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Traffic flow optimization with QoS constrained network admission control ⋆

Alfréd Csikós * **Hamed Farhadi** **,**** **Balázs Kulcsár** ** **Themistoklis Charalambous** ***,** **Henk Wymeersch** **

* *Institute for Computer Science and Control, Hungarian Academy of Sciences, Budapest, Hungary. E-mail:* csikos.alfred@sztaki.mta.hu.
** *Department of Signals and Systems, Chalmers University of Technology, Gothenburg, Sweden. E-mails:* {farhadi, kulcsar, thecha, henkw}@chalmers.se.
*** *Department of Electrical Engineering and Automation, School of Electrical Engineering, Aalto University, Espoo, Finland.*
**** *John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA, USA.*

Abstract: The paper proposes a control design method in order to gate input flow to a protected urban vehicular network such that travel time Quality of Service (QoS) constraints are preserved within the network. In view of the network to be protected (also called the region), two types of queues are distinguished: *external* and *internal*. While external queues represent vehicles waiting to enter the protected network, an internal queue can be used to describe the network's aggregated behaviour. By controlling the number of vehicles entering the internal queue, the travel time within the network subject to the vehicular conservation law and the Network Fundamental Diagram (NFD) can be subsequently controlled. The admission controller can thus be interpreted as a mechanism which transforms the unknown arrival process governing the number of vehicles entering the network to a regulated process, such that prescribed QoS requirements on travel time in the network and upper bound on the external queue are satisfied. The admission control problem is formulated as a constrained convex optimization problem and a Model Predictive Control (MPC) problem. A case study demonstrates the benefits of the admission control mechanisms proposed.

Keywords: Traffic control; traffic flow; admission and perimeter control; network fundamental diagram; travel time; Quality of Service.

1. INTRODUCTION

Traffic congestion has become a major issue, especially in big cities, since it results in - among others - delays, pollutant emissions, higher energy expenditure and accidents (see, for example Bigazzi and Figliozzi (2012) and references therein). Intelligent Transportation Systems via control and coordination of traffic flows has been of vital importance, in order to appropriately use finite road capacity both in under- or over-saturated traffic conditions. One efficient urban approach is to adapt traffic lights at signalized intersections. While such mechanisms may perform well for undersaturated conditions, oversaturated conditions cannot be efficiently handled. The influence of an intersection to its topological neighbours is of critical importance, but the traffic dynamics in such a microscopic level in the network have not yet been well understood.

The above limitations have triggered the need to control the admission of vehicles in a traffic network to avoid oversaturated conditions and to look at the traffic dynamics from a macroscopic point of view. Towards this end, the concept of NFD, often called Macroscopic Fundamental Diagram (MFD), has been adopted as a basis for the derivation of traffic control strategies. The theory was first proposed in Godfrey (1969) and further developed in Daganzo and Geroliminis (2008) and Helbing (2009) (its application to experimental data is analyzed in Mahmassani et al. (1987); Geroliminis and Daganzo (2008); Ampountolas and Kouvelas (2015)).

In particular, details of individual intersections are not required to describe the congestion level and evolution of the traffic network; instead, under the assumption that the traffic network and the traffic inflow are homogeneous, they can all be aggregated to a single queue (internal queue) whose length affects various properties of the network. It is therefore desirable to aim at controlling the number of vehicles in the network by means of gating the input flow to the network; this is also known as perimeter flow control; see, e.g., Daganzo (2007). The state of the network can be easily monitored via loop detector data in real-time while the computational complexity of gating the input is low, making it an easy and realistic concept to implement.

