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Design of the optimal motions of autonomous vehicles in intersections through neural networks

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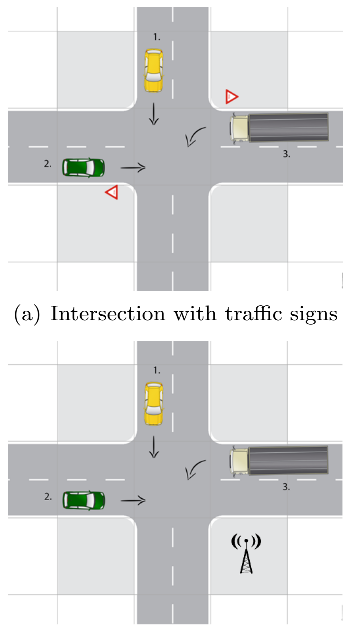
Abstract: The handling of vehicle interactions is a challenge in the research into the traveling of autonomous vehicles. This paper focuses on collision-free motion design of autonomous vehicles to guarantee their minimum traveling time in intersections. First, a decision logic of the order of the vehicles in intersections is proposed. Based on the decision logic a constrained nonlinear optimization method is also proposed, with which the minimum traveling time of the vehicles without their collision is guaranteed. Since the on-line solution of the nonlinear optimization task can be numerically complex, a neural network based approximation of the optimal solution is developed. The efficiency of the method with various intersection scenarios is shown in the CarSim simulation environment.

Keywords: autonomous vehicles, neural networks, intersections, constrained nonlinear optimization

1. INTRODUCTION AND MOTIVATION

The interactions of autonomous vehicles in smart cities are an important research field in the autonomous vehicles. Since in urban areas a large number of intersections are found, there is a huge potential in the appropriate control of vehicles approaching the intersections. The coordinated control of the autonomous vehicles provides a more flexible solution for vehicle interactions in intersections, which is able improve the effectiveness and, simultaneously, the safety of the traffic system.

Figure 1 illustrates an example concerning autonomous vehicles crossing in intersections. A conventional intersection with traffic signs is illustrated in Figure 1(a). In the scenario three vehicles are approaching the intersection. *Vehicle 1* has right of way against the other two vehicles since it is coming on a higher-order road and the other two vehicles have give way sign. *Vehicle 2* wants to go straight ahead, while *Vehicle 3* wants to turn left. In the scenario both *Vehicle 2* and *Vehicle 3* must be decelerated and stopped if necessary. It leads to increasing energy and fuel consumptions and loss of time.



(a) Intersection with traffic signs

(b) Intersection w/o traffic signs

Fig. 1. Illustration of intersection scenarios

However, without traffic signs (Figure 1(b)) in the scenario it is possible to modify the motion profile of the

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vehicles to achieve minimum loss of time and energy and fuel consumptions reduction. The initial velocity, the acceleration and/or deceleration requirements and the fuel characteristics determine the appropriate crossing order in the intersection.

The optimal control of the motions of autonomous vehicles in intersections has several advantages. In the real traffic scenario the autonomous vehicles are traveling together with human-driven vehicles and pedestrians. It is a necessary condition to set the traffic rules clearly, which determine the motions of the various types of travelers. Consequently, these traffic rules reduce the possibilities of autonomous vehicles and, moreover, the behavior of the humans must be incorporated in the control of the intersection, , see e.g., Li et al. (2016).

Moreover, the architecture in the control strategy of the autonomous vehicles in intersections should be varied. A centralized coordination of the vehicles in the intersection results in an efficient operation of the traffic system. However, it requires enhanced infrastructure, e.g. V2V and V2I communications Wuthishuwong and Traechtler (2013). Moreover, the autonomous vehicles must have a decision logic in themselves, with which the interactions are handled. Thus, the control problem of the smart intersections has both centralized and individual components.

The performances in intersections can also lead difficulties in the control design. For example, the minimum traveling time of the vehicles can cause the deceleration of the slow heavy vehicles to provide priority for the fast passenger cars. However, the deceleration and acceleration maneuvers of the heavy vehicles require significant amount of energy and fuel and result in decreases emission in intersections. Since there is a contradiction between the minimization of the energy consumption and that of traveling time, in the control of the autonomous vehicles a balance between the performances must be guaranteed.

