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# Questions regarding vehicle safety and the mathematical analysis of safety in large scale networks using positive dynamic systems and probability theory methods

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*Abstract* This paper introduces the mathematical modeling, safety analysis and development of diagnostic methods. Its aim is to establish a new, accredited research laboratory for the testing and quality assurance of hybrid and electric driven vehicles. The task is very complex and it puts special emphasis on environmental and safety issues.

*Keywords:* analysis of driven vehicles, diagnostic procedures, nonlinear modeling methods, consideration of safety and complex traffic circumstances, laboratory developments.

## 1.INTRODUCTION

This paper presents a theoretical research approach that uses complex mathematical modeling. It aims to recognize and determine the critical points in the road network carrying high risk from the car accidents' point of view. The results can be used in the definition of the necessary preventive measures and for the same end, in the implementation of new intelligent controls at the identified critical points. Another important area of application is the creation of a large scale database, in which the network specific accident risk values dependent on time and place, weather and seasonality are available. This can be particularly useful in the automatic supply of information for the increasingly widespread autonomous vehicles, when a large number of non-autonomous vehicles traveling in the network at the same time.

The accident events have two main sources: one is the driver error-vehicle failure - the failure rate of vehicles varies by country, but is typically an order of magnitude smaller than the number of errors committed by the drivers [12] - and the other is the condition of the network and its environment. (E.g. the glaring sunbeams, the main cause is the rising and setting sun, which cause an average of 3900 road accidents in the UK every year, of them 28 fatal. Potholes; The poor quality of rural roads. Unfavorable weather, rain, fog, snow, hail. Approximately 10% of the fatal road accidents are caused by slippery roads.)

Unfortunately, the event of an accident can be caused by the drivers anytime and anywhere in much different ways. (E.g. the loss of control on the vehicle caused by a wrong reaction of the driver. 65% of the fatal road accidents in the UK are caused by the above. Speeding. Cell phone use. The distraction of the driver's attention by passengers, that exceeds the number of cell phone use accidents. Sleeping-driver accidents particularly during the dawn hours. This accident type is ten times more common among men under the age of 30 than among similar-aged females. Sneeze; very serious threat because the drivers close their eyes involuntarily and at an average highway speed they can run even up to 15-20 meters with closed eyes, during which practically anything can happen.)

In the general case the accident event is caused by the driver error-vehicle failure event ($E_0$) and the deterioration of condition events ($E_1$, $E_2$,..., $E_N$) occurring during the trip at the network point and its environment together. Finally, the accident risk resulting from the overall accident probability in a given section is greatly influenced by the vehicle density ($\varrho$) and speed ($v$).

In case of the analysis points important to us, however, the accident risk is influenced by the cross-sectional vehicle flow volume per unit time, $q= \varrho\, v$.

## 2. OWN RESEARCH TRENDS

Our research embraces the complex dynamic-environmental-traffic effects [21], [11] relating to vehicles as well as their modeling, testing and new principled laboratory analysis [22]. In terms of models we equally examine the conventional non-linear network dynamics [6],[7] dealt with the IDM dynamics, [23] examined them in traffic procedures as well. When discussing network dynamics we suggest to use the methodology based on the theory of positive systems, where the model basically is a macroscopic model. The conditions of controllability and observability of the positive systems [1],[2],[3],[4],[5],[10] cannot be clearly deduced from known methods of general systems [20]. The problem is especially evident if we demand a non-negative range of values not only for the states but also for the interfering signals as well. For this reason describing public traffic processes clearly as a positive system is not a trivial task. The directing task in this case means that you have to direct the system in such way from one state to another that the fact that the states can only take up non-negative values during the direction remains to be true [24]. The modeling environment elaborated by us (despite the fact that we constructed a macroscopic model) is suitable for describing the actual "getting there" process, as well, leaving from an arbitrary stating point to any other reachable point of the system taking into consideration real traffic processes, traffic lights, congestions, parking etc. Besides the optimal directing of vehicle groups and route proposal, this procedure is also important in other fields. For example in the field of intelligent vehicles and also in the field of dynamic analysis [15] environmental load, testing of vehicles [16], [17] because it is possible to do very fast calculations for a large number of vehicles in various times and places. We have developed an intelligent model-making system that uses a computerised algebraic method for the complex, non-linear, dynamic modeling. Using this we minimise the time required for model-design on the human side. Thanks to the available IT equipment and several electronic and electromechanic parts built into vehicles, targets set for complexity are almost fully achievable today [9].

