i \qquad \forall\, k,i \tag{8}$$

$$x_{i,t}^k \leq L_{i,t} \qquad \forall\, k,i,t \tag{9}$$

$$\alpha_i^k + \beta_{i,t}^k \geq U_{i,t} - Q_t^k \qquad \forall\, k,i,t \tag{10}$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\left(U_{i,t}x_{i,t}^k \; - \; Q_t^k x_{i,t}^k\right) = \; = \sum_{i=1}^{N} M_i\alpha_i^k \; + \; \sum_{i=1}^{N}\sum_{t=1}^{T} L_{i,t}\beta_{i,t}^k \quad \forall\, k \tag{11}$$

$$U_{i,t},\; L_{i,t},\; M_i,\; x_{i,t}^k,\; \beta_{i,t}^k,\; \varepsilon_t^k \geq 0 \qquad \forall\, k,i,t \tag{12}$$

This formulation minimizes the total error of the approximation (5), where the error is defined as the absolute difference between the measured consumption and the cumulated consumption implied by the model (6)-(7). Primal constraints define the total load (8) and the maximum load per period (9) similarly to that in direct problem of the consumer. The corresponding dual constraints are formulated in (10). Equality (11) states that the value of the primal objective must equal the dual objective. Finally, constraint (12) states that all variables are non-negative, except for α_i^k, which is unconstrained.

Formulation (5)-(12) corresponds to a *quadratically constrained quadratic program* (QCQP), since terms $U_{i,t}x_{i,t}$ in the primal objective, as well as $M_i\alpha_i^k$ and $L_{i,t}\beta_{i,t}^k$ in the dual objective in line (11) are quadratic. All other constraints and the objective are linear. Since constraint (11) is non-convex, no efficient exact solution approach can be expected for this problem. Therefore, an SLP [12], [13] solution approach was adopted. This approach iteratively builds local LP approximations of the original problem, solves them using standard LP techniques, and then modifies the actual QCQP solution according to the optimal LP solution. This technique can be expected to show good computational performance on problems where most constraints are linear, which also holds for the above formulation. At the same time, it is an iterative heuristic that may be trapped in local minima. This solution approach was implemented in FICO Xpress 7.8 using the SLP package.

IV. Experimental Evaluation

A. Design of Experiments

Computational experiments analyzed the effectiveness of the proposed approach on randomly generated data. Problem instances were generated by fixing $T = 12$ and varying N from 1 to 5 and K from 25 to 200. For each value of N, an initial instance was created by combining loads with different characteristics, i.e., given parameters $U_{i,t}^*$, M_i^*, and $L_{i,t}^*$. These parameter values will be referred to as the *original* parameter values.

Then, a set of K historic samples were artificially computed by solving the direct problem using the original parameter values and a randomized tariff Q^k, resulting in preliminary consumption values $\dot{z}^k$. Then, a random perturbation was applied to the preliminary values to reflect the requirement that the approach is expected to work in applications were the consumer model is only an imperfect characterization of the true consumer behavior. Hence, the historic samples contained consumption values $z_t^k = \dot{z}_t^k(1 + U(-\pi, \pi))$, where $U(a, b)$ stand for the continuous uniform random distribution between a and b. The value of π was varied between 0 and 0.2, where $\pi = 0$ stands for the pure theoretic case where the applied consumer model describes precisely the behavior of the consumer, and increasing values of the π correspond to less and less accurate consumer models.

The proposed approach was evaluated by eliciting the consumer parameters from the generated historic samples, and then solving a reference problem, i.e., an instance of the consumer's direct problem, both the original parameter values ($U_{i,t}^*$, M_i^*, and $L_{i,t}^*$) and the elicited parameter values ($U_{i,t}$, M_i, and $L_{i,t}$). The *root mean square error* (RMSE) was computed on the resulting load curves of the cumulated consumption.