Several papers have produces various results concerning intersections. A framework for the intersection control, which is based on the queuing theory was presented by Tachet et al. (2016). A model predictive control based intersection control using centralized approach for two vehicles was presented by Riegger et al. (2016). The quadratic programming method is the possibility of real time implementation compared to the convex optimization using space coordinates, see Murgovski et al. (2015). An autonomous intersection management system in which the connection of intersections was handled by multi-agent viewpoint was presented by Dresner and Stone (2008); Hausknecht et al. (2011). Another multi-agent solution, which is based on a heuristic optimization algorithm, was presented by Zohdy and Rakha (2012). The objective of the research is to reduce total time delay for the entire intersection, while collisions are prevented. A mixed-integer linear programming based method for the coordination of vehicles in the intersection was published by Fayazi et al. (2017).

In this paper an optimal motion design for the autonomous vehicles in the intersection is proposed. As a contribution of the paper, the form of the constrained nonlinear optimization problem is presented. It is approximated by neural networks for computational reasons. The paper also proposes a solution for the control of the intersection in which there are only autonomous vehicles. The intersection with two roads crossing is also examined, while the vehicles are driven into straight, left or right directions. The proposed solution provides a centralized coordination between the vehicles, while traveling time is minimized and collisions are avoided. The control of the intersection is based on the neural network method, which is a novel approach.

The paper is organized as follows. In Section 2 the optimization problem together with the performance and the order of vehicles is formulated. Section 3 presents the method using neural networks. In Section 4 the efficiency of the method is illustrated through high-fidelity CarSim simulation examples. Finally, Section 5 summarizes the contributions of the paper and the future challenges.

2. OPTIMIZATION OF VEHICLES IN THE INTERSECTION

In the optimization procedure applied to the intersection it is necessary to find the motion profile for all autonomous vehicles which guarantees safe approaching for them. The optimization procedure contains two tasks to be solved:

- First, it is necessary to find the appropriate order of the vehicles. In this paper the goal of the intersection control is to find the minimum traveling time for each vehicle. Thus, it is required to find the order of vehicles with which the minimum traveling time for all vehicles is guaranteed.
- Second, the kinematics of the vehicles for the given vehicle order must be determined, e.g., the acceleration/deceleration or the velocity profile for each autonomous vehicle.

2.1 The determination of the vehicle order

The determination of the vehicle order is a key problem in the optimization of autonomous vehicles in the intersection. If there are N number of vehicles in the intersection, the number of the possible orders is $N!$. It means that $N!$ number of autonomous vehicle motion profiles can be designed, which results in different traveling times of the autonomous vehicles. All of these motion profiles have local minimums, which are connected to the current vehicle order. Consequently, the intersection control, which guarantees minimum traveling time leads to a nonlinear optimization problem of the vehicle orders. The goal of the first layer is to find the vehicle order which can lead to the global optimum solution. Furthermore, the results of the vehicle order are the appropriate initial conditions in the motion optimization, which guarantees to find the global minimum traveling time in the second layer.

In this paper, the determination of the vehicle order is based on a rule which is defined through the experience of a large number of simulation examples. The vehicles crossing the intersection have right a way based on their initial velocity at the entrance $v_i(k)\big|_{k=1}$ and the length of their route L_i. The route of vehicle i depends on its approaching intention, such as straight motion, left or right turning. The order of the vehicles is decided through the following rate:

$$\mathcal{T}_i = \frac{L_i}{v_i(k)\big|_{k=1}}. \quad (1)$$

The rate $\mathcal{T}_i$ is computed for all $i = 1 \dots N$ vehicles. The $\mathcal{T}_i$ values are ordered increasingly, which determines the order of the vehicles. For example a vehicle with small initial velocity and/or long route inside of the intersection does not have right a way against the fast and/or short route vehicles. Although the presented rule is based on simulation experience, it results in the global minimum traveling time in most of the scenarios. The order of the vehicles has been built in the optimization of the intersection through the initial conditions.

2.2 Optimization problem

The optimization method is based on a division of the route, which is illustrated in Figure 2. The route of the vehicle i in the intersection is divided into M equidistant segments with index j and the acceleration $a_{i,j}$ along this route is assumed to vary linearly. The aim of the optimization is to compute the control inputs $a_{i,j}$ for each i vehicle and j segment, which guarantees the minimum traveling time for the vehicles and collisions are avoided. There are two performances:

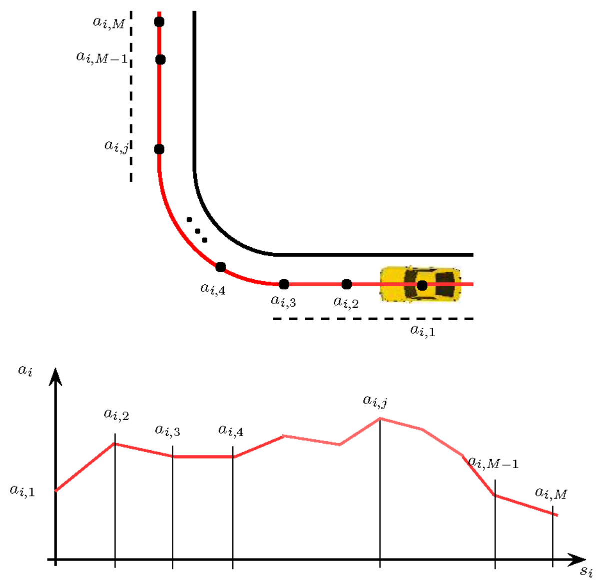


Fig. 2. Division of the vehicle route

- The performance of the control task is defined as the minimum traveling time for each vehicle. It is represented by the sum of traveling times of all vehicles such as
$$J = \sum_{i=1}^{N} t_i \quad (2)$$
where t_i is the traveling time for vehicle i, which is the time value between the entrance and the exit of the route. Moreover, N represents the number of the vehicles in the intersection.
- The avoidance of collision is guaranteed by the safe distance s_{safe}, which must be held by the vehicles. The distance $e_{i,l}$ between the vehicles i and l is computed through their positions $(x_i, y_i), (x_l, y_l)$, which results that
$$e_{i,l} = \sqrt{(x_i - x_l)^2 + (y_i - y_l)^2} > s_{safe} \quad (3)$$
The parameter s_{safe} must be selected according to the velocity of the vehicles.

The optimization tasks (2) and (3) require the model of the vehicles, with which both the t_i and $e_{i,l}$ values can be calculated. For this reason a simplified discrete time longitudinal model with with sampling time T is formulated in the following way:

$$v_i(k+1) = v_i(k) + T a_i(k), \quad (4a)$$
$$s_i(k+1) = s_i(k) + T v_i(k) + \frac{T^2}{2} a_i(k), \quad (4b)$$

where index i is related to the vehicle order. The current acceleration command $a_i(k)$ comes from the control input sequence $a_{i,j}$ through interpolation and depending on the (x_i, y_i) position of vehicle i. The motion equations of the vehicles are rearranged to a state-space form

$$\begin{bmatrix} v_i(k+1) \\ s_i(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ T & 1 \end{bmatrix} \begin{bmatrix} v_i(k) \\ s_i(k) \end{bmatrix} + \begin{bmatrix} T \\ T^2/2 \end{bmatrix} a_i(k) \quad (5)$$

which can be formed in the state space representation form:

$$x_i(k+1) = A x_i(k) + B u_i(k), \quad (6)$$

where $x_i(k)$ represents the state $x_i(k)$ and $u_i(k) = a_i(k)$ is the control input of the system, which is linked to $a_{i,j}$. The position of the vehicle is also determined by the lateral motion. However, it is assumed that the autonomous vehicles follow the curvature of the intersection. Thus, the lateral motions of the vehicles are not influenced by the other vehicles, e.g. overtaking is not allowed. Consequently, the positions of the vehicle $(x_i(k), y_i(k))$ can be computed from their initial positions, the distances $s(k)$ and the motion directions of the vehicles.

In the optimization the model (6) is used to calculate the minimum traveling time of the vehicles, with which collision of the vehicles is avoided. The optimum problem is formulated based on the performance of the system (2), the constraint (3) and the vehicle model (6) as

$$\min_{a_{i,j}:\forall i \in N, j \in M} \sum_{i=1}^{N} t_i \quad (7)$$

such that

$$e_{i,l}(k) > s_{safe} \qquad \forall i, l \in N, \forall k \quad (8a)$$
$$x_i(k+1) = A x_i(k) + B u_i(k) \qquad \forall i \in N. \quad (8b)$$

The result of the optimization is significantly influenced by the order of the vehicles. The initial conditions of $a_{i,j}$ are determined from the order of the vehicles. The initial conditions are selected to guarantee the predefined vehicle order, e.g. the first vehicles have the highest $a_{i,j}$ initial value, while the last vehicles have the lowest $a_{i,j}$.

