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Modelling and analysis of mixed traffic flow with look-ahead controlled vehicles ⋆

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Abstract: The paper presents the modelling of a mixed traffic flow in which look-ahead controlled vehicles with a speed control are driven together with conventional vehicles. Since the speed profile of the lookahead control may differ from that of the conventional vehicle, the structure of the traffic flow changes. The paper analyses the impact of vehicles applying look-ahead control strategy on the traffic flow. In a simulation-based analysis it is shown that the fundamental diagram of the traffic network changes. A new mixed-traffic model incorporates the nonlinearity of the traffic dynamics using a polynomial formula, in which the look-ahead cruise control is considered with a parameter-dependent form. Using these nonlinear forms the impact of the look-ahead vehicles on the stability of the traffic is examined through controlled invariant sets. In this analysis the sum-of-squares programming method is applied. The results are demonstrated through simulation examples using the VISSIM simulation environment.

Keywords: Mixed-traffic, Look-ahead control, Traffic modelling and analysis

1. MOTIVATION AND INTRODUCTION

The autonomous control of vehicles has several advantages, such as advanced accident avoidance, improved agility of the vehicle and stress free traveling for the passengers. However, the autonomous vehicles may have an impact on traffic dynamics, which results from the smooth driving and the accurate following of the traffic rules. Moreover, the automation of the vehicles requires the existence of intelligent transportation systems, which are able to provide information about the traffic environment. Therefore, the automation of the vehicles is in strong relation with the control of the traffic systems.

The relations and connections between autonomous vehicles and the traffic environment have been examined in several papers. The traffic flow model, which is characterised by the autonomous vehicles, is proposed in Li and Ioannou [2004]. The impacts and benefits of the cooperative cruise control on the traffic flow are examined in Arem et al. [2006], while the role of the vehicle automation on the energy and emission is presented in Barth et al. [2013]. The aspects of optimal control design for traffic flow on motorways in the presence of vehicle automation and communication are shown in Roncoli et al. [2015]. Autonomous vehicle control facilitates the communication between the vehicles and the infrastructure. A multiple-stack architecture of the vehicle cooperation with different layers is proposed in Lin and Maxemchuk [2012], Wei and Dolan [2009], Halle and Chaib-draa [2005]. A hierarchical decomposition strategy for handling context information is proposed in Fuchs et al. [2007].

A recent technology in the optimisation of the vehicle motion is the look-ahead control. It can be considered as an extension of adaptive cruise control with road and traffic information.The purpose of the look-ahead control is to design the speed of a vehicle in order to reduce driving energy and fuel consumption while keeping traveling time, see e.g., Passenberg et al. [2009], Hellström et al. [2009], Saerens et al. [2013].

The contribution of the paper is the formulation of traffic dynamics, which incorporates both conventional vehicles and look-ahead control vehicles. A new mixed-traffic modelling approach is based on the multi-class traffic models concerning the look-ahead control vehicles. The paper analyses the impact of vehicles applying look-ahead control strategy on the traffic flow. The mixed-traffic model incorporates the nonlinearity of the traffic dynamics using a polynomial formula, in which the look-ahead cruise control is considered with a parameter-dependent form. Using these nonlinear forms the impact of the look-ahead vehicles on the stability of the traffic is examined through controlled invariant sets.

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The paper is organised as follows. Section 2 summarizes the principles of the look-ahead control. Section 3 demonstrates the effect of look-ahead vehicles on the traffic speed and the energy consumption based on VISSIM simulation examples. Section 4 presents the modelling of the mixed traffic based on the multi-class methods. Section 5 proposes the stability analysis on the system using the SOS programming.

2. PRINCIPLES OF THE LOOK-AHEAD CONTROL

In this section a brief summary of the look-ahead control is presented. Since the aim of the paper is to propose an analysis of the impact of look-ahead cruise control on the traffic flow, only the most important details of the method are presented here. A detailed discussion of the look-ahead control strategy can be found in Németh and Gáspár [2013].

