2018 International Conference on Mechanical, Electronic and Information Technology (ICMEIT 2018)
ISBN: 978-1-60595-548-3

Uncertainty, Stability and Robustness of Time-Delay Systems

Ruth BARS, Csilla BÁNYÁSZ and László KEVICZKY

Department of Automation and Applied Informatics,
Budapest University of Technology and Economics,
Institute for Computer Science and Control,
Hungarian Academy of Sciences, Hungary

Keywords: Robustness, Stability, Performance, Time-delay uncertainty.

Abstract. Time delay generally represents the transport delay in a real process. It may deteriorate significantly the properties (stability and transient performance) of a closed-loop control process. But uncertainty in the knowledge of the time delay may cause instability and influences the robustness of the control. Control algorithms are generally very sensitive to delay mismatch. In this paper the robust design of the YOULA parameterized controller is investigated considering delay mismatch. Stability region is given providing design method to ensure stability and the required performance.

Introduction

Identification, control even adaptive algorithms usually assume the apriori knowledge of the process time-delay. This knowledge is sometimes very uncertain and the mismatch coming from a lack of precision in mathematical modeling of the plant and/or changes in the plant parameters with time can result instability. It would be desirable to know how the time-delay mismatch influences the basic robustness and performance behaviors of the closed-loop control.

Some controller design methodologies, mostly for discrete-time systems, include the time-delay of the plant also into the parameters [1,2]. Unfortunately relatively few papers (e.g., [3-6]) can be found dealing with the influence of the accuracy of the apriori knowledge or estimate of the time-delay, which is sometimes called the time-delay mismatch problem. Our paper investigates the influence of the time-delay uncertainty on the robust stability and performance.

The framework how this issue will be discussed is the *generic two-degree of freedom* (*GTDOF*) system topology [7] which is based on the *YOULA-parameterization* [8] providing all realizable stabilizing regulators (*ARS*) for open-loop stable plants and capable to handle the plant time-delay. The advantage of this approach is that it is easy to calculate the "best" reachable optimal regulator depending on the applied H_2 and/or H_∞ norms as criteria. The drawback is that this methodology can be applied only for open-loop stable plants.

A *GTDOF* control system is shown in Fig. 1, where y_r, u, y and w are the reference, process input, output and disturbance signals, respectively. The optimal *ARS* regulator of the *GTDOF* scheme [9] is given by

$$R_\mathrm{o} = \frac{P_\mathrm{w} K_\mathrm{w}}{1 - P_\mathrm{w} K_\mathrm{w} S} = \frac{Q_\mathrm{o}}{1 - Q_\mathrm{o} S} = \frac{P_\mathrm{w} G_\mathrm{w} S_+^{-1}}{1 - P_\mathrm{w} G_\mathrm{w} S_- z^{-d}} \tag{1}$$

where

$$Q_\mathrm{o} = Q_\mathrm{w} = P_\mathrm{w} K_\mathrm{w} = P_\mathrm{w} G_\mathrm{w} S_+^{-1} \tag{2}$$

is the associated optimal *Y-parameter* [10] furthermore

$$Q_\mathrm{r} = P_\mathrm{r} K_\mathrm{r} = P_\mathrm{r} G_\mathrm{r} S_+^{-1} \quad ; \quad K_\mathrm{w} = G_\mathrm{w} S_+^{-1} \quad ; \quad K_\mathrm{r} = G_\mathrm{r} S_+^{-1} \tag{3}$$

assuming that the process is factorable as

$$S = S_+ \bar{S}_- = S_+ S_- z^{-d} \tag{4}$$

where S_+ means the inverse stable (*IS*) and S_- the inverse unstable (*IU*) factors, respectively. z^{-d} corresponds to the discrete time-delay, where d is the integer multiple of the sampling time. Here P_r and P_w are assumed stable and proper transfer functions (reference models). An interesting result was [11] that the optimization of the *GTDOF* scheme can be performed in H_2 and H_∞ norm spaces by the proper selection of the serial G_r and G_w embedded filters.

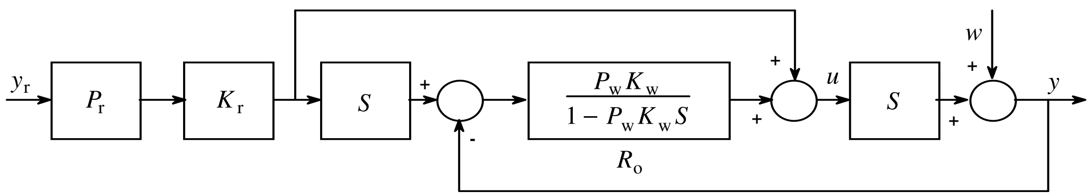

Figure 1. The *generic TDOF* (*GTDOF*) control system.