The result of the optimization is the sequence a_i for all vehicles. The solution of the task (7) is based on an optimization algorithm, which is able to handle the nonlinear constraints, see e.g. Gill et al. (1981); Coleman and Li (1996). In practice, the optimum solution is computed using Matlab/Simulink and CarSim, in which the models of the vehicles are formulated. It represents the constraint

(8b). The cost of the optimization (7) is computed from the simulation, which it is performed by a candidate $a_{i,j}$ sequence. During the simulation the fulfillment of collision avoidance constraint (8a) is checked. Finally, the candidate values $a_{i,j}$, which guarantee the safe distance (8a) and the minimum traveling time (7) are selected.

3. NEURAL NETWORKS IN THE OPTIMAL MOTION CONTROL

The optimization task (7) results in a global optimal solution for the vehicle order in the intersection. Although it guarantees the optimal solution, the computation time of the task can be high due to the nonlinear constraints. Therefore, in this section an approximation of the optimal solution through neural networks is presented.

The goal of the neural network generation is to find a function $\mathcal{F}$ in which the output values are the acceleration commands $a_{i,j}$ and the input values are the initial positions of the vehicles $s_i(1)$, the initial velocities $v_i(1)$ and the approaching intentions D_i of the vehicles. Thus, the function $\mathcal{F}$ defines the following relationship:

$$a_{i,j} = \mathcal{F}(s_i(1), v_i(1), D_i), \tag{9}$$

where D_i represents straight, turning left or right motions. Since the optimization problem can only be solved for given $s_i(1), v_i(1), D_i$ values, the positions and velocities of the vehicle are gridded in the possible ranges. Solving the optimization problem (7) on the gridpoints, training data for the neural network fitting are generated.

In the fitting of the neural networks an input layer, hidden layers and an output layer are defined. The nodes in the input layer receive the input values $s_i(1), v_i(1)$. The approaching intention D_i is not considered in the neural network, since different networks for all D_i scenarios are generated. The operation of the control in intersection is incorporated in the nonlinear functions of the hidden layers. Moreover, the role of the output layer is to generate the acceleration commands $a_{i,j}$.

The neural network results in an approximation of the optimal solution and it has several additional advantages.

- Since the training data can be computed offline, it is also possible to generate the neural network offline. The neural networks can be implemented into the control of the autonomous vehicles in intersections and make fast computation possible.
- The function $\mathcal{F}$ is continuous, which means that any kind of $s_i(1), v_i(1)$ values can be chosen, depending on the current traffic scenario.
- During the motion of the autonomous vehicles it is assumed that the required $a_{i,j}$ acceleration command is realized. However, in practice the dynamics of the vehicles can result in differences in the motion from the kinematic model (6). Moreover, disturbances can also modify the motions of the vehicles. It means that the acceleration commands $a_{i,j}$ are required to be checked during the motion of the autonomous vehicles. The continuous function $\mathcal{F}$ makes it possible to recompute the acceleration commands, depending on the current $s_i(k), v_i(k), D_i(k)$. The recomputing process generates a feedback in the control of the intersection, which is illustrated in Figure 3.

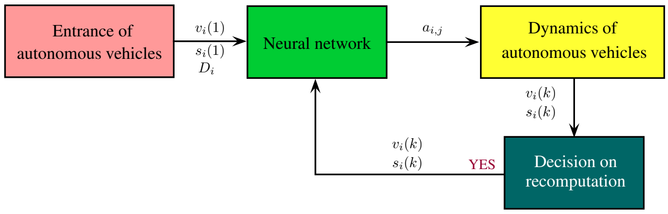


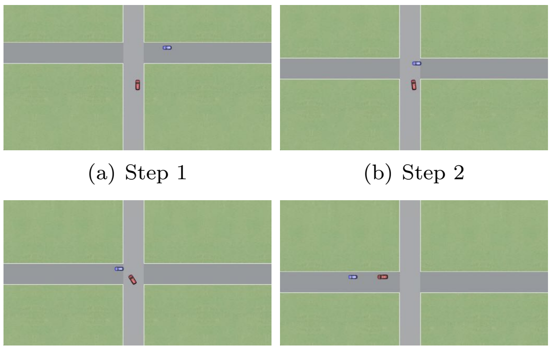
Fig. 3. Recomputing process in the intersection

4. ILLUSTRATION OF THE CONTROL METHOD

In the following section the efficiency of the presented method is illustrated through CarSim simulation examples. The purpose of the simulations is to present the guaranteeing of the minimization of traveling time and to show the appropriate handling of the neural network.