The road ahead of the vehicle is divided into n number of segments, of which lengths can be different. The lengths of the segments are determined by the topography of the road: the road grade must be constant along each section, which is assumes to be known. Furthermore, the speed limits on the road segments are also considered to be known. These speed values are called $v_{ref,0}, v_{ref,1}, ..., v_{ref,n}$ reference speeds of the sections. The acceleration of the vehicle is considered to be constant along a road segment, which results in constant longitudinal control force F_{li}. However, the acceleration of the vehicle may change in the different intervals. $F_{di,r} = mg \sin \alpha_i$ is the force resistance from the road slope, $F_{di,o}$ represents other resistances such as rolling resistance and aerodynamic forces.

In the look-ahead cruise control it is not necessary to exactly guarantee $\dot{\xi}_i^2 \rightarrow v_{ref,i}^2$ for all i segment points. The priorities of each condition are represented by the prediction weights $\gamma_1, \gamma_2, ..., \gamma_n$. For example, if γ_3 has a high value, then $\dot{\xi}_3^2 \rightarrow v_{ref,i}^2$ condition has a high priority. Furthermore, it is defined a weight Q, which determines the tracking requirement of the current reference speed $v_{ref,0}$. By increasing Q the momentary speed becomes more important while further road conditions become less important.

The proposed speed, which exploits the road slopes and considers the reference speeds, is calculated in the following form:

$$\dot{\xi}_0 \rightarrow \sqrt{\vartheta - 2s_1(1-Q)(\ddot{\xi}_0 + gsin\alpha)}, \tag{1}$$

where value ϑ depends on the road slopes, the reference speeds and the weights

$$\vartheta = Qv_{ref,0}^2 + \sum_{i=1}^{n} \gamma_i v_{ref,i}^2 + \frac{2}{m} \sum_{i=1}^{n} s_i F_{di,r} \sum_{j=i}^{n} \gamma_j. \tag{2}$$

The equation shows that the proposed speed $\dot{\xi}_0$ depends on the weights (Q and γ_i). The proposed speed poses two optimisation problems: the longitudinal force must be minimised and the deviation from the reference velocity must be minimised. The minimization of the longitudinal control force $F_{l1}^2 \rightarrow \min$ leads to a quadratic optimization problem (Optimization 1). In this criterion the road inclinations and speed limits are taken into consideration by using appropriately chosen weights $\bar{Q}, \bar{\gamma}_i$. The minimization of the difference between the current velocity and the reference velocity $|v_{ref,0} - \dot{\xi}_0| \rightarrow \min$ leads to the optimal solution (Optimization 2), which is achieved by selecting the weights: $\breve{Q} = 1$ and $\breve{\gamma}_i = 0$, since in this case the vehicle tracks the predefined speed.

The two optimization criteria lead to different optimal solutions and a balance between the performances must be achieved. Performance weight R_1 ($0 \leq R_1 \leq 1$) is related to the importance of the minimisation of the longitudinal control force F_{l1}, while performance weight R_2 ($0 \leq R_2 \leq 1$) is related to the minimisation of $|v_{ref,0} - \dot{\xi}_0|$. There is a constraint on the performance weights: $R_1 + R_2 = 1$. Thus a balance between the optimisations tasks can be achieved by selecting the following performance weights:

$$Q = R_1\bar{Q} + R_2\breve{Q} = 1 - R_1(1 - \bar{Q}) \tag{3a}$$

$$\gamma_i = R_1\bar{\gamma}_i + R_2\breve{\gamma}_i = R_1\bar{\gamma}_i, \quad i \in \{1, .., n\} \tag{3b}$$

The equations show that the prediction weights depend on R_1. Normally the driver sets weight R_1 based on his goals and requirements, thus he creates a balance between energy saving and traveling time.