Robust Stability Conditions for *GTDOF* Control Systems

Be M the model of the process. Assume that the process and its model are factorizable as

$$S = S_+ \bar{S}_- = S_+ S_- z^{-d} \,;\, M = M_+ \bar{M}_- = M_+ M_- z^{-d_\mathrm{m}} \tag{5}$$

where S_+ and M_+ mean the inverse stable (*IS*), $\bar{S}_-$ and $\bar{M}_-$ the inverse unstable (*IU*) factors, respectively. z^{-d} and z^{-d_m} correspond to discrete time delays, where d and d_m are the integer multiple of the sampling time, usually $d = d_\mathrm{m}$ is assumed. (To get a unique factorization it is reasonable to ensure that $\bar{S}_-$ and $\bar{M}_-$ are monic, i.e., $\bar{S}_-(1) = \bar{M}_-(1) = 1$, having unity gain.) It is important that the inverse of the term z^{-d} is not realizable, because it would mean an ideal predictor z^d. These assumptions mean that $\bar{S}_- = S_- z^{-d}$ and $\bar{M}_- = M_- z^{-d_\mathrm{m}}$ are uncancelable invariant factors for any design procedure. Introduce the additive

$$\Delta = S - M \,;\;\; \Delta_+ = S_+ - M_+ \;\;;\, \Delta_- = S_- - M_- \tag{6}$$

and the relative model errors

$$\ell = \frac{\Delta}{M} = \frac{S - M}{M} \quad ; \quad \ell_+ = \frac{\Delta_+}{M_+} \quad ; \quad \ell_- = \frac{\Delta_-}{M_-} \tag{7}$$

It is easy to show that the characteristic equation using the *ARS* regulator is (for $d = d_\mathrm{m} = 0$)

$$M_+ M_- = 0 \tag{8}$$

if a $Q = \hat{Q}(M_+ M_-)^{-1}$ parameter is applied, i.e., if someone tries to cancel both factors. This means that the zeros of the *IU* factor will appear in the characteristic equation and cause instability. This is why these zeros (and the time delay itself) are called invariant uncancelable factors.

Introducing the model based, nominal complementary sensitivity function

$$\hat{Z} = \frac{\hat{R} M}{1 + \hat{R} M} = \hat{Q} M \tag{9}$$

the well known robust stability condition $\left\|\hat{Z}\ell\right\|_\infty < 1$ for the *ARS* regulator gives $\left\|\hat{Q} M \ell\right\|_\infty < 1$, i.e.,

$$\left|\hat{Q}M\right| < \frac{1}{|\ell|} \qquad \text{or} \qquad |\ell| < \frac{1}{\left|\hat{Q}M\right|} \qquad \forall\omega \tag{10}$$

Thus the robust stability strongly depends on the model M and how the model-based *Y-parameter* $\hat{Q}$ is selected.

Consider the practical form of the optimal regulator (using M in Eq.1) of the *GTDOF* system based on the available model M of the process

$$\hat{R} = \frac{P_{\mathrm{w}} G_{\mathrm{w}} M_{+}^{-1}}{1 - P_{\mathrm{w}} G_{\mathrm{w}} M_{-} z^{-d_{\mathrm{m}}}} = \frac{\left(P_{\mathrm{w}} G_{\mathrm{w}} M_{+}^{-1}\right)}{1 - \left(P_{\mathrm{w}} G_{\mathrm{w}} M_{+}^{-1}\right)\left(M_{+} M_{-} z^{-d_{\mathrm{m}}}\right)} = \frac{\hat{Q}}{1 - \hat{Q}M} \tag{11}$$

where

$$\hat{Q} = P_{\mathrm{w}} G_{\mathrm{w}} M_{+}^{-1} \text{ and } R_{\mathrm{o}} = R(M = S) \tag{12}$$

is the nominal *Y-parameter* depending on the model of the plant, which gives back Eq.2 as $\hat{Q}\big|_{M=S} = Q_{\mathrm{o}} = P_{\mathrm{w}} G_{\mathrm{w}} S_{+}^{-1}$. The dependence on the inverse stable part is direct and visible, however, G_w generally depends on the inverse unstable part. We can now state that $\hat{R}$ is also an *ARS* controller (but do not forget that only for the model M and not for the true process S).

Analyze the basic robust stability condition Eq.10 obtained for *ARS* regulators in case of the *generic scheme*, where the optimal regulator is given by Eq.10 and $\hat{Q} = P_{\mathrm{w}} G_{\mathrm{w}} M_{+}^{-1}$ from Eq.11. We get

$$\left|\hat{Q}M\ell\right| = \left|P_{\mathrm{w}} G_{\mathrm{w}} M_{+}^{-1} M \ell\right| = \left|P_{\mathrm{w}} G_{\mathrm{w}} M_{-} z^{-d} \ell\right| = \left|P_{\mathrm{w}} \ell\right| \tag{13}$$

where $\left|G_{\mathrm{w}} M_{-}\right| = 1$ can be ensured for a monic M_{-} by the optimization of G_{w}, furthermore $\left|z^{-d}\right| = 1$, thus

$$\sup_{\omega} |\ell| \leq 1 / \left|P_{\mathrm{w}}\right| \text{ or } \|\ell\|_{\infty} \leq 1 / \left\|P_{\mathrm{w}}\right\|_{\infty} \tag{14}$$

Because the right hand side of this inequality depends only on P_{w}, which is the reference model for the regulatory property of the *GTDOF* system, this means that this is a special controller structure, where the performance of the closed-loop is directly influenced by the robustness limit (via the selected P_{w}).

