AR and ARMA Spectral Analysis of Suspension System of a Commercial City Bus

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Abstract—Concerning the increasing demand for intelligent and efficient urban vehicle systems with low cost maintainability and high passenger comfort, reliable methods are needed to model and to evaluate the imposed performances. The measurements, vibrations emerging on the wheels and the body, that has ben taken on a city bus are analyzed in the frequency domain. In this paper a parametric spectral analysis (AR/ARMA method) of the suspension system of a commercial city bus is presented. The goal of the analysis is to find the right structure for the systems. Parametric methods used in this paper justify and extend the results obtained by non-parametric ones and provide more accurate results for vibration analysis. One of the main conclusions of the investigations is that the quarter-car model structure based on first principles does not reflect the true frequency domain behavior of the system. Thus the identification of the physical model must be complemented with a suitable uncertainty modeling and classification.

I. Introduction

Recently, there has been a growing demand for vehicles with better driving characteristics in which efficiency, maintainability and performance are ensured. Concerning vertical dynamics the accelerations generated by variations in road surface induces discomfort of passengers on one hand side and also may cause harm in freight and further impairs of the road surface. Thus the primary task of vehicle suspension system is to provide a more comfortable ride and in a same time a reduced effect on the road. Assessing technical conditions of the road network and of fleets of public vehicles in a city provides an important support to operators in making decisions on the scheduling and harmonization of maintenance works.

In order to fulfill these goals a two step process was followed: in the data acquisition step measurements of the suspension systems are performed during the normal everyday operation of the fleet of vehicles. Then, in the second step, a vibration analysis is executed that helps to objectively evaluate both the road quality and the technical conditions of the vehicles.

Concerning the analysis phase a mathematical model of the nominal vibration mechanism is to be formulated first, which serves as a reference for evaluating both the suspension system of a specific vehicle, and also the conditions of road surface. With repeated experiments a set of nominal models are obtained from which a so called nominal uncertainty model set can be constructed, [3], [6]. The road network is divided into segments and the modeling is repeated for every segment during normal operation of the vehicles. When comparing the models with the nominal uncertainty model set we encounter differences caused by road surface variations, unmodeled dynamics of the suspension system excited by the lateral and longitudinal accelerations and roll and pitch motions of the vehicle and variations in the mass of load. The effects of lateral/longitudinal/roll and pitch dynamics should be filtered out from the vertical measurements by an estimation of these disturbing motions while measurements taken on the same segments with multiple vehicles can be compared in order to separate the effects of the road and the effect of change in the technical conditions of the vehicles.

As a first step toward model construction, a non-parametric spectral analysis of the suspension system was carried out and presented in [10]. Non-parametric power spectral density (PSD) estimation methods, like periodogram and Welch's method, provide respectively a detailed and a subjective estimation. In the latter case the choice of window width, window function and the amount of overlapping strongly influence the obtained PSD. Nevertheless, non-parametric PSD estimates provide a first impression on the measurements and the physical system.

In this paper parametric spectral analysis is presented. It enables a more objective analysis due to a broader class of validation tools. Two systems are considered as in [10]. The input in both cases is a white nose process representing road excitation and other disturbances. The outputs are the vertical acceleration of the wheel axis and the chassis, respectively. Autoregressive (AR) and autoregressive moving average (ARMA) models are considered to find the right structure for the systems. The right order of the models are estimated based on Akaike's information criterion and viewing the pole-zero map of the models. PSD of the outputs are computed and compared with the results in [10]. Parametric methods used in this paper justify and extend the results obtained by non-parametric methods, help objectively distinguish peaks that are present in the periodogram and may be averaged in the Welch's method, and in this way, provide more accurate results for vibration analysis.

The paper is organized as follows. Preliminaries on parametric spectral analysis are presented in Section II. The measure-

ment setting, experimental conditions and data preprocessing are briefly described in Section III. Parametric spectral analysis of the suspension systems is presented in Section IV. The main conclusions of the paper are summarized in Section V.

II. PRELIMINARIES ON PARAMETRIC SPECTRAL ANALYSIS

The goal of spectral estimation is to describe the frequency-distribution of the power contained in a signal, based on a finite set of data. The power spectral density (PSD) of a *wide-sense stationary random process* x_t is mathematically related to the autocorrelation sequence by the discrete Fourier transformation (DFT), see [5], [7]. There are various methods of spectrum estimation. In *nonparametric methods* the PSD is estimated directly from the signal. The simplest such method is the periodogram, other nonparametric techniques are Welch's method, [11], and the multitaper method, [9], both reduce the variance of the periodogram.

Parametric methods estimate PSD from a signal that is assumed to be output of a linear system driven by white noise. Examples are the Yule-Walker and the Burg method. These methods estimate the PSD by first estimating the parameters of the linear system that hypothetically generates the signal. They tend to produce better results than classical nonparametric methods when the data length of the available signal is relatively short. Parametric methods also produce smoother estimates of the PSD than nonparametric methods, but are subject to error from model misspecification.

The most commonly used linear system model is the all-pole model, a filter with all of its zeroes at the