4.1 Intersection with 2 vehicles

In the first simulation scenario two autonomous vehicles are in the intersection as illustrated in Figure 4. The intention of the blue vehicle (*Vehicle 2*) is to drive straight in the intersection, while the intention of the red vehicle (*Vehicle 1*) is to turn left. Thus, there is a conflict between the vehicles, because both of them are required to exit in the same lane of the intersection.

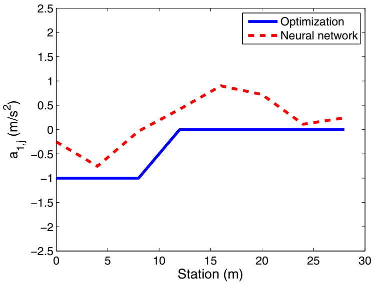


(a) Step 1 (b) Step 2 (c) Step 3 (d) Step 4

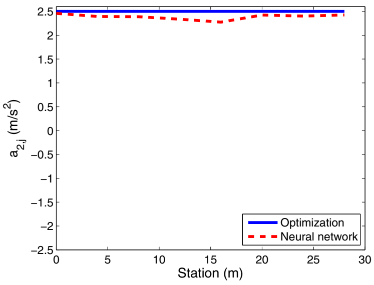
Fig. 4. Results of the simulation with two vehicles

The result of the simulation scenario using the proposed neural network method is shown in Figure 4. The vehicles at the entrance are shown in Figure 4(a). Since the $\mathcal{T}$ value of the blue vehicle is higher than the red vehicle due to its significantly higher velocity, the blue vehicle has right of way against the red vehicle. Figure 4(b)-(c) show the critical moments of the vehicle interaction. It is shown that the distance between the vehicles is reduced. However, the previously defined $s_{safe} = 2.5m$ is not violated, because the minimum distance between the vehicles is $2.83m$. The end of the maneuvers is presented in 4(d).

The results of the constrained nonlinear optimization method and those of the neural network based approximation are compared in Figure 5. It is shown that the selection of $M = 8$ is close enough to each other, which represents the acceptance of the neural network fitting. Moreover, the impacts of $a_{i,j}$ on the velocities are illustrated in Figure 6. The results of the velocity signals show that the motion of the vehicles in the optimal solution and

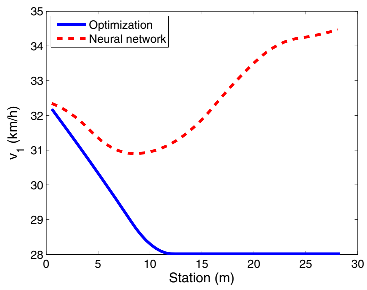


(a) Vehicle 1

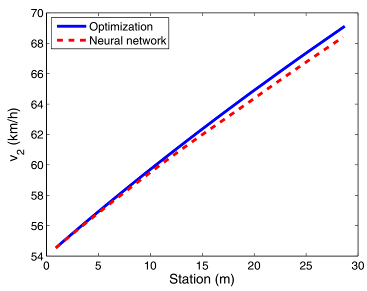


(b) Vehicle 2

Fig. 5. Acceleration commands of the vehicles



(a) Vehicle 1



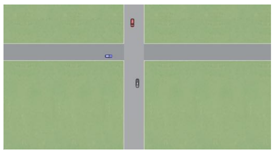
(b) Vehicle 2

Fig. 6. Velocity profiles of the vehicles

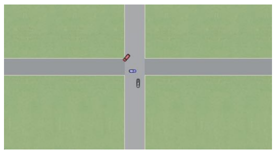
in its approximation are close to each other. Although in the case of *Vehicle 1* the velocity error slightly increases due to the error in the neural network fitting. Despite of this error the minimum distance between the vehicles is modified only by $0.02m$, which means that the error is acceptable.

4.2 Intersection with 3 vehicles

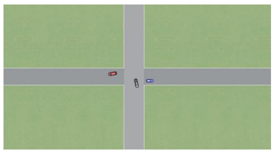
The second simulation presents a more complex scenario, in which three vehicles are involved, see Figure 7. The blue *Vehicle 2* wants to travel straight, the red *Vehicle 3* wants to turn right, while the black *Vehicle 1* wants to turn left and move forward in the same direction as *Vehicle 3*. The motions of the vehicles show that the scenario with three vehicles contain several conflicts.