Computation of the Relative Model Error

Let us compute the relative model error ℓ for an *IS* plant, where the model uncertainty comes only from a time-delay mismatch. The delay-free term is assumed to be known exactly, so $\bar{M}_{-} = 1$ and $M_{+} = S_{+}$. In this case

$$\ell = \ell_{\mathrm{d}} = \frac{\Delta}{M} = \frac{S - M}{M} = \frac{S_{+} z^{-d} - S_{+} z^{-d_{\mathrm{m}}}}{S_{+} z^{-d_{\mathrm{m}}}} = z^{-\left(d - d_{\mathrm{m}}\right)} - 1 \tag{15}$$

Assume an equivalent continuous time plant with time-delay τ and a model with time-delay τ_{m}. The analogous equivalence means

$$\ell = \ell_{\tau} = e^{-\Delta\tau s} - 1 \tag{16}$$

where $\Delta\tau = \tau - \tau_{\mathrm{m}}$. The robust stability condition Eq.14 for the continuous time case is now

$$\sup_{\omega} |l_{\tau}| = \sup_{\omega} \left|e^{-j\Delta\tau\omega} - 1\right| \leq 1 / \left|P_{\mathrm{w}}(j\omega)\right| \tag{17}$$

For the sake of simplicity assume a first order reference model now

$$P_{\mathrm{w}} = \frac{1}{1+s\mathrm{T}_{\mathrm{w}}} \quad ; \quad P_{\mathrm{w}}(j\omega) = \frac{1}{1+j\omega\mathrm{T}_{\mathrm{w}}} \tag{18}$$

which means an $1/T_{\mathrm{w}}$ bandwidth design goal for the resulting closed-loop. Using the first order reference model Eq.18 the inequality to be solved for $\Delta\tau$ is

$$\sup_{\omega} \left|e^{-j\Delta\tau\omega} - 1\right| \leq \left|1 + j\omega T_{\mathrm{w}}\right| \tag{19}$$

which has the solution as a robust stability (*RS*) condition

$$\ell_{\tau} = \left|\frac{\Delta\tau}{\tau}\right| = \left|1 - \frac{\tau_{\mathrm{m}}}{\tau}\right| < \frac{\pi}{\sqrt{3}} \frac{T_{\mathrm{w}}}{\tau} = 1.82 \frac{T_{\mathrm{w}}}{\tau} \tag{20}$$

This inequality is one of our major result. The solution of the inequality Eq.19 can be easily followed on Fig. 2.

It is interesting to mention that using the first order TAYLOR expansion of the exponential term one can get a good approximation of Eq.19 and a sufficient but not necessary condition for small deviations

$$\ell_{\tau} = \left|\frac{\Delta\tau}{\tau}\right| = \left|1 - \frac{\tau_{\mathrm{m}}}{\tau}\right| < \frac{T_{\mathrm{w}}}{\tau} \tag{21}$$

The interpretation of Eq.20 and Eq.21 is very simple: for small T_{w}, which means high closed-loop performance, the model time delay τ_{m} must be close to the true delay τ. So it is obtained that the admissible time-delay mismatch is limited by the inverse of the performance. It could be furthermore very interesting how this limit influences the robustness of the loop, see the next section.

There is a simple, however, a somewhat virtual way to increase the robust stability limit Eq.20 by a higher order cutting filter form of the reference model

$$P_{\mathrm{w}} = \frac{1}{\left(1+s\mathrm{T}_{\mathrm{w}}\right)^{n}} \quad ; \quad P_{\mathrm{w}}(j\omega) = \frac{1}{\left(1+j\omega\mathrm{T}_{\mathrm{w}}\right)^{n}} \tag{22}$$

Following the same procedure how Eq.20 was obtained from Eq.19, a more general *RS* form can be derived

$$\ell_{\tau} = \left|\frac{\Delta\tau}{\tau}\right| = \left|1 - \frac{\tau_{\mathrm{m}}}{\tau}\right| < a(n) \frac{T_{\mathrm{w}}}{\tau} \tag{23}$$

where the increasing coefficient $a(n)$ is plotted in Fig. 3.

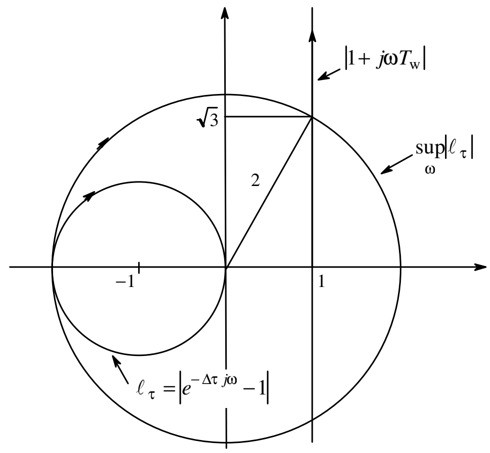

Figure 2. Simple graphics helping to understand the solution of inequality Eq.19.