Daganzo (2007) first used the NFD to synthesize a controller that maximizes the network outflow, thus comprising a starting point for using the NFD theory for controlling traffic flow. Several works followed the developed control strategies based on NFD to maximize the capacity of homogeneous traffic networks. In this case, a

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single-region model with one NFD represents the dynamics of the network appropriately. The paper by Hajiahmadi et al. (2013) formulates the optimal control problem as a mixed integer linear optimization problem, with two types of controllers: perimeter controllers and a switching controller of fix-time signal plans. However, the solution to the problem cannot be used in real time. For alleviating this problem, a Proportional-Integral (PI) controller is proposed by Keyvan-Ekbatani et al. (2012) for real-time gating, with an application to the network of Chania, Greece. By modeling the dynamics of the external queues, the perimeter problem is solved with a Nonlinear MPC in Csikós et al. (2015). In Haddad (2017) a model for multi-region networks is introduced. The flow characteristics of the the urban regions are modeled by the NFD functions, while aggregate boundary queue dynamics for both regions are modeled by input-output balance differential equations.

While perimeter flow control has recently received a lot of attention from a control theoretic perspective, further QoS requirements for the system, such as average time delay in the network, have not been considered. In this work, similarly to the classic perimeter control problem, the objective is to optimize network performance through the maximization of network throughput. However, we additionally include QoS requirements, adopting the service indicators of communication networks (see, for example, Klessig and Fettweis (2014); Liu et al. (2014.); Le et al. (2012) and references therein) to (a) keep the travel time spent in the network below a certain threshold, and (b) avoid, if possible, the blockage at the entrance of external queues. These QoS requirements are incorporated as constraints into the system. The problem emanating from our objective and constraints, is first formulated as a constrained convex optimization problem and it is solved via internal point methods. If the NFD is assumed to have a quadratic form, the constrained convex optimization problem can be transformed into a QP problem and solved efficiently. Next, we also cast and solve the problem as an MPC one. The performance of our approaches is demonstrated via a case study and compared to that of the simple PI controller.

2. NOTATION AND PRELIMINARIES

The system dynamics is modeled through the conservation of vehicles for both the internal and external queues. The first state equation gives the time evolution of the number of vehicles in the protected/controlled network (representing the evolution of "internal queues") over a sample step of length T, that is,

$$N_{k+1} = \left[N_k + T \left(\sum_{i \in \mathcal{I}} q_k^{in,i} - \sum_{j \in \mathcal{O}} q_k^{out,j} \right) \right]^+, \quad (1)$$

where $[\cdot]^+$ is the maximum between zero and its argument, N_k denotes the number of vehicles, $q_k^{in,i}$ and $q_k^{out,j}$ denote the inflow at link i and outflow at link j at sample step k in unit [veh/h], respectively. $\mathcal{I}$ denotes the set of entrance queues and $\mathcal{O}$ denotes the set of exit links.

Let $q_k^{in} \triangleq \sum_{i \in \mathcal{I}} q_k^{in,i}$ and $q_k^{out} \triangleq \sum_{j \in \mathcal{O}} q_k^{out,j}$, equation (1) can be abstracted to a single internal queue, i.e.,

$$N_{k+1} = \left[N_k + T \left(q_k^{in} - q_k^{out} \right) \right]^+. \quad (2)$$

The network outflow is modelled through the NFD concept. The total regional circulating flow $Q(N)$ is approximated by Edie's generalized definition of flow, i.e., the weighted average of link flows multiplied with link lengths. If we assume that the average trip length Υ in the network is constant and the average link length is given by l, then the output (throughput) of the network can be expressed as follows Daganzo (2007):

$$q_k^{out} = \frac{l}{\Upsilon} Q\big(N_k\big). \quad (3)$$

Output flow q_k^{out} is the estimated rate at which vehicles complete trips per unit time either because they finish their trip within the network or because they move outside the network. This function describes steady-state behavior of single-region homogeneous networks if the input to output dynamics are not instantaneous and any delays are comparable with the average travel time across the region Kulcsar et al. (2015).