## 3. THE PATTERN ANALYSIS

With respect to the research carried out by us, we can differentiate between static and dynamic types. The pattern analysis belongs to the static, while the quasi-accident analysis falls within the group of dynamic analyses. This type of analysis means not only a further thorough analysis of road accidents but also new research activities aiming at automatizing accident measuring and recording exact physical parameters. Each intersection and road-section has an accident pattern frequency, a distribution and a weighted distribution. As regards the research of accident prevention it is important to determine the set of critical patterns. As all these patterns belong to past accidents, their joint structure can provide us with significant information. With the knowledge of the above, the graph of traffic network can be investigated with respect to the environmental parameters [18].

## 4. QUASI-ACCIDENTS: DYNAMIC ANALYSIS DEVELOPMENT

We would like to emphasis the importance of detecting quasi-accidents alongside the trajectories and further developing this analysis, detection could be made automatic.

A quasi-accident is such an event where actual situations, physical changes have happened to the road, environment or the vehicle but this has not yet resulted in an accident. These situations are higher in number than the actual accidents and they give us warnings and help us detect danger. How can accident risk be detected? Where and why are hey most likely to happen? The most effective way of preventing accidents is to eliminate causes of accidents before the accident actually happens. For prevention a system approach accident prevention process analyser can be built in the system. An up to date, unified traffic network IT database and a regulated organisational system of contacts on top of this offers several advantages and new opportunities. During its development it is very important to use info-communicational assets and that the information is simply and fast accessible. From the classic data supply cycle the valuable information is missing that contain significant circumstances about quasi-accidents. This examination integrates traffic processes and the examination of the dynamic processes of vehicles travelling on the networks. In this way the complex analysis is done by taking into consideration the real traffic processes in a unified dynamic system for 3D hybrid vehicles as well as conventional vehicles that are suitable for a stochastic dynamic calculus using the emission blocks belonging to them. When calculating environmental load emissions are examined using real traffic simulations.

## 5. DYNAMIC ANALYSIS DEVELOPMENT

Expected results: We can perform the analysis of complex dynamic effects by developing a simulator. For this the process-models of traffic network developed by us are available.

Joining the measurements of the simulator and the real traffic creates objective conditions for the following analyses:

a.) Observation of network processes
b.) Detecting critical places.
c.) The analysis of the joint probability of occurrence of environmental states.
d.) The analysis of safety critical effects in the traffic. Analysis of human qualities.

### 5.1 Traffic model for analysis

A very important new structural result from the modeling point of view is that the dynamic model of the road network is made up of the multitude of the same elements and the co-domain of each $x_i$ state parameter value is located in the interval of [0,1]. Therefore, the parking lots may also be treated as generalized sections of the model and are dynamic components of the network as well as the lanes [8].

Another important new structural result is that, regardless of the map-graph, a unified hyper-matrix structure can be specified for the mathematical modeling of large-scale road network processes, which for a network located not necessarily in a singly connected domain describes the entire system of relations between network elements (internal-internal, external-internal, internal-external and external-external relations).

The new description of the dynamic model of the system is the base for the calculation and control of the system processes.