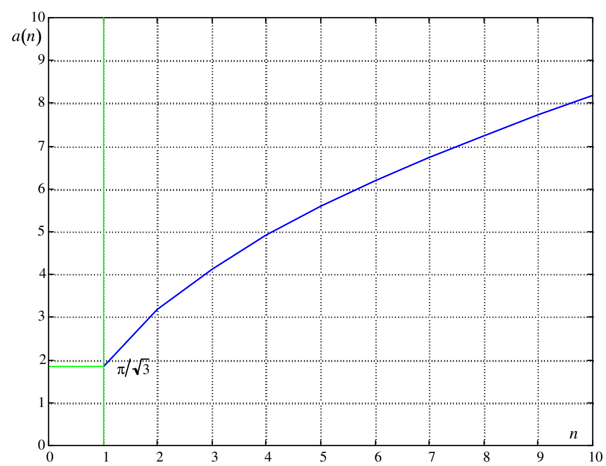

Figure 3. The $a(n)$ function.

Performance, Robustness and Time-Delay Mismatch

Detailed investigation of the above mentioned limiting behavior needs further numerical computations. Simple calculations give that the sensitivity function of the *GTDOF* system with *IS* plant, having time-delay mismatch for the discrete-time case is (assuming $G_\mathrm{w} = 1$)

$$E = \frac{1 - P_\mathrm{w} z^{-d_\mathrm{m}}}{1 + l\, P_\mathrm{w} z^{-d_\mathrm{m}}} = \frac{1 - P_\mathrm{w} z^{-d_\mathrm{m}}}{1 + l_\mathrm{d} P_\mathrm{w} z^{-d_\mathrm{m}}} \tag{24}$$

and the continuous time equivalent follows as

$$E = \frac{1 - P_\mathrm{w} e^{-s\tau_\mathrm{m}}}{1 + l\, P_\mathrm{w} e^{-s\tau_\mathrm{m}}} = \frac{1 - P_\mathrm{w} e^{-s\tau_\mathrm{m}}}{1 + l_\tau P_\mathrm{w} e^{-s\tau_\mathrm{m}}} \tag{25}$$

For P_w given by Eq.18 the sensitivity function Eq.25 becomes

$$E = \frac{1 + s\mathrm{T_w} - e^{-s\tau_\mathrm{m}}}{1 + s\mathrm{T_w} + P_\mathrm{w}\left(e^{-s\tau} - e^{-s\tau_\mathrm{m}}\right)} \tag{26}$$

The well-known NYQUIST stability margin (the simplest robustness measure) is defined by

$$\rho_m = \rho_{min}(R) = \min_{\omega} |\rho(\omega, R)| = \min_{\omega} |1+RS| = \min_{\omega} |1+Y(j\omega)| = \frac{1}{|E|_\infty} \tag{27}$$

which is the distance between the point $(-1+0j)$ and the closest point of the open-loop transfer function $Y(j\omega)$. The reciprocal value of the norm is $|E|_\infty$. Unfortunately there is no simple analytical solution to obtain how the closed-loop robustness depends on the time-delay mismatch and on the performance. It is, however, possible to compute the graphical plot of a complex functional relationship $\rho_m = \rho_{min}(\tau_m/\tau, T_w/\tau)$ with the help of MATLAB.

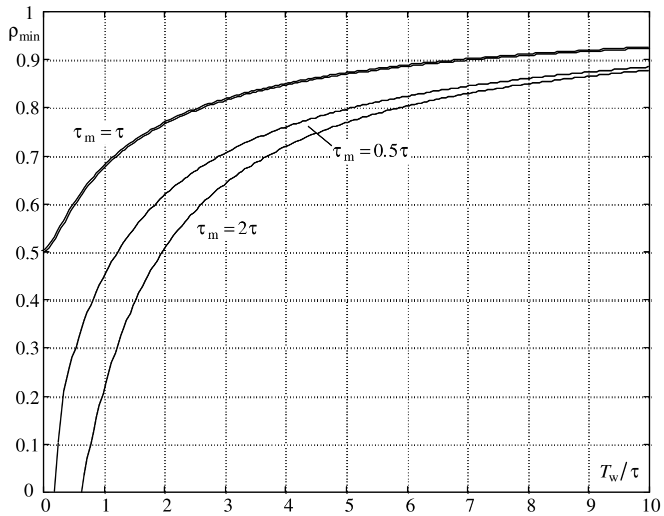

Figure 4. Function $\rho_{min}(T_w/\tau)$ for $\tau_m = 0.5\tau, \tau, 2\tau$.

As a result Fig. 4 shows the function $\rho_{min}(T_w/\tau)$ for $\tau_m = 0.5\tau, \tau, 2\tau$. For the ideal $\tau_m = \tau$ (no mismatch) case ρ_{min} depends only on our design goal (T_w) and on the plant time-delay (τ), more exactly on their relative value T_w/τ. The best robustness measure is $\rho_{min}(0) = 0.5$ for cases when the reference model P_w requires a very fast transient response from the time-delay process and the measure is $\rho_{min}(\infty) = 1$, if τ is negligible comparing to the time lag of P_w. It can be well seen that either under- or over-estimation of the time-delay causes considerable decrease of the robustness. Virtually ρ_{min} is more sensitive for over-estimation. (The left ends of the plots correspond to the robust stability limit.) While the no mismatch case provides an all stabilizing property for any performance requirement, in case of a non zero time-delay mismatch one can always expect the violation of the robustness stability limit for higher performance design.