Network inflow q_k^{in} is considered to be the controlled input of the system that follows the admission control policy. This flow depends on the entrance queue state, network state, QoS requirements, and the network NFD. The admittance into the network is described through a simple queuing model, for entrance gate i, by:

$$L_{k+1}^i = \left[L_k^i + T \left(\lambda_k^i - q_k^{in,i} \right) \right]^+, \quad (4)$$

where L_k^i is the queue length of the external queue and λ_k^i denotes the uncontrolled arrival rate at time k. We assume the arrival rate is an unknown, deterministic and bounded demand sequence. Summing all external queues $i \in \mathcal{I}$,

$$L_{k+1} = \left[L_k + T \left(\lambda_k - q_k^{in} \right) \right]^+, \quad (5)$$

where $L_k = \sum_{i \in \mathcal{I}} L_k^i$ and $\lambda_k = \sum_{i \in \mathcal{I}} \lambda_k^i$. Note, that only a controlled number of vehicles enter the network, and no disturbance flows are present, i.e. we assume to gate all external flows entering the network for sake of simplicity. Regarding the overall system, however, λ_k is considered as disturbance.

3. PROBLEM STATEMENT

Similar to the classic perimeter control problem, the objective is to optimize network performance through the maximization of network throughput. Moreover, the network performance is characterized by the QoS requirements set. These QoS requirements are usually stated as stochastic values, e.g., the expected value of time delay, blockage probability of external queues.

In this work, QoS indicators are handled as deterministic values. By specifying upper/lower bounds for the indicators, hard constraints can be given for the system. For the traffic networks, two QoS requirements are considered:

- The average time delay in network should be less than a given threshold.
- The blockage of external queues should be avoided.

Average time delay in network

This indicator is modeled by the following formula:

$$\Delta(N_k) = \frac{l}{v(N_k)} - \frac{l}{v_{free}}, \quad (6)$$

where l denotes the average link length of the network ($l = M^{-1} \sum_{i=1}^{M} l_i$), for links $i \in 1, ..., M$ while $v(N_k)$ and v_{free} denote the actual and free link travel speed of the network, respectively.

According to Edie (1963), the average network speed can be expressed by using the generalized network-wide traffic variables:

$$v(N) = \frac{\text{TTD}(N)}{\text{TTS}(N)}, \tag{7}$$

where $TTD(N)$ and $TTS(N)$ denote the Total Travel Distance (TTD) and Total Time Spent (TTS) in the network, given by the definitions $\text{TTD}(N)=Q(N)T\sum_{i=1}^{M} l_i$ and $\text{TTS}(N)=NT\sum_{i=1}^{M} l_i$, respectively. Substituting the generalized definitions, the following formula is obtained for average network speed:

$$v(N_k) = \frac{Q(N_k)}{N_k}. \tag{8}$$

Note, that $Q(N_k)$ is chosen, such that $v(N_k)$ is an invertible function. In fact, it is intuitive that as the number of vehicles in the network N_k increases, the average speed of the network is expected to decrease. By assuming a continuously differentiable concave NFD over the eventual interval on N and network flow uniformity, invertibility of $v(N_k)$ is therefore a direct consequence of the NFD model.

The free travel speed can be approximated by the following formula:

$$v_{free} = \lim_{N_k \to 0^+} \frac{Q(N_k)}{N_k} \overset{(a)}{=} \lim_{N_k \to 0^+} \frac{\partial Q(N_k)}{\partial N_k}, \tag{9}$$

where (a) is due to L'Hôpital's rule.

Let Δ_{nom} denote the delay in the network when a vehicle travels with v_{free} and it is equal to l/v_{free}. We require that the average time delay in the network is smaller than a threshold value, herein denoted by Δ_{tr}, i.e., $\Delta(N_{k+1}) \le \Delta_{tr}$. Note that the required average time delay will be higher than the time needed when a vehicle travels with v_{free}, i.e., $\Delta_{tr} > \Delta_{nom}$.

Blockage of external queues

A deterministic approach is followed in which the aim is to avoid queue blockage, i.e., $L_k \le L_{cap}$ needs to be satisfied for all k, where L_{cap} denotes the capacity of the external queue (in fact the sum of all capacities of all external queues).