B. Results with a Single Load Type

For instances with a single type of load ($N = 1$), the proposed approach showed very promising performance. The elicited parameter values allowed us to reproduce the load curves derived from the original parameter values with a very good accuracy. The load curves derived from the original parameter values and from the elicited parameter values with only a few samples ($K = 25$) and a high perturbation ($\pi = 0.2$) are compared in Fig. 1. This image captures the *worst* elicitation result in case of $N = 1$, while for other scenarios, the curves match each other even closer. The RMSE of the elicitation for different combinations of K and π is

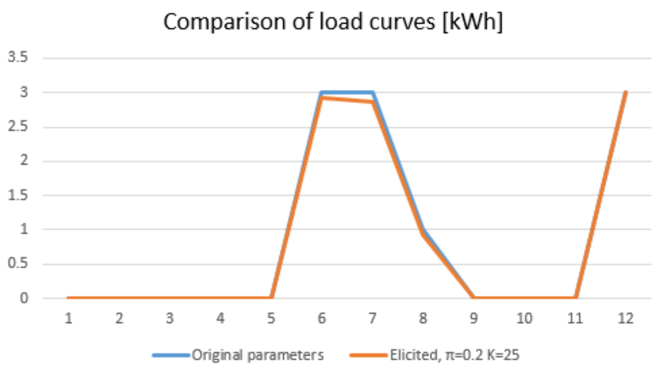

Fig. 1. Comparison of the load curves over time for $N = 1$ with the original parameters values and the elicited parameters ($K = 25$, $\pi = 0.2$).

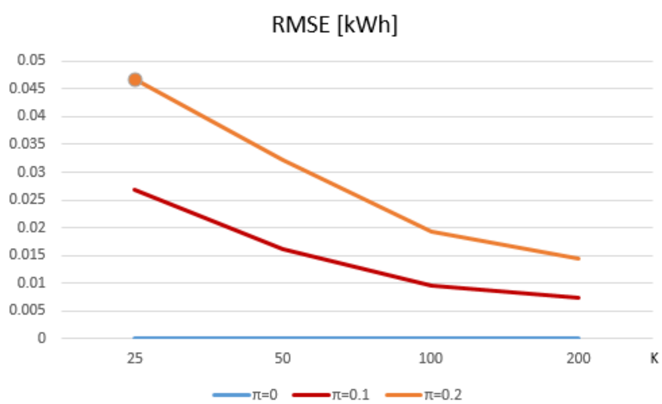

Fig. 2. RMSE of the elicitation for $N = 1$ over different values of K and π. The orange dot highlights the scenario for which the load curve was analyzed in the previous figure.

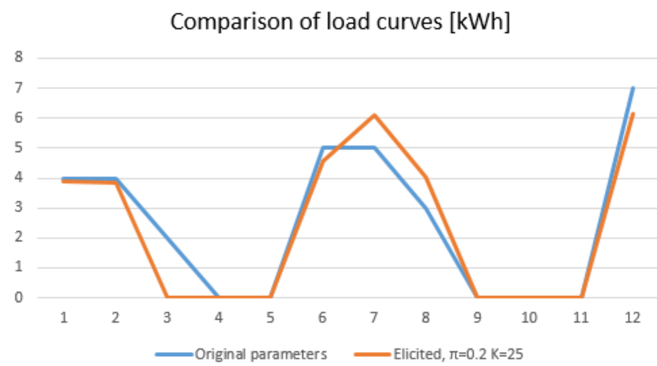

Fig. 3. Comparison of the load curves over time for $N = 3$ with the original parameters values and the elicited parameters ($K = 25$, $\pi = 0.2$).

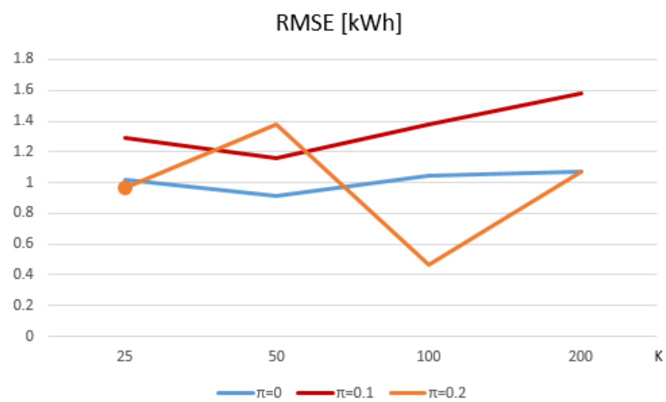

Fig. 4. RMSE of the elicitation for $N = 3$ over different values of K and π. The orange dot highlights the scenario for which the load curve was analyzed in the previous figure.

depicted in Fig. 2. An orange dot indicates the combination of K and π applied in the previous comparison of the power curves. A very positive result is that the error of the elicitation converges to zero as the number of samples K increases. Computation times were below 3 seconds even for the largest instances with $K = 200$.