It may be more reasonable to plot the function $\rho_{min}(\tau_m/\tau)$ parametrized by T_w/τ as Fig. 5 shows (our second major result). One can see how the robustness is extremely sensitive for high performance requirement, when T_w/τ is small and how this sensitivity decreases when T_w/τ is large for low performance design. It is also interesting to observe, that for small mismatch the over-estimation of the delay gives higher ρ_{min}, however, for large mismatch ρ_{min} is somewhat more sensitive, as it is shown in Fig. 5.

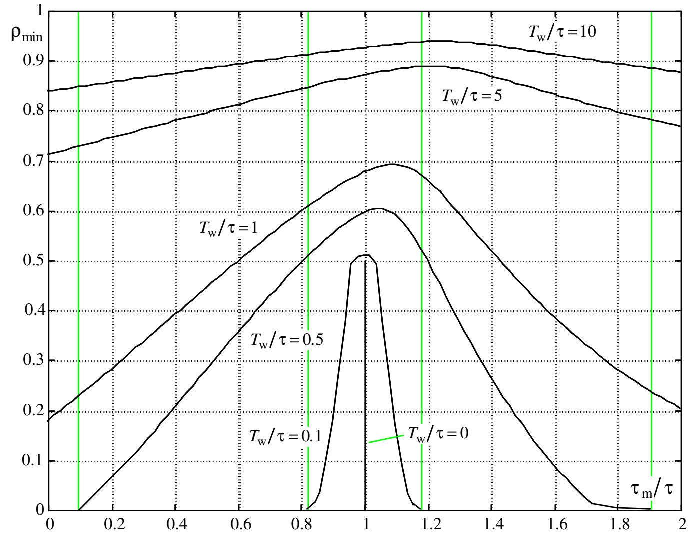

Figure 5. The function $\rho_{min}(\tau_m/\tau)$ parametrized by T_w/τ.

In a relatively wide range of T_w/τ, the over-estimation of the time-delay by τ^*/τ improves (i.e. increases) the ρ_{min} to ρ^*_{min} according to the maxima of the curves observable in Fig. 5. The over-estimation is less than 25% and the improvement is marginal, less than 5% as Fig. 6 shows.

If we assume that the time-delay mismatch is less than 20% in a practical case, the robustness degradation is always less than 10% for $T_w/\tau \geq 0.5$, which can be well seen in Fig. 6. So if we want to speed up the open-loop process to a time constant, which is considerably less than the delay, then it can only be done using a quite accurate knowledge of the time-delay. Contrary, if someone can expect a considerable variation in the time delay then only a less demanding (slower) design is more reliable and robust.

The above results strengthen the conservative practical design experience that the time-delay is practically equivalent to an *IU* zero, i.e. invariant.

It is interesting to summarize the complex relationship between performance, robustness and time-delay uncertainty and indicate an acceptable area as Fig. 7 shows.

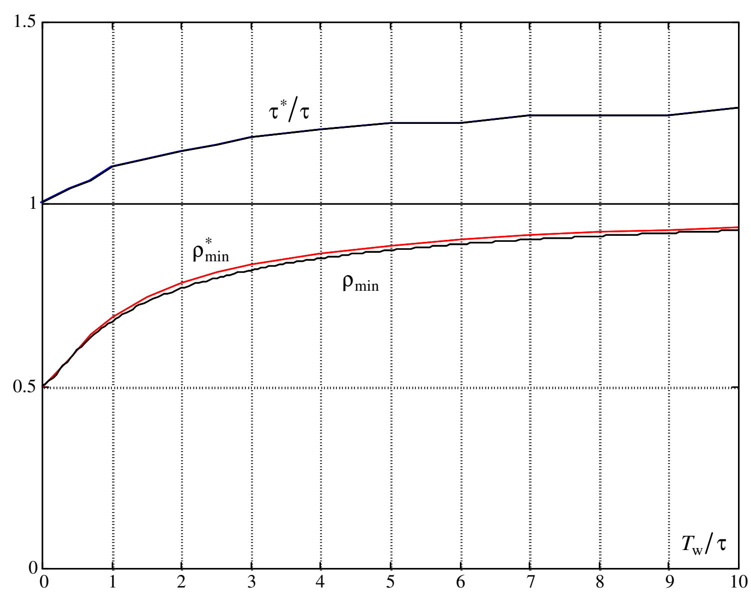

Figure 6. Influence of the time-delay over-estimation.

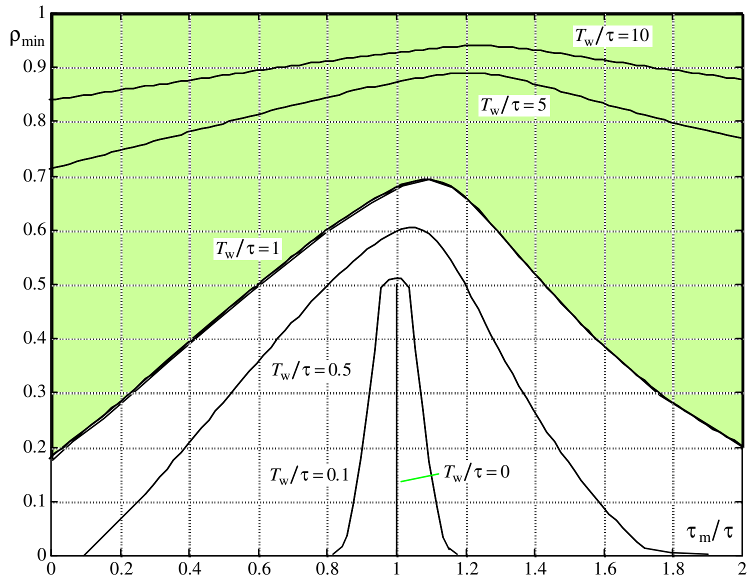

Figure 7. The shaded area is suggested for acceptable good deal between performance, robustness and time-delay mismatch.