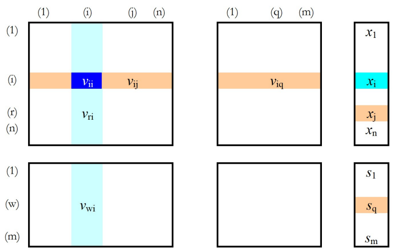
In the referred contributions there is discussion about a constructed network, which is bounded by a closed curve. [23]. It contains n sectors of internal network and m sectors from external network. We assume that the external sectors have direct connection to internal sectors and their state is known by measurements. The differential equation system is the following:

$$\dot{x} = \langle \boldsymbol{L} \rangle^{-1} \left[ \boldsymbol{K}_{\underline{11}}(\boldsymbol{x},\boldsymbol{s})\, \boldsymbol{x} + \boldsymbol{K}_{12}(\boldsymbol{x},\boldsymbol{s})\, \boldsymbol{s} \right] \qquad (1)$$

where $x \in \Re^n$, $\forall\ x_i \in [0,1]$, $(i=1,2,\ldots,n)$, $\dot{x} \in \Re^n$, $s \in \Re^m$, $\forall\ s_i \in [0,1]$, $(i=1,2,\ldots,m)$, $L = \mathrm{diag}\{l_1, \ldots, l_n\}$, $l_i$ length of road sections in the main diagonal $(\forall l_i > 0,\ i=1,2,\ldots,n)$, $K_{11} \in \Re^{nxn}$, $K_{12} \in \Re^{nxm}$.

The operation of the network is determined by $K_{11}$ and $K_{12}$ relational matrices. These matrices assign the existence of the relationship between every sector of the system, and at the same time they represent the differential equation system describing the dynamic operation of the sections, so thus the constricted network.

$\boldsymbol{K}_B =$




(2)

*Figure 3. The relational matrices of the i-th inner sector*

The determination of the vehicle density in the i-th inner sector, according to the (1) continuous model:

$$\dot{x}_i(t) = \frac{1}{l_i}\left[\left[\sum_{j=1;(j\neq i)}^{n} v_{ij} \cdot x_j(t) + \sum_{q=1}^{m} v_{iq} \cdot s_q(t) - \left(\sum_{r=1;(r\neq i)}^{n} v_{ri} + \sum_{w=1}^{m} v_{wi}\right) x_i(t)\right]\right] \qquad (3)$$

.

Where: (4)

$$v_{ij} = S(x_i(t)) \cdot V(x_i(t), x_j(t), \underline{e}_i, \underline{e}_j) \cdot E(x_j(t)) \cdot \alpha_{ij}(x(t),t) \cdot \beta_{ij}(x(t),t) \cdot \gamma_{ij}(x(t),t) \cdot$$

$$v_{iq} = S(x_i(t)) \cdot V(x_i(t), s_q(t), \underline{e}_i, \underline{e}_q) \cdot E(s_q(t)) \cdot \alpha_{iq}(x(t),t) \cdot \beta_{iq}(x(t),t) \cdot \gamma_{iq}(x(t),t) \cdot$$

$$v_{ri} = S(x_r(t)) \cdot V(x_r(t), x_i(t), \underline{e}_r, \underline{e}_i) \cdot E(x_i(t)) \cdot \alpha_{ri}(x(t),t) \cdot \beta_{ri}(x(t),t) \cdot \gamma_{ri}(x$$

$$v_{wi} = S(s_w(t)) \cdot V(s_w(t), x_i(t), \underline{e}_w, \underline{e}_i) \cdot E(x_i(t)) \cdot \alpha_{wi}(x(t),t) \cdot \beta_{wi}(x(t),t)$$

in case of fulfillment of the following relationship:

$$\sum_{r=1;(r\neq i)}^{n} \alpha_{ri} + \sum_{w=1}^{m} \alpha_{wi} = 1$$

$$v_{ii} = -\left[\left(\sum_{r=1;(r\neq i)}^{n} v_{ri} + \sum_{w=1}^{m} v_{wi}\right)\right] \qquad (5)$$