Remark 1. Note that for very large arrival rate λ_k it is not possible to guarantee that both QoS requirements are fulfilled. •

4. MAIN RESULTS

In this section, the problem is first cast as an optimization problem. Then, after algebraic manipulations, we restate our constraints (QoS requirements) as upper and lower bounds of the internal queue length.

4.1 Optimization problem

The control aim is to maximize the network outflow (3) such that the specified QoS conditions are satisfied. The outlined QoS conditions can be formalized as follows:

- For the time delay, $\Delta(N_{k+1}) \le \Delta_{tr}$ is given This condition is used to guarantee a QoS on the travel time vehicles spend in the region. It gives an upper bound for the internal queue N and thus the inflow to the region.
- External queue blockage is avoided if $L_{k+1} \le L_{cap}$. It leads to a lower bound for the internal queue, and indirectly for the inflow to the region.

Additionally, a constraint can be formalized for the admissible flow as follows:

$$0 \le q_k^{in} \le min(\lambda_k + L_k/T, g_{max}s), \tag{10}$$

where the maximal green time of the entering links is calculated as $g_{max} = \sum_{i\in\mathcal{I}} g_{max,i}$, with $g_{max,i}$ denoting the maximal green time of input link i. The saturation flow of input links is assumed to be constant (for simplicity of exposition), and it is denoted by s. This constraint is not restricting the operation of the network and it basically states that the inflow cannot be less than zero or more than the external queue which can be injected into the network.

The optimization problem can then be formulated as follows:

$$\max_{q_k^{in}} \quad Q(N_{k+1}) \tag{11a}$$

$$\text{subject to:} \quad \Delta(N_{k+1}) \le \Delta_{tr} \tag{11b}$$

$$L_{k+1} \le L_{cap} \tag{11c}$$

$$0 \le q_k^{in} \le min(\lambda_k + L_k/T, g_{max}s) \tag{11d}$$

$$N_{k+1} = \left[N_k + \left(q_k^{in} - q^{out}(N_k)\right)\right]^+ \tag{11e}$$

$$L_{k+1} = \left[L_k + \left(\lambda_k - q_k^{in}\right)\right]^+. \tag{11f}$$

We hereby suggest the following optimal delay-aware traffic control policy.

Proposition 1. Given a single-step control horizon with constraints (11b)-(11f) on state variables N_{k+1} and L_{k+1} and q_k^{in}. Optimization problem (11) can be relaxed to a convex optimization problem:

$$\max_{N_{k+1}} \quad Q(N_{k+1}) \tag{12a}$$

$$\text{subject to:} \quad N_{k+1}^{lb} \le N_{k+1} \le N_{k+1}^{ub} \tag{12b}$$

from which once the optimization problem (12a) is solved, the optimal control input q_k^{in} can be calculated by (2). •

Proof 1. The upper and lower bounds are obtained as follows. By substituting the speed function $\Delta(N_k)$ from (6) into (11b), a constant lower bound can be derived for the speed, i.e.,

$$v^{lb,delay} = \frac{l}{\Delta_{tr} + \Delta_{nom}}. \tag{13}$$

The constant upper bound for the internal queue is obtained by inverting the speed function:

$$N^{ub,delay} = v^{-1}\left(\frac{l}{\Delta_{tr} + \Delta_{nom}}\right). \tag{14}$$

Substituting the upper bound for controlled inflow q_k^{in} from (11d) into the equality constraint (11e) a non-constant upper bound emerges and it is given by

$$N_{k+1}^{ub,que} = q_k^{in,ub} + N_k - q^{out}(N_k). \tag{15}$$

As a result, the applied upper bound for the decision variable is given as the minimum of the upper bounds found in (14) and (15), i.e.,

$$N_{k+1}^{ub} = \min(N_k^{ub,que}, N^{ub,delay}). \tag{16}$$

Lower bound for N can be obtained by substituting (11e) and (11f) to (11c), i.e.,

$$N_{k+1}^{lb,block} = N_k - q_k^{out}(N_k) + L_k + \lambda_k - L_{cap}. \tag{17}$$