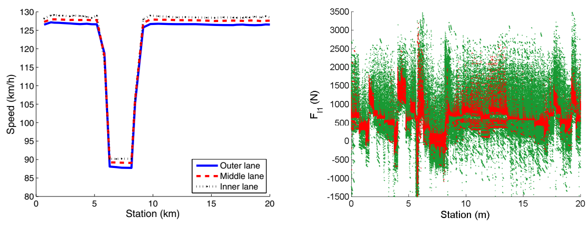
3. ANALYSIS OF THE LOOK-AHEAD CONTROL IN THE TRAFFIC

In this research the traffic system model and the measurements of a test network are built in the VISSIM microscopic traffic simulation system. As a demonstration example, a $20km$ segment of the Hungarian M1 highway between Budapest and Tatabánya with 3 lanes is modeled in VISSIM, considering the terrain characteristics, the speed limits and the three-lane characteristic of the road. The speed limit on the section is $130km/h$, although there are further speed limits.

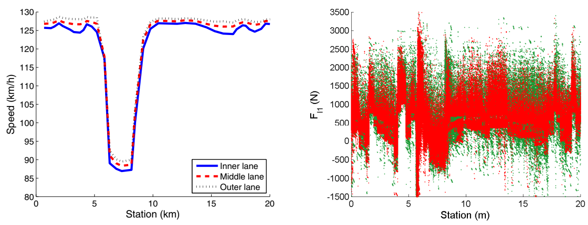
In the following several traffic scenarios are presented and compared. The purpose of the simulations is to show the effect of some parameters on the average traffic speed and the traction force. Three parameters are chosen, such as the inflow of the vehicles on the highway section (q_{i-1}), the ratio of the look-ahead vehicles in the entire traffic (κ), and the R_1 parameter of the look-ahead vehicles. The simulations show the respective average speeds in the outer, middle and inner lanes.

In the first scenario $q_{i-1} = 3000$ veh/h is the value of the vehicle inflow on the highway section. The ratio of the look-ahead vehicles is set at $\kappa = 1\%$, and $R_1 = 0.7$ weight is chosen. It means that few vehicles have look-ahead control, thus the traffic flow is not significantly influenced. Figure 1(a) illustrates the average speeds of the vehicles in the road sections in each lane and shows the traction forces of the look-ahead control vehicles (red) and conventional vehicles (green). The average speeds of the lanes are close to the maximum speed limit. However, in the outer lane (where the slowest vehicles are) the traffic speed is slightly decreased due to the look-ahead vehicles. The traction forces significantly differ because the look-ahead vehicles are able to reach their energy-efficient speed profile and this results in 8.6% force saving for the look-ahead vehicles.

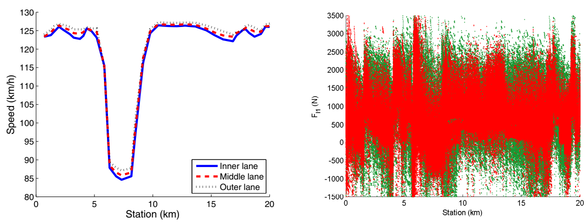
In the second scenario the ratio of the look-ahead vehicles is increased to $\kappa = 20\%$. In this case the look-ahead vehicles have a high impact on the traffic flow. Figure 1(b) shows that the average speed in the lanes significantly varies. Furthermore, the speed profiles of the vehicles without look-ahead control are also influenced by the look-ahead vehicles. The traction forces of the look-ahead vehicles are closer to the conventional vehicles. Thus, due to the increased traffic, the not all of the look-ahead vehicles are able to guarantee the fuel-economy motion. Furthermore, the traction force of the conventional vehicles slightly decreases. Thus, in this scenario the look-ahead vehicles have a low impact on the traffic flow: the speed profile and the traction force of the vehicles without look-ahead control are not modified significantly.