**Summary of the characteristics applied in the large road network model**

- In our model $0 \le x_i(t) \le 1$; $(i=1,\ldots,n)$ normalized vehicle density is used as state parameter. The total length of the vehicles in one stage or section is divided by the length of the stage or section. This calculation can also be used in the case of parking lots, so the parking lots are generalized sections of the model as well.
- The object of the model is a positive non-linear system. In the network the medium flows with variable speed and according to the defined time-dependent distribution factors are denoted by $\alpha_{ij}(t)$. The medium is represented by road vehicles. The speed depends on the vehicle density, the maximum value of which is limited in all sections. In addition, the speed function is influenced by weather and visibility conditions, road geometry, quality and width.
- $\beta_{ij}(t)$ denotes the obstruction $(0 \le \beta_{ij}(t) < 1)$ or facilitation $(1 < \beta_{ij}(t))$ occurring at the transition between certain sections.
- $0 \le u_{ij}(t) \le 1$ switching function operation takes into account the effect of traffic lights operating at the section transitions.
- Vehicles are exchanged between the parallel running sections (lanes), as well as between parking lots and sections in the network. This transfer is taken into account by the proportionality function $0 \le \gamma_{ij}(t)$ or $0 \le \gamma_{ij}(x_i(t), x_j(t), t)$.
- Internal prohibiting automatisms operate on the network, as well: vehicles cannot be transferred from j to i, if i is full, $x_i(t)=1 \Rightarrow S(x_i(t))=0$. Vehicles also cannot be transferred from j to i, if j is empty $x_j(t)=0 \Rightarrow E(x_j(t))=0$. These conditions are easy to follow applying the normalized state parameters, which provide that in the model vehicles cannot be taken from empty sections (the density does not enter into the negative range), and also cannot be

transferred to sections where the density has already reached 1.

- The network is analysed on a not necessarily singly connected domain bounded by a closed curve "G". In the external sections, which are in direct transfer relation with a network section, the normalized traffic density $0 \leq s_i(t) \leq 1$; $(i=1,\ldots,m)$ is measured.
- The traffic model is the so-called macroscopic model.
- The mathematical model is a non-linear, non-autonomous differential equation system.

The bibliography of speed-density relationship offers several functions, e.g: the [14] linear and the [13] logarithmic forms. These functions are stochastic relations based on measurements, wherein $V_{Max}$ and other constants are derived from results of regression methods. The variable x denotes the vehicle density on a section, and $v(x)$ is the expected average speed of the vehicles processing on the section, depending on x. The classical literature does not deal with the definition of the environmental vector, but the speed is determined not only by vehicle density, but by other environmental parameterization as well; this refinement can be implemented with the modification of $V_{\text{Max}}$, or via the modification of the function itself considering the weather, visibility, road quality, width of the road. These environmental, seasonal factors can be represented in the environmental parameter vector *e* in the relational matrix: $V=v(x,\underline{e})$. Consider an applicable $V=v(x,\underline{e})$ function:

$$v(x,\underline{e}) = \frac{e_4 \cdot V_{Max}}{e_3 + e_2 \cdot \left(\frac{x}{1-x^{e_5}}\right)^{e_1}} \qquad (6)$$

In this case, the parameter vector **e** contains 5 parameters:

$$\underline{e}=[e_1, e_2, e_3, e_4, e_5]$$

The following table demonstrates the favorable and unfavorable parameter domains.

| $e_i$ | Meaning of the parameter | Unfavorable cases | Favorable cases |
|---|---|---|---|
| $e_1$ | Road quality | Bad: $e_1$=0.1 - 0.3 | Good: $e_1$=3 – 4 |
| $e_2$ | Curly road | Lot of curves: $e_2$=3 - 4 | Few curves: $e_2$=0.1-0.2 |
| $e_3$ | Slippery road | Bad, slippery: $e_3$=1.2 - 4 | No slippery: $e_3$<1 |
| $e_4$ | Safety,visibility | Bad conditions: $e_4$=0.5 - 0.7 | Good conditions: $e_4$>1 |
| $e_5$ | Width of road | Narrow: $e_5$=0.1 - 0.2 | Wide: $e_5$>4 |

*Table 3: demonstration of the **e** parameter vector*

The internal domain is located between the two distinct domains, in most of the cases the practical parameter comes from this internal interval.
The borders of the intervals are empirical values, in a given case the coordinates of the $\underline{e}=[e_1, e_2, e_3, e_4, e_5]$ parameter vector are determined via regression analysis after the speed–density measurement.