Simulation Examples

Example 1.

The continuous plant is given by the transfer function

$$S(s)=\frac{1}{(1+2s)(1+4s)(1+6s)}e^{-10s}$$

For the YOULA parameterized design separate the transfer function to invertible and non-invertible parts. The non-invertible part of the process is the dead time. The inverse of the invertible part, which is equal to its model:

$$S_+(s)=M_+=\frac{1}{(1+2s)(1+4s)(1+6s)}$$

Let us choose now the disturbance filter as $P_w=1/(1+5s)^3$ and the reference filter as $P_r=1/(1+8s)^3$. P_w has to be of the same or higher order than P_r. The YOULA parameter then is

$$Q=P_w M_+^{-1}=\frac{(1+2s)(1+4s)(1+6s)}{(1+5s)^3}.$$

In the choice of P_w the condition of robustness, $T_w/\tau \geq 0.5$ discussed above was taken into consideration.

It is expected, that from relationship

$$\left|1-\frac{\tau_m}{10}\right| < a(3)\frac{1}{10} \approx 0.4$$

acceptable behavior will be reached within mismatch $6<\tau_m<14$

Figure 8 shows the step response and the disturbance rejection of the control system when there is no mismatch between the time delay of the system and its model and in the mismatched cases when the time delay of the model is 6 sec and 14 sec, respectively. A step disturbance of amplitude 0.5 acts at time point 150 sec. It is seen that the control system is robust for these uncertainties in the time delay.

Further simulations show that with this disturbance filter the control system tolerates even much bigger uncertainties in the time delay.

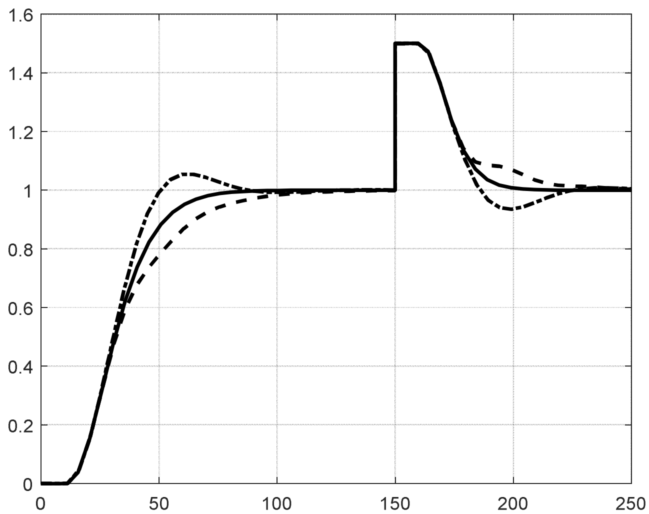

Figure 8. Step response of the control system with the YOULA controller.

Full line: accurate model; dash-dotted line: $\tau_m = 6$, dotted line: $\tau_m = 14$

Example 2.

The continuous plant is given by the transfer function

$$S(s) = \frac{1}{(1+2s)(1+4s)(1+6s)} e^{-10s}$$

The plant is sampled with sampling time $T_s = 2$ sec and a zero order hold is applied at its input. Let us design a YOULA parameterized controller. Analyze the effect of different filters.

The pulse transfer function of the plant is

$$G(z) = \frac{0.017792\,(z+2.396)(z+0.167)}{(z-0.7165)(z-0.6065)(z-0.3679)} z^{-5}$$

Let us separate the pulse transfer function into invertible and non-invertible parts. The dead time cannot be inverted. The zero outside the unit circle cannot be inverted either as it would cause unstable behavior between the sampling points. The second zero is supposed to be in the "good" region" considering Fig. 7. It usually can be cancelled, or if not, it is possible to derive another version of the control algorithm. In the terms of the shift operator z^{-1} the separation of the pulse transfer function becomes as follows:

$$G_-\left(z^{-1}\right) = \frac{\left(1+2.396z^{-1}\right)z^{-1}}{3.396} z^{-5}$$

(Its static gain has to be 1.)

$$G_+\left(z^{-1}\right) = \frac{0.017792 \cdot 3.396\left(1+0.167z^{-1}\right)}{\left(1-0.7165\,z^{-1}\right)\left(1-0.6065\,z^{-1}\right)\left(1-0.3679\,z^{-1}\right)}$$

Let us apply now the sampled continuous filters used in the previous example: $P_w = 1/(1+5s)^3$ and $P_r = 1/(1+8s)^3$. Their pulse transfer function is

$$P_r(z) = \frac{0.021615\,(z+3.098)(z+0.2218)}{(z-0.7788)^3}$$

and

$$P_{\mathrm{w}}(z)=\frac{0.007926(z+2.774)(z+0.1978)}{(z-0.6703)^3}$$

The YOULA parameter with the filters is

$$Q=P_{\mathrm{w}}G_{+}^{-1}=\frac{0.0079263(z+2.774)(z+0.1978)}{(z-0.6703)^3}\times\frac{(z-0.7165)(z-0.6065)(z-0.3679)}{0.017792\cdot 3.396(z+0.167)z^2}$$

See some results in Fig. 9.