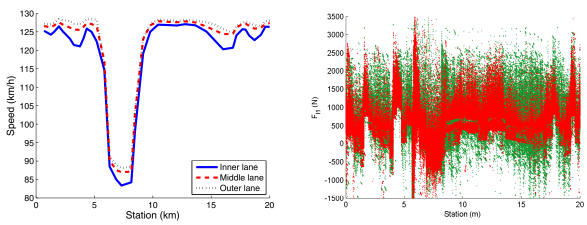
(a) $q_{i-1} = 3000\ veh/h$, $\kappa = 1\%$, $R_1 = 0.7$



(b) $q_{i-1} = 3000\ veh/h$, $\kappa = 20\%$, $R_1 = 0.7$



(c) $q_{i-1} = 5000\ veh/h$, $\kappa = 20\%$, $R_1 = 0.7$



(d) $q_{i-1} = 3000\ veh/h$, $\kappa = 20\%$, $R_1 = 0.9$

Fig. 1. Average speeds and traction forces

The purpose of the third scenario is to illustrate the effect of the increase q_{i-1}. The $q_{i-1} = 5000\ veh/h$ traffic inflow on the highway section represents rush hour traffic. In this scenario the look-ahead vehicles must adapt more to the conventional vehicles, which leads to the slight increase in the force for look-ahead vehicles, see Figure 1(c). However, the conventional vehicles must also adapt to the motion of the look-ahead vehicles, which results in the reduction of forces in the conventional vehicle and the force requirement of the conventional vehicles is reduced by 6.1%. Thus, in increased traffic the look-ahead control has a significant impact on the entire traffic and the look-ahead vehicles have a benefit the entire traffic in terms of force requirement.

The fourth simulation scenario shows the results of the increase in R_1. Since R_1 is higher, the energy-efficient cruising of the look-ahead vehicles has a priority. Compared to the results of the second scenario, the average speed of the vehicles in the outer lane significantly decreases, see Figure 1(d). In the further lanes the average speed is close to the $R_1 = 0.7$ scenario. It means that most of the look-ahead vehicles have moved into the outer lane. As a result, the traction force of the look-ahead vehicles is smaller, the saving of force requirement is 11.9% compared to the conventional vehicles.

The force requirements are summarized in Table 1.

Table 1. Results of the analyses

q_{i-1}	$\kappa[\%]$	R_1	Mean of force $[N]$		Saving [%]
			conventional	look-ahead	
3000	1	0.7	678.1	622.1	8.6
3000	20	0.7	676.2	629.4	7.2
5000	20	0.7	662.8	623.8	6.1
3000	20	0.9	674.8	598.7	11.9

4. MODELLING OF THE MIXED TRAFFIC DYNAMICS

4.1 The mixed traffic model

The analysis of the mixed traffic has proposed that the look-ahead vehicles have a significant impact on the average speed and the energy consumption of the entire traffic. In this section the effect of the look-ahead vehicles is formulated in the dynamics of the traffic.

Since the traffic contains two different classes of vehicles, their dynamics may be different. In the case of the look-ahead vehicles the selection of the current speed depends on several factors, such as the forthcoming terrain, road conditions, speed limits. However, the conventional vehicles move with the maximum speed, if possible. Thus, the traffic is the mix of the flow of two classes of vehicles.

There are several methods, in which the modelling of the multi-class traffic is presented Pasquale et al. [2015], Wageningen et al. [2015], Benzoni-Gavage and Colombo [2003], Daganzo [2002], Nair et al. [2012]. In the following the method of Benzoni-Gavage and Colombo [2003] is specified to the problem of mixed traffic with look-ahead vehicles.

The model formulates the traffic density ρ_i of segment i as the weighted sum of density of each vehicle class:

$$\rho_i = \sum_u \eta_u \rho_{i,u}, \tag{4}$$

where η_u is the passenger car equivalent of each class and index $_u$ represents the different vehicle classes. In the examined traffic scenario $u = 1$ represents the conventional vehicles, while $u = 2$ is the look-ahead controlled vehicles. The value of η_u is computed as $\eta_u = L_u/L_1$, where L_u is the gross vehicle length of class u, and L_1 is for the vehicle class 1. In the mixed traffic of look-ahead and conventional vehicles the difference of the classes is in the speed selection, while $L_1 = L_2$ is considered. The steady-state traffic speeds of each vehicle classes $V_{i,u}$ are formed as:

$$V_{i,u} = \beta_{i,u} V_{i,1}, \tag{5}$$

which also defines the class-specific fundamental relation compared to class 1. The steady-state speed-density relation is expressed as