Note, that $N^{lb,block}$ may take negative values. Hence, the applied lower bound is given as:

$$N_{k+1}^{lb} = \max(0, N_k^{lb,block}). \tag{18}$$

Remark 2. Due to the min and max functions in the constraint description, we have nonlinear constraints that are usually simplified by a mixed integer formulation. In our approach, time-varying constraints are applied, and assuming a polynomial NFD with a global maximum (e.g., a quadratic function), the maximization of discharge flow leads to a convex optimization problem with new constraints to be solved in each step. As a result, the mixed integer formalization is no longer needed. •

Remark 3. Regarding the overall network that involves the external and internal queues, the QoS requirements define a modified capacity of the system through the time varying interval of bounds. As noted in Remark 1, for a very large arrival rate λ_k it is not possible to guarantee that both QoS requirements are fulfilled. This can be seen from (18), where as λ_k increases the lower bound becomes higher, and hence for large λ_k our lower bound may become higher than the upper bound. One of the main advantages of our method, is that it is able to detect when this situation occurs. In such situations, we need to prioritize between the QoS conditions. In our scheme, priority is given to the vehicles in the protected network, i.e., violation of the upper bound, which corresponds to guaranteeing the average time delay in the network, is not permitted. Hence, when the lower bound becomes equal to or even exceeds the upper bound, at that time step the solution of the problem N_{k+1} is the upper bound itself, and no optimization is required to be solved. The maximum arrival rate λ_k that can be handled by the network is found by restricting the lower bound of the vehicles in the network to be smaller than or equal to the upper bound, i.e., $N^{lb}_{k+1} \leq N^{ub}_{k+1}$. Thus,

$$\lambda_k \leq \lambda_k^{\max} \triangleq N^{ub}_{k+1} + L_{cap} - N_k + q^{out}_k(N_k) - L_k.$$

For any value above $\lambda_k^{\max}$, by choosing N_{k+1} to be the solution to the optimization, we relax the constraint of having $L_{k+1} \leq L_{cap}$ for the external queues in order to keep the network flow at its maximum and avoid compromising the travel delay in the network. •

4.2 An MPC approach

MPC is well suited to this problem, since it is a direct constraint handling method that can be implemented over a finite prediction horizon. Optimization problem (11) is now adapted to MPC framework. First, an equality constraint is involved for the disturbance: throughout the control horizon, λ_k is considered constant. Furthermore, the decision variable of the optimization is the vehicle inflow q^{in}_k instead of the internal queue N_{k+1}, and distinctively, bounds are defined for the states and the control input. The cost function is also extended. The first term implies the optimization of discharge flow of the protected network. The demand matching in the second term is given to avoid an unnecessary suppression of inflow. The first two terms thus lead to a balanced control of the internal and external queues. The third term is applied to suppress input oscillations. Hence, the optimization problem for the MPC framework is given by

$$\min_{[q^{in}_k,..,q^{in}_{k+N_c}]} \sum_{\ell=1}^{N_c} \{-Q(N_{k+\ell}) + \|q^{in}_{k+\ell} - \lambda_{k+\ell}\|_2^2 + \|q^{in}_{k+\ell} - q^{in}_{k+\ell-1}\|_2^2\} \quad (19a)$$

$$\text{subject to:} \quad N_{k+1} = N_k + \left[q^{in}_k - q^{out}(N_k)\right] \quad (19b)$$

$$L_{k+1} = L_k + \left[\lambda_k - q^{in}_k\right] \quad (19c)$$

$$\lambda_{k+\ell} = \lambda_k, \forall \ell \in 1, ..., N_c \quad (19d)$$

$$0 \leq L_{k+\ell} \leq L_{cap} \forall \ell \in 1, ..., N_c \quad (19e)$$

$$0 \leq q^{in}_{k+\ell} \leq min(\lambda_{k+\ell} + \frac{L_{k+\ell}}{T}, g_{max}s) \quad \forall \ell \in 1, ..., N_c. \quad (19f)$$

The controller solves a convex optimization problem in a rolling horizon manner Grüne and Pannek (2011).