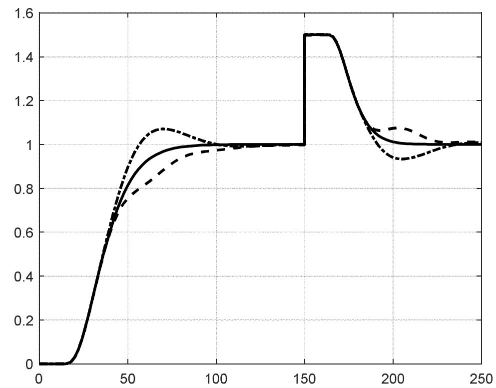

Figure 9. Step response of the discrete control system with the YOULA controller
Full line: accurate model; dash-dotted line: $\tau_{\mathrm{m}}=6$, dotted line: $\tau_{\mathrm{m}}=14$.

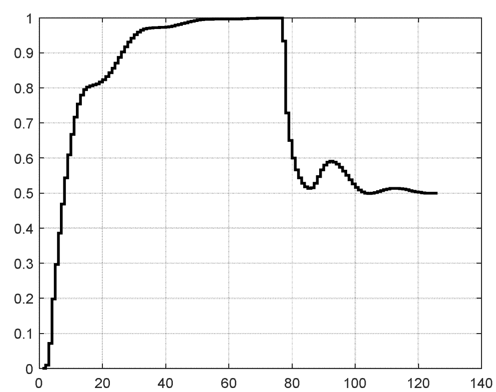

Figure 10. The course of the discrete control signal in case of mismatch
(The x scale shows the sampling instants.).

Figure10 demonstrates the course of the discrete control signal for the case of $\tau_{\mathrm{m}}=14$. Let us note that the plant is continuous, so the output signal is shown also between the sampling instants.

Figure 11 shows two cases with big time delay mismatch, when there is no time delay considered in the model (full line) and when the time delay in the model is $\tau_{\mathrm{m}}=20$ (dash-dotted line). Even in these cases the control system remains stable.

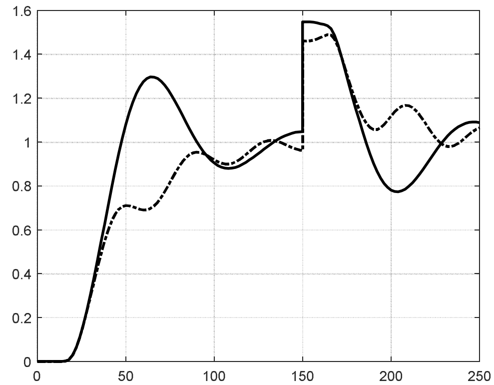

Figure 11. Step responses in cases of big time delay mismatch
Full line: no time delay in the model, dash-dotted line: $\tau_m = 20$.

Example 3.

The continuous non-minimumphase plant is given by the transfer function

$$S(s) = \frac{1-s}{(1+2s)(1+3s)}$$

The step response of the plant is shown on Fig.12. This transfer function can be considered as a first-order PADE approximation of the time delay system given by transfer function $S_{\mathrm{appr}}(s) = \frac{e^{-10s}}{1+5s}$. The step response of this system is also shown in the figure.

Let us realize a YOULA parameterized control system, where the model is the approximating time delay model.

Let us choose a first-order filter $P_{\mathrm{w}} = 1/(1+10s)$. In the choice of P_{w} the condition of robustness, $T_{\mathrm{w}}/\tau \geq 0.5$ discussed above was taken into consideration.

It is expected, that from relationship $\left|1 - \frac{\tau_{\mathrm{m}}}{\tau}\right| < 1.82 \cdot \frac{T_{\mathrm{w}}}{\tau}$ acceptable behavior will be reached within mismatch $0.9 < \tau_{\mathrm{m}} < 19.1$.

Be the reference filter also $P_{\mathrm{r}} = 1/(1+10s)$. The YOULA parameter then is

$$Q = P_{\mathrm{w}} M_{+}^{-1} = \frac{1+5s}{1+10s}.$$

Figure 13 shows the reference step response and the disturbance rejection for the nominal model (full line), for the case when the time delay in the model is 1 sec (dash-dotted line), and when the time delay in the model is 15 sec.

It is seen that the control can be designed based on the time delay approximation of the non-minimum phase system, and the robustness considerations can be taken into account in the design of the time constant of the disturbance filter.

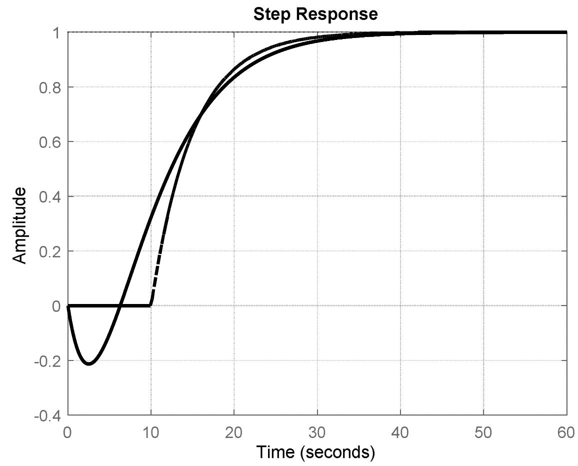

Figure 12. Step response of a non-minimum phase system and its approximation with time delay.