$$V_{i,u}(\rho) = v_{i,u,max} \left[1 - \left(\frac{\rho_i}{\rho_{i,max}} \right)^{l_c}, \right]^{m_c} \tag{6}$$

where l_c, m_c are model specific parameters and $\rho_{i,max}$ represents the maximum density, where the model is valid, see Pasquale et al. [2015]. Moreover, $v_{i,u,max}$ is the maximum (free) traffic speed, related to $\rho_i \to 0$. Considering that l_c and m_c are constant for the look-ahead and the conventional vehicle classes, $V_{i,1}$ and $V_{i,2}$ differ only in their maximum traffic speed. Thus, the scale $\beta_{i,u} = \beta_{i,2}$ is defined as Benzoni-Gavage and Colombo [2003]:

$$\beta_{i,2} = \frac{v_{i,2,max}}{v_{i,1,max}}. \tag{7}$$

Based on the previous relations the law of conservation for the entire traffic can be written as

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i} \left[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k) \right], \tag{8}$$

where k denotes the discrete time step index and T is the discrete sample time step. q_i and q_{i-1} denote the traffic flow of segment i and $i-1$, r_i and s_i are the sum of ramp inflow and outflow values. The length of the segment is denoted by L_i. In (8) the flows $q_{i-1}(k)$ and $s_i(k)$ are measured values, while $r_i(k)$ is a controlled inflow, which is also known. However, the outflow $q_i(k)$ of segment i depends on several factors, see e.g. Messmer and Papageorgiou [1990], Treiber and Kesting [2013]. In this paper the determination of the outflow is based on the fundamental relationship $q_i(k) = \rho_i(k) v_i(k)$, where $v_i(k)$ is the actual traffic speed Ashton [1966]. In steady-state case $v_i(k) = V_i(k)$, it is equal to

$$q_i(k) = \rho_i(k) V_i(k). \tag{9}$$

Through (8) the dynamics of the mixed traffic is formulated using (4) as

$$\begin{aligned}
\rho_i(k+1) &= \rho_{i,1}(k+1) + \rho_{i,2}(k+1) = \rho_{i,1}(k) + \rho_{i,2}(k) + \\
&+ \frac{T}{L_i} \Big[-q_{i,1}(k) - q_{i,2}(k) + q_{i-1,1}(k) + q_{i-1,2}(k) + \\
&+ r_{i,1}(k) + r_{i,2}(k) - s_{i,1}(k) - s_{i,2}(k) \Big] = \\
&= \rho_i(k) + \frac{T}{L_i} \Big[-Q_i(k) + q_{i-1}(k) - s_i(k) + r_i(k) \Big].
\end{aligned} \tag{10}$$

In (10) the flow $q_{i-1}(k) - s_i(k) + r_i(k)$ is measured. Furthermore, the outflow $Q_i(k) = q_{i,1}(k) + q_{i,2}(k)$ is formed based on (9)

$$\begin{aligned}
Q_i(k) &= \rho_{i,1}(k) V_{i,1}(k) + \rho_{i,2}(k) V_{i,2}(k) = \\
&= V_{i,1} \big(\rho_{i,1}(k) + \beta_{i,2} \rho_{i,2}(k) \big) = \\
&= V_{i,1} \big(\rho_{i,1}(k) + \rho_{i,2}(k) + \beta_{i,2} \rho_{i,2}(k) - \rho_{i,2}(k) \big) = \\
&= V_{i,1} \big(\rho_i(k) - \rho_{i,2}(1 - \beta_{i,2}) \big)
\end{aligned} \tag{11}$$

The form of $Q_i(k)$ proposes the important consequences of the mixed traffic. In the case of traffic with only conventional vehicles ($\rho_{i,2} \equiv 0$) the outflow is $Q_i(k) = V_{i,1} \rho_i(k)$. If the rate of the look-ahead vehicles κ increases, it results in the increase in $\rho_{i,2}$. Since $\beta_2 < 1$, the overall outflow $Q_i(k)$ decreases. Thus, the high κ results in reduced $Q_i(k)$. Moreover, the increase in the look-ahead parameter R_1 leads to the reduction of $v_{i,2,max}$ and $\beta_{i,2}$. It results in the increase in $(1 - \beta_{i,2})$, which decreases Q_i. Therefore, the increase in κ and R_1 leads to the reduction of $Q_i(k)$.