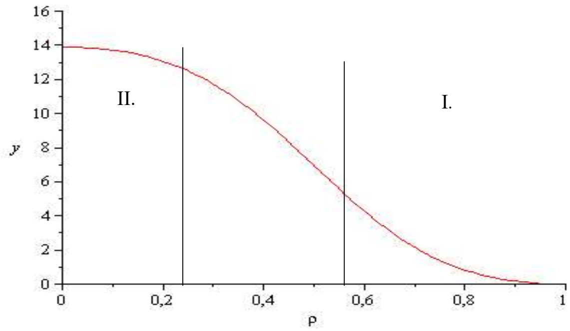



*Figure 4. v(x) speed-density function with $e_1=2$; $e_2=1$; $e_3=1$; $e_4=1$; $e_5=1$ parameters*

## 6. CONNECTION OF ACCIDENT PROBABILITIES AND TRAFFIC DATA FOR THE CALCULATION OF NETWORK RISKS

### 6.1 The conditional probability of accident occurrence in the model

We analyse the occurrence of accidents at certain points of a trajectory. If an accident occurred at a point, let B denote this event. At this point, all the possible events causing the accident must be taken into account, let $E_0, E_1, \ldots E_i, \ldots, E_N$ denote these events, respectively. In this wise any detected $E_i$ event (e.g. road defect, curve, icing, etc.) can be connected to a single accident occurrence probability $P(B|E_i)$ taking into account the conditional probability: $P(B|E_i)=P(B\,E_i)/P(E_i)$.

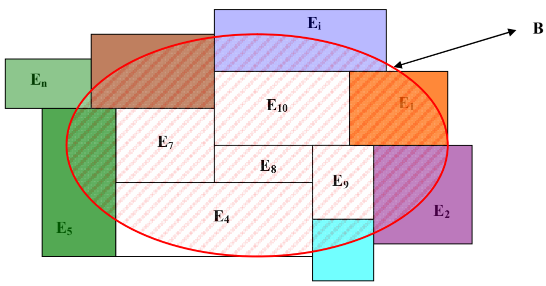



*Figure 5. Consideration of multiple possible events leading to the accident*

Let $A_i = B|E_i$ (i=0,1,2, ... , N) denote the conditional events in short. Then it is known, that the following inequality holds for arbitrary $A_0$, $A_1$, $A_2$, ...., $A_N$ events:

$$P\left(\sum_{i=0}^{N} A_i\right) \leq \sum_{i=0}^{N} P(A_i)$$

On this basis, if the the accident probability at the given point is approximated by the sum of the conditional probabilities $P(B|E_i)$, (i=0,1,2, ... ,N), the defined risk will not underestimate the danger, therefore it can be safely used.

6.2 The impact of network dynamics on risk

In the course of the determination of sectors the network lanes were divided into parts so that the homogeneity within individual sectors is ensured. On that basis, we may assume the same environmental and traffic conditions within the sector at all points and that the unified mathematical modeling is provided for the individual sectors. When determining the starting and ending point of the sector, **the geometry of the network**, the intersections, the entry or exit points, and the starting or termination points of the lanes are **taken into account**.

During the refinement of the division points the **facilities affecting traffic** (signals, pedestrian crossings, lane narrowings, speed limits) and **environmental impacts** (change in road quality, change in road width, elevation changes, etc.) **were taken into account**.

Based on our dynamic model it is possible to determine the **vehicle densities and speeds** in the sectors of the network at an arbitrary point in time.