5. SIMULATION ANALYSIS

The proposed control algorithm is applied for a test system, simulated in Simulink assuming a quadratic NFD of the form $Q(N_k) = aN_k^2 + bN_k$. The model parameters are given in Table 1.

Parameter	Value	Parameter	Value
a	-0.1	g_{max}	200s
b	40	v_{nom}	40 km/h
T	60 s	Δ_{nom}	13.5 s
l/Υ	0.025	Δ_{tr}	$5\Delta_{nom}$
s	0.5 veh/s	L_{cap}	200 veh

Table 1. Model parameters

5.1 Control scenarios

In this work, we investigate the performance of 3 controllers:

1. QP controller. For a quadratic NFD, the optimization problem (12a) becomes a QP problem.

2. MPC controller. Following a manual tuning, the control horizon for the MPC in (19a) is chosen as $N_c = 6$.

3. PI controller. The control rule is similar to the one applied in Keyvan-Ekbatani et al. (2012):

$$q^{in,PI}_k = q^{in}_{k-1} + K_I(N_k - N_{k-1}) + K_P(N_{opt} - N_k), \quad (20)$$

where the optimal internal queue $N_{opt} = \arg\max Q(N)$, whereas K_P and K_I are the control design parameters, obtained by manual tuning. The design resulted in the following values: K_I=0.3, K_P=0.085. Input saturation is applied for the controller in the following form:

$$q^{in,sat}_k = min(q^{in,ub}_k, q^{in,PI}_k). \quad (21)$$

where $q^{in,ub}_k$ is given in eq. (10).

5.2 Simulation results

Case study 1

First, a scenario, in which a sinusoid arrival rate is simulated, is analyzed featuring all three controllers. The simulation results of a 2-hour-long scenario are plotted in Figs. 1-3. Fig. 1 (top plot) depicts the arrival rate and the entrance flows of the different control situations. In case of no control, the network gets congested around 1000s and the entering flow starts to decrease drastically. The three controllers (PI, MPC, QP) however manage to avoid congestions in the protected network (see bottom plot in Fig. 1 and Fig. 3). Here, the PI control shows a fundamentally different behaviour to that of the MPC and QP controllers. The PI controller cannot avoid the blockage of the external queue. The reason for this is that the state bounds (11b) and (11c) cannot directly be applied to the proposed PI controller. However, the bound for the input signal (11d) is satisfied due to the input saturation (21). This leads to high gradients in control signal at 4200s and 6600s.

The MPC and QP approaches are very similar regarding the input signal and the states, the former having oscillations occasionally (partially due to the improper future information on the disturbance in the MPC scheme). The performance of the controlled systems can be best observed in Fig. 2, bottom plot. The discharge flow is similar in the cases of the MPC and the QP controllers. In case of

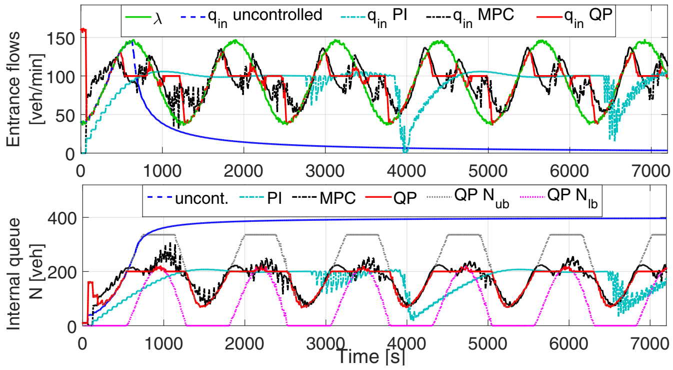

Fig. 1. Entrance flows (top) and internal queues (bottom) with respect to time. In the figure for the entrance flows (top), we have the arrival rate λ_k and the corresponding entrance flows q_k^{in} for the uncontrolled case and all the controlled cases. In the figure for the internal queue states (bottom), we have the internal queue for all cases and the upper and lower bounds obtained for the QP optimization problem.