In the following, the previous consequences are built-in the formulation of $Q_i(k)$. In practice, the relationship between $Q_i(k)$ and $\rho_i(k)$ can be characterised using historic measurements Gartner and Wagner [2008], such as

$$Q_i(k) = \mathcal{F}(\rho_i(k)) \tag{12}$$

In this paper the function $\mathcal{F}(\rho_i(k))$ is formed in a polynomial, which depends on κ and R_1. The selection of the polynomial formulation is motivated by the form of the steady-state speed-density relation, see (6):

$$\mathcal{F}(\rho_i(k), R_1, \kappa) = \sum_{j=1}^{n} c_j(R_1, \kappa) \rho_i(k)^j \tag{13}$$

where the coefficients in the polynomial are formed as $c_j(R_1, \kappa) = \sum_{l=1}^{m} \left(d_l R_1^l \kappa^l \right)$, in which d_l values are constants. Finally, the mixed-traffic model for look-ahead and conventional vehicles is transformed as:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i} [-\mathcal{F}(\rho_i) + q_{i-1}(k) + r_i(k) - s_i(k)] \tag{14}$$

where $q_{i-1}(k), r_i(k), s_i(k)$ are considered to be known (measured or controlled variable). Moreover, the exact knowledge of the complete operational fundamental diagram assumes the measurements of all the above traffic variables, see Papageorgiou and Vigos [2008].

4.2 Determination of the fundamental diagram

In the following the fundamental diagram of the mixed-traffic is determined through simulations. Figure 2 demonstrates an example of the measurements and the fundamental diagram $Q_i(k) = \mathcal{F}(\rho_i(k))$ of the system. The simulations have been performed with different traffic densities, related to $R_1 = 0.7$ and $R_1 = 0.9$ look-ahead parameter values. The example illustrates the measurements on all of the lanes, i.e. outer, middle and inner lanes. Polynomial functions are fitted to the different scenarios, as shown in Figure 2. The figure illustrates that R_1 has a significant impact on the fundamental diagram. If R_1 is increased, then the outflow Q_i is reduced due to the decreased vehicle speed. However, the critical density, which is related to the maximum of the fundamental diagram, is higher. Thus, the selection of R_1 has an impact on the outflow and the critical density, which is in relation to the stability of the traffic dynamics.

The function $\mathcal{F}(\rho_i(k))$ is determined based on numerous VISSIM simulation results of the highway section. As an example, in Figure 3 the shape of the fundamental diagram, depending on κ and R_1, is illustrated.

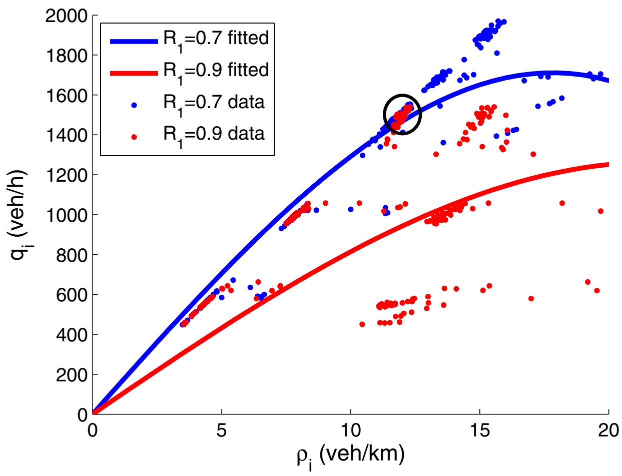
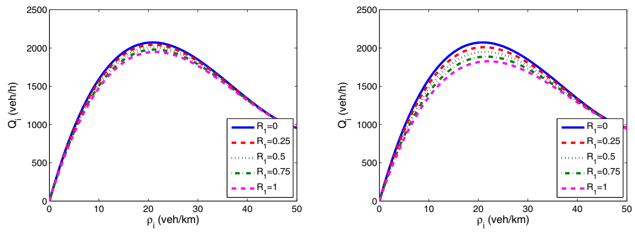