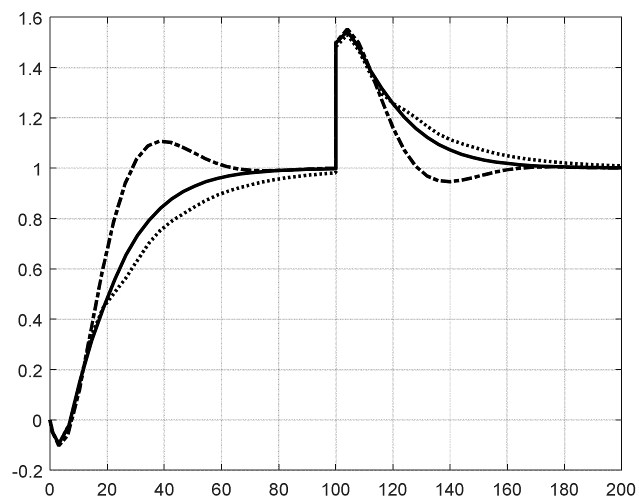

Figure 13. Step response of the continuous control system with the YOULA controller of the non-minimum phase plant Full line: accurate model; dash-dotted line: mismatched model: $\tau_m = 1$, dotted line: $\tau_m = 15$.

Summary

Real processes frequently contain time delay. If e.g. a proportional process contains several time constants it can be approximated by a first order lag and a time delay. Transportation processes also are modelled with time delay. In closed-loop control the controller design methods calculate the parameters of the controller taking into consideration the control specifications and the model of the process. These methods assume an apriori known time delay. But in practical applications time delay uncertainty, i.e. mismatch between the time delay of the process and its model has always to be assumed. Control methods as e.g. *PID* control or dead beat control are very sensitive to time delay mismatch which may cause instability or bad transient performance. YOULA parameterized controller design provides possibilities for robust performance. It has to be mentioned that other controller design methods are special cases of YOULA parameterization. Therefore robust design of YOULA parameterized controllers was discussed here. It was described how time delay mismatch influences the robustness degradation and the reachable closed-loop performance.

A new necessary and sufficient inequality condition for robust stability is derived for the maximum allowable time-delay mismatch and a simpler sufficient condition is also given for a first and an n-th order reference model.

The relationship of robustness, performance and time-delay uncertainty is represented by a graphical plot helping to make an acceptable compromise between contradictory criteria.

The investigations show that bandwidth higher than the bandwidth of the delay term ($T_w < \tau$) can be reached only for a considerably lower robustness and at the same time a much more accurate knowledge of the time-delay is necessary. So the acceptable performance domain means $T_w \geq \tau$.

We found that a certain slight overestimation of the time-delay improves the robustness, but a higher overestimation causes considerable robustness degradation again.

Simulation examples demonstrate the effectiveness of the robust design method.

References

[1] H. Kurz, W. Goedecke, Digital parameter adaptive control of processes with unknown constant or time-varying deadtime. *Automatica*, 21, (1981) 625-638.

[2] Cs. Bányász, L. Keviczky, Recursive time delay estimation method. *International J. of Systems Science*, 25, 11, (1994) 1857-1865.

[3] R.D. Hocken, S.V. Salehi, J.E. Marshall, Time-delay mismatch and the performance of predictor control schemes. *Int. J. Control*, 38, 2, (1983) 433-447.

[4] A. De Paor, A modified Smith predictor and controller for unstable processes with time delay. *Int. J. Control*, 41, 4, (1985) 1025-1036.

[5] K. Yamanaka, E. Shimemura, Effects of mismatched Smith controller on system response. *10th IFAC World Congress*, Munich, Germany, (1987) 316-321.

[6] YA. Z. Tzypkin, M. Fu, Robust stability of time-delay systems with an uncertain time-delay constant. *Int. J. Control*, 57, 4, (1993) 865-879.

[7] L. Keviczky, Combined identification and control: another way. (Invited plenary paper.) *5th IFAC Symp. on Adaptive Control and Signal Processing*, *ACASP'95*, Budapest, Hungary, (1995) 13-30.

[8] J.M. Maciejowski, *Multivariable Feedback Design*, Addison Wesley, (1989).

[9] L. Keviczky, Cs. Bányász, An iterative redesign technique of reference models: How to reach the maximal bandwidth? *11th IFAC Symp. on System Identification SYSID'97*, Fukuoka, Japan, (1997) 619-624.

[10]L. Keviczky, Cs. Bányász, Direct relationships of performance, robustness measures and amplitude constraint. CDC'2002 Las Vegas, USA, (2002) 4160-4161.

[11]L. Keviczky, Cs. Bányász, Optimality of two-degree of freedom controllers in H_2- and H_∞-norm space, their robustness and minimal sensitivity. *14th IFAC World Congress*, F, Beijing, PRC, (1999) 331-336.