the MPC control, the same QoS bounds are applied as in the QP case. Therefore, the constant bounds are always preserved. However, the time-varying state bounds of the MPC optimization for the internal queue are not necessarily equal to the QP bounds (12b) as state constraints in the MPC case are set separately, and additionally, an equality constraint is given for the disturbance (19d). Hence, the QP state bounds are not expected to be satisfied by the MPC controller (e.g., between 1200s and 1800s); see Fig. 2, bottom plot. Focusing on the external queue size for the

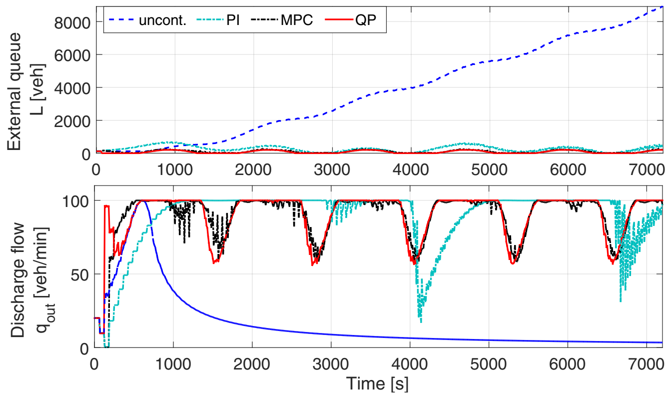

Fig. 2. External queue length (top) and discharge flow (bottom) for the controlled case. For the uncontrolled case, the network becomes congested and its discharge flow reduces dramatically (bottom) causing the external queue length to grow large (top). For the controlled cases, the QP and MPC controllers have similar performance.

controlled cases for which it remains bounded, it is shown in Fig. 3 that, while both the QP and MPC approached respect the bounds, for the PI there is no saturation to prevent it from exceeding the upper bound L_{cap} of the external queue.

As it can be seen, the MPC and QP control show similar performance. The main advantage of the QP solution over the corresponding MPC solution is its low computational demand. The computation times[1] of the different controllers are summarized in Table 2.

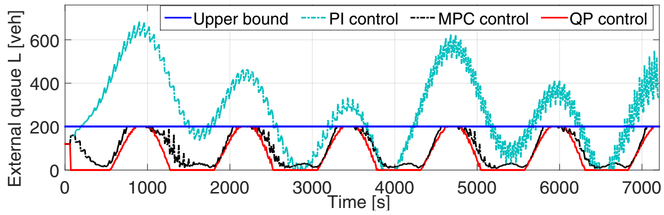

Fig. 3. External queue length for the controlled case. While both the QP and MPC approached respect the bounds, for the PI there is no saturation to prevent it from exceeding the upper bound L_{cap} of the external queue.

Method	MPC	QP
Comp. time [s]	1.649	0.091

Table 2. Computation time of a sample step

Case study 2

In the second scenario, the load of the overall system is not changed. However, the threshold value on time delay is reduced to Δ_{tr}=2.25Δ_{nom}. By this change, the operation of the QP controller can be analysed in cases when the lower state bound exceeds the upper bound. This situation leads to an infeasible problem for the MPC controller, as it cannot satisfy the conflicting constraints of the internal and external QoS state bounds. Therefore, the analysis focuses on the behaviour of the QP solution.

The simulation results of the scenario are plotted in Figs. 4-6. The top plot of Fig. 4 depicts the arrival rate and the uncontrolled and controlled entrance flows. Similarly to case study 1, the network gets congested around 1000s with no control (see Figs. 4 and 5).