Fig. 2. Fundamental diagram of the traffic network



(a) $\kappa = 10\%$ (b) $\kappa = 30\%$

Fig. 3. Polynomial approximation of the diagram

5. STABILITY ANALYSIS ON THE MIXED TRAFFIC SYSTEM

In this section the impact of κ and R_1 on the stability of the mixed-traffic system is analysed. The state space representation of the mixed-traffic system (14) is given in the following form:

$$x(k+1) = f(R_1(k), x(k)) + g_1 u_{max}(R_1(k), \kappa(k)) + g_2 d(k) \tag{15}$$

where $f(\rho, x(k))$ is a matrix, which incorporates smooth polynomial functions and $f(R_1, 0) = 0$ and the state of the system is $x(k) = \rho_i(k)$. $u_{max}(R_1(k), \kappa(k))$ is the function of the maximum controlled inflow $r_i(k)$ and $d_i(k) = q_{i-1} - s_i$ includes the measured disturbances of the system. The stability analysis is based on the computation of the controlled invariant set using the Sum-of-Squares (SOS) programming method Tan and Packard [2008]. In the following, the stability analysis is discussed briefly. The overall description of the method is found in Németh et al. [2015], which is modified to the mixed-traffic scenario.

The parameter-dependent Control Lyapunov Function is chosen in the next form:

$$\mathrm{V}(R_1(k), \kappa(k), x(k)) = V(x(k)) \cdot b(R_1(k), \kappa(k)) \tag{16}$$

where $b(R_1(k), \kappa(k))$ is an intuitively chosen parameter-dependent basis function.

The existence of $\mathrm{V}(R_1(k), \kappa, x(k))$ is transformed into set-emptiness conditions. Moreover, the domains of $R_{1,min} \leq R_1(k) \leq R_{1,max}$ and $\kappa_{min} \leq \kappa(k) \leq \kappa_{max}$ are also formulated in the set-emptiness conditions. Using the generalized S-Procedure Jarvis-Wloszek [2003] the set-emptiness conditions can be transformed to SOS existence problem. The description of the transformation steps is found in Németh et al. [2015].

As a result, an optimisation problem is derived, in which the SOS conditions must be guaranteed. The optimization problem is to find an $u_{max}(R_1(k), \kappa(k))$ solution and feasible $\mathrm{V}(R_1(k), \kappa(k), x(k))$ for the following task:

$$\max \; u_{max}(R_1(k), \kappa) \tag{17}$$

over $\quad s_{1\ldots7} \in \Sigma_n;\; V(x(k)), b(R_1(k), \kappa(k)) \in \mathcal{R}_n$
such that

$$\begin{aligned} &-\Big((V(f(R_1(k), \kappa, x(k)) + g u_{max}(R_1(k), \kappa(k))) - \\ &\quad - V(x(k))) \cdot b(R_1(k), \kappa) + \nu \cdot V(x(k)) \Big) - \\ &\quad - s_1 \Big(V(x(k)) \cdot b(R_1(k), \kappa) - (1-\varepsilon) \Big) - \\ &\quad - s_2 \Big(1 - V(x(k)) \cdot b(R_1(k), \kappa) \Big) - s_3 x(k) - \\ &\quad - s_4 \left(R_1(k) - R_{1,min} \right) - s_5 \left(R_{1,max} - R_1(k) \right) - \\ &\quad - s_6 \left(\kappa(k) - \kappa_{min} \right) - s_7 \left(\kappa_{max} - \kappa(k) \right) \in \Sigma_n \end{aligned} \tag{18}$$

where the set of SOS polynomials in n variables is defined as:

$$\Sigma_n := \left\{ p \in \mathcal{R}_n \,\middle|\, p = \sum_{i=1}^{t} f_i^2,\; f_i \in \mathcal{R}_n,\; i = 1, \ldots, t \right\} \tag{19}$$

The result of the optimisation (17) defines the maximum Controlled Invariant Set, in which the system is stable under the function $u_{max}(R_1(k), \kappa)$. Inside of the Controlled Invariant Set the mixed-traffic system is stable.