C. Results with Multiple Load Types

In contrast to the above findings, results with multiple load types ($N \geq 2$) are more ambiguous. Qualitatively, the power curves stemming from the elicited parameter values give a good approximation of the original power curve (Fig. 3), but the errors are considerably higher (Fig. 4). Furthermore, the errors do not converge to zero with the increase of the number of samples, which is clearly a negative result. Computation took a couple of minutes for the largest instances ($N = 5$, $K = 200$).

Additional validation experiments confirmed that the original values for $U_{i,t}$, M_i, and $L_{i,t}$ incur a (close-to-)optimal solution of the mathematical program (5)-(12), and also result in an order of magnitude lower errors than displayed above. This means that the proposed mathematical model is an appropriate and effective formulation of the parameter elicitation problem. At the same time, the SLP solution approach readily offered by the commercial solver could not deliver a high-quality solution of the mathematical program, but stopped with a considerable optimality gap. This implies that future research must address the development of a more efficient solution approach for the proposed model.

V. Conclusions and Future Research

This paper proposed a novel computational approach to eliciting parameters of electricity consumer models from historic data, by the application of inverse optimization and SLP techniques. While the approach was illustrated on a specific consumer model that involves multiple controllable loads, the approach itself is rather generic: it can be applied to arbitrary consumer models formulated as linear programs, which is the typical representation in the literature. Initial computational experiments on randomly generated test instances showed promising results. The proposed mathematical formulation effectively captured the parameter elicitation problem, and the SLP solution approach achieved favorable results on the special case of the problem with $N = 1$. The approach also delivered reasonable results for the generic case with an arbitrary N, but the convergence properties of the algorithm did not meet our expectations.

Accordingly, future work will first focus on the development of more efficient computation techniques for parameter elicitation. These techniques need to be adapted to and validated on different consumer models, such as consumers with controllable loads, batteries, HVAC, etc. In the long run, the ultimate objective is the assessment of the appropriateness of these consumer models for DRM by validation on real test data. We believe that this is a crucial step towards the practical application of game theoretic models for DRM.

REFERENCES

[1] M. Vallés, A. Bello, J. Reneses, and P. Frías. Probabilistic characterization of electricity consumer responsiveness to economic incentives. *Applied Energy*, 216:296–310, 2018.

[2] S. Maharjan, Q. Zhu, Y. Zhang, S. Gjessing, and T. Basar. Demand response management in the smart grid in a large population regime. *IEEE Trans. Smart Grid*, 7(1):189–199, 2016.

[3] M. Zugno, J. M. Morales, P. Pinson, and H. Madsen. A bilevel model for electricity retailers' participation in a demand response market environment. *Energy Economics*, 36:182–197, 2013.

[4] M. Yu and S. H. Hong. Supply-demand balancing for power management in smart grid: A Stackelberg game approach. *Applied Energy*, 164:702–710, 2016.

[5] A. Kovács. On the computational complexity of tariff optimization for demand response management. *IEEE Transactions on Power Systems*, 33(3):3204–3206, 2018.

[6] A. Kovács. Bilevel programming approach to optimizing a time-variant electricity tariff for demand response. In *IEEE Int. Conference on Smart Grid Communications*, pages 674–679, 2016.

[7] D. Livengood and R. Larson. The energy box: Locally automated optimal control of residential electricity usage. *Service Science*, 1(1):1–16, 2009.

[8] C. Develder, N. Sadeghianpourhamami, M. Strobbe, and N. Refa. Quantifying flexibility in EV charging as DR potential: Analysis of two real-world data sets. In *2016 IEEE International Conference on Smart Grid Communications (SmartGridComm)*, pages 600–605, 2016.

[9] J. Jargstorf, C. De Jonghe, and R. Belmans. Assessing the reflectivity of residential grid tariffs for a user reaction through photovoltaics and battery storage. *Sustainable Energy, Grids and Networks*, 1:85–98, 2015.

[10] R. K. Ahuja and J. B. Orlin. Inverse optimization. *Operations Research*, 49(5):771–783, 2001.

[11] C. Heuberger. Inverse combinatorial optimization: A survey on problems, methods, and results. *Journal of Combinatorial Optimization*, 8:329–361, 2004.

[12] R. H. Byrd, N. I. M. Gould, J. Nocedal, and R. A. Waltz. An algorithm for nonlinear optimization using linear programming and equality constrained subproblems. *Mathematical Programming, Ser. B.*, 100(1):27–48, 2003.

[13] F. Palacios-Gomez, L. Lasdon, and M. Engquist. Nonlinear optimization by successive linear programming. *Management Science*, 28(10):1106–1120, 1982.