Both of them affect the risk of accidents in the sector. The modeling takes into account all real states. It must, however, be pointed out that in terms of traffic analyses two typical accident types can be distinguished:

**Type I, high vehicle density accidents:** In practice it shows up that the higher the vehicle density is, the higher the frequency of accidents becomes, since at the same driver capability distribution the incidence of the weaker drivers is greater; in addition to that the neural load is greater due to the congestions as well. Because of the high density the speed is lower and therefore in this case minor accidents are typical. See Fig. 4, density range I.

**Type II, high speed accidents:** When the vehicle speeds are greater, the densities are lower, but at the same time that increases the risk arising from the delay in reaction time, the carelessness and the incorrect assessment of environmental parameters. This also increases the likelihood of accidents, in addition to that in this case primarily the severe accidents are typical. See Fig. 4, density range II.

Since a road section, or in terms of traffic a cross-section is analysed in case of a trajectory, we try to find the answer that how it influences the increase in the number of accidents and the risks.

At the cross section the volume of traffic is determined by q, the number of passing vehicles per unit of time. In cases different form the average density the same traffic volume can be present at a higher ($\varrho_2$) and at a lower density ($\varrho_1$), namely ($q_1 = q_2$) as seen in Fig. 6, so the knowledge of only that can pose uncertainty. In our case the model and the simulation always clearly identify the vehicle densities and thus the corresponding speeds, so the risks can be clearly calculated.

In summary, the mathematical model clearly assigns dynamic accident risks, defined as the product of the estimated accident probabilities and the calculated cross-sectional traffic volume, to road sections belonging to road length parameters. It can perform a much more comprehensive classification of roads than the previous methods and can induce effective safety measures depending on traffic, location, environment, weather and seasonalities.

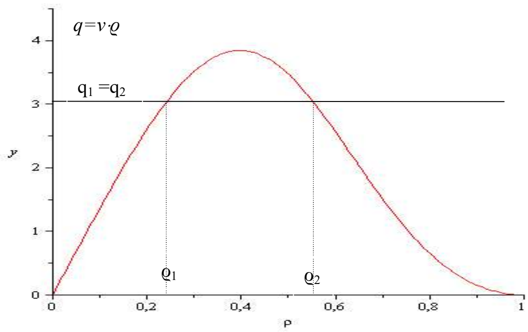



*Figure 6. Traffic volume as a function of density*

## 7. CONCLUSIONS

The applied dynamic network model in our research uses the narrowed down network traffic model [23]. There are new possibilities for developing and validating an accelerated method for the analysis of environmental loads alongside the city trajectories. From the large scale model used by us, based on prior validating we are able to extract the progression profiles which are, just like in reality, complicated and complex and consist of a sequence of accelerations, decelerations and frequent stops.

Innovative solutions in the field of traffic control serve to ensure optimal traffic services and warnings for safety applications and danger. Approaches, research methods and developments used by us aim to progress in finding new solutions to complex control problems occurring in complicated city traffic networks. An up to date, unified traffic network IT database and a regulated organisational system of contacts on top of this offers several advantages and new opportunities. During its development it is very important that the information is simply and fast accessible. In the newly formed system it is a basic principle to utilise the experiences of the already functioning systems e.g. European Transport Information System. In case of electric and hybrid-electric driven vehicles it is important to uncover the connection between modeling and laboratory measurements. The to be prepared network ITS means such a variable network that simultaneously evaluates and regards the traffic, the environmental loads and the safe and energy-effective operation in its control. The technology based on the calculating cloud and the application of the modern GNSS in large volume data processing means a sudden rise in quality and speed. The use of modern detailed dynamic maps is of great significance in the operation of the new monitoring-system, expert system and the public information system. The project and the adjoining researches [19] result in the elaboration of such new industrial diagnostic methods that are expanded onto the safety effect analysis. Equipment development is realised through the co-operation with our international research and industrial partners. These results simultaneously have a positive effect on the development of new diagnostic systems, on the public road transportation in cities as well as on safety.

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