The stability domain is computed based on the results of the traffic model (14), which are derived from the VISSIM simulations. The aim of the analysis is to illustrate the effects of R_1 and κ on the stability of the traffic. The result of the Controlled Invariant Set-based optimization (17) is the function $u_{max}(R_1(k), \kappa)$. It means that the inequality $q_{i-1} + r_i - s_i \leq u_{max}(R_1(k), \kappa)$ must be guaranteed to avoid the instability of the system. The instability results in the rapid increase in ρ_i, which causes congestion. Considering the traffic model (14) with the fundamental diagram (Figure 3), the optimization process leads to the following result:

$$u_{max} = 2073 - 496 \cdot R_1 \cdot \kappa \tag{20}$$

The function is illustrated in Figure 4. It demonstrates that the increases in R_1 and κ have a disadvantageous effect on u_{max}. This has two consequences on the traffic control strategy:

- it is necessary to limit the controlled inflow r_i to guarantee $q_{i-1} + r_i - s_i \leq u_{max}(R_1(k), \kappa)$, or
- it is required to influence the look-ahead control strategy of the vehicles through R_1, which results in the modification of the fundamental diagram.

Figure 5 illustrates the Controlled Invariant Set $V = 1$. The stability of the system can be guaranteed between V_{-} and V^{-}. Figure 5 demonstrates that the stability range varies depending on κ and R_1. However, V is in the stable range in the entire κ and R_1 range, the choice of the Controlled Lyapunov Function is suitable.

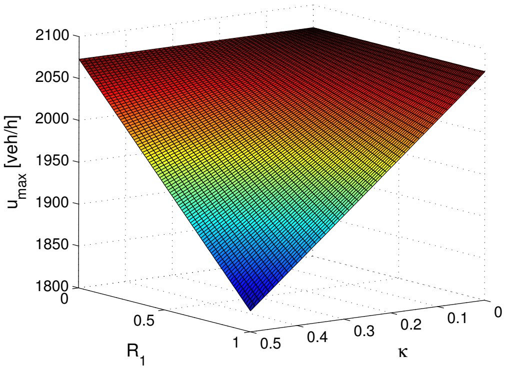


Fig. 4. Result of the stability analysis

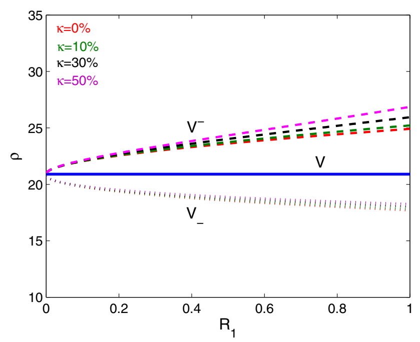


Fig. 5. Controlled Invariant Set of the system

6. CONCLUSIONS

In the paper the mixed traffic flow, in which both look-ahead controlled vehicles with a speed control and conventional vehicles are driven, has been analysed. The paper analyses the impact of vehicles applying look-ahead control strategy on the traffic flow. Three parameters concerning the look-ahead control and the traffic flow have been examined. The number of vehicles in the traffic network, the ratio of look-ahead vehicles in the traffic and the energy-efficiency scaling parameter of the look-ahead control are considered. In a simulation-based analysis it is shown that the fundamental diagram of the traffic network changes. The mixed-traffic model incorporates the nonlinearity of the traffic dynamics using a polynomial formula, in which the look-ahead cruise control is considered with a parameter-dependent form. Using these nonlinear forms the impact of the look-ahead vehicles on the stability of the traffic is examined through controlled invariant sets. Stability requires the length of the queue on the inflow ramps to be small, which results in the reduction in the waiting time of the inflow vehicles. The analysis is based on the sum-of-squares programing method.

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