At certain points, the lower bound N_{k+1}^{lb} reaches the upper bound N_{k+1}^{ub} - first around 950s. In such cases, N_{k+1}^{lb} is neglected and N_{k+1}^{ub} is accepted as the problem solution. By this, the QoS indicator for time delay is preserved in favour of the protected network, while the external queue is blocked (see Fig. 5). When the arrival rate decreases, the controller is able to resolve the blockage of the external queue.

6. CONCLUSIONS AND FUTURE DIRECTIONS

6.1 Conclusions

In this paper we proposed an admission control mechanism that maximizes network outflow while specified QoS requirements are satisfied. These QoS requirement were incorporated as constraints into the system. First, a single-step predictive, convex constrained optimization problem was formulated, and an algorithm was developed ensuring throughput maximization subjected to network travel time constraint guarantees. Next, the problem was formulated as an MPC one. The performance of our approaches was demonstrated via case studies and compared to that of the simple PI controller. The case studies illustrate that the proposed mechanisms have improved performance in terms

[1] On an Intel i5 2.4 GHz 8GB RAM computer.

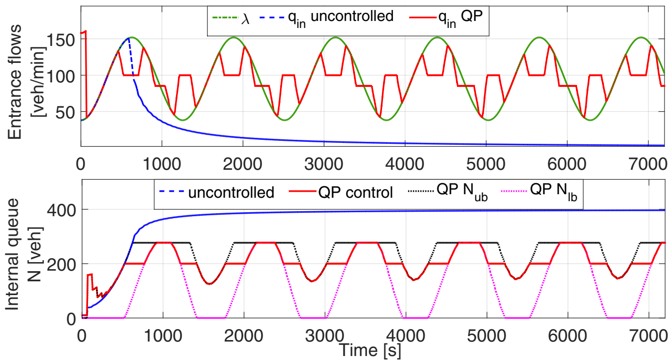

Fig. 4. Entrance flows (top) and internal queues (bottom) with respect to time. In the figure for the internal queue states (bottom), we have the equalized upper and lower bounds at times of extreme queues.

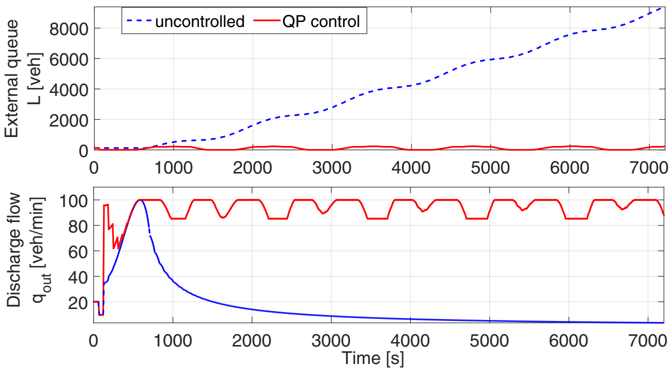

Fig. 5. External queue length (top) and discharge flow (bottom) for the controlled case. For the controlled case, the QP controller maintains an acceptable flow discharge rate in spite of the compromised operation.

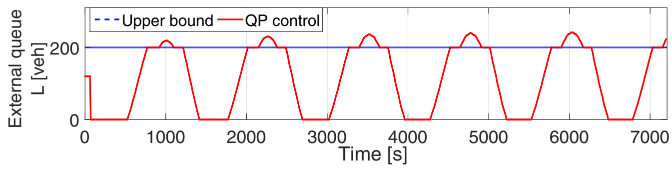

Fig. 6. External queue length for the controlled case. At certain points, the QoS condition for external queue blockage needs to be sacrificed to satisfy the QoS of the protected network.

of network throughput, average time delay and external queue length.

6.2 Future Directions

Since the shape of the NFD is affected by different factors, it is important to study the problem under uncertain traffic flow description. Towards this end, Kulcsar et al. (2015) proposed an $\mathcal{L}_2$ optimal control design, and Haddad and Shraiber (2014) a robust control one, based on the Linear Parameter-Varying (LPV) model structure. However, none of these approaches incorporated QoS requirements, which is part of our ongoing research.

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