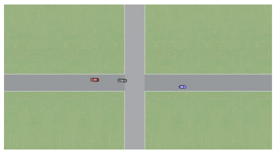
(a) Step 1



(b) Step 2



(c) Step 3



(d) Step 4

Fig. 7. Results of the simulation with three vehicles

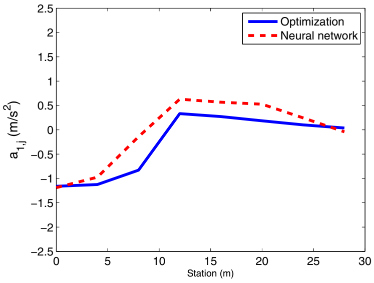
Similarly to the previous scenario, Figure 7 presents the different moments of the simulation. The initial positions of the vehicles are shown in Figure 7(a). The blue *Vehicle 2* has right a way in the intersection, the red *Vehicle 3* is the next in the order, while the black *Vehicle 1* is the last one. Figure 7(b) illustrates the conflict between *Vehicle 2* and *Vehicle 1*, while Figure 7(c) is about the interaction of *Vehicle 3* and *Vehicle 1*. The results of the maneuvers are shown in Figure 7(d). The illustration shows that the control of autonomous vehicles is also safe in the case of three vehicles.

The acceleration commands with the solution of the constrained nonlinear optimization and with the neural network approximation for all vehicles are found in Figure 8. Since the signals are rather close to each other, the fitting of the neural network has been successful. Moreover, the neural network solution results only in $0.03m$ reduction in the minimum distance between the vehicles. The resulting velocity profile of the vehicles is presented in Figure 9. It shows that the optimization and the neural network result in similar velocity profiles with small differences.

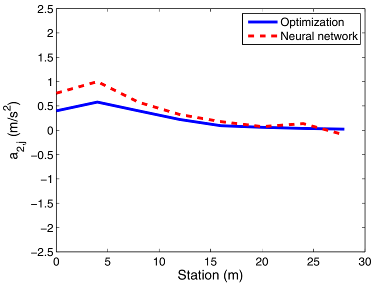
5. CONCLUSIONS

In the paper the optimal motions of autonomous vehicles for intersections have been presented. The proposed method guarantees the minimum traveling time without collisions of the vehicles. The motion design of the vehicles leads to a nonlinear constrained optimization problem, whose solution has been approximated using neural networks. The simulation scenarios with 2 and 3 autonomous vehicles show that the proposed design method is able to guarantee the optimal traveling time in the intersection, while collisions are avoided. The neural network based approximation leads to acceptable results, and it reduces the minimum distances only slightly.

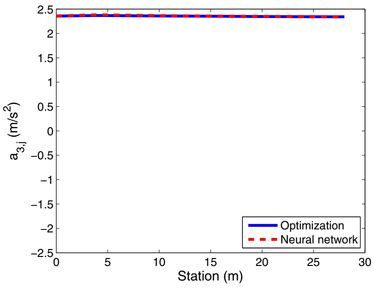
In future research the optimization problem will be analyzed in further scenarios, such as energy consumption or emission of the vehicles. A further challenge in the motion design of the autonomous vehicles is the handling



(a) Vehicle 1



(b) Vehicle 2



(c) Vehicle 3

Fig. 8. Acceleration commands of the vehicles

of mixed traffic scenarios, in which human-driven vehicles and autonomous vehicles are traveling together.

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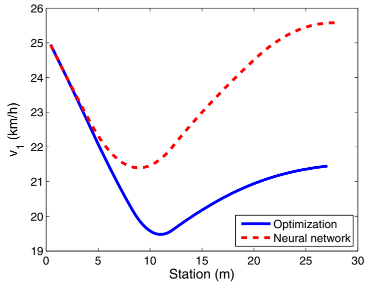
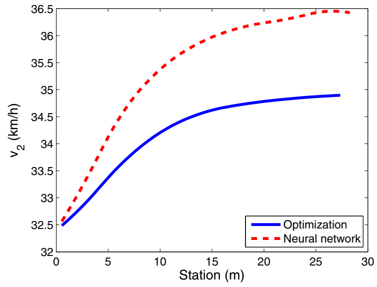
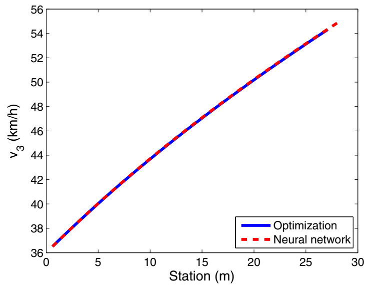


(a) Vehicle 1



(b) Vehicle 2



(c) Vehicle 3

Fig. 9. Velocity profiles of the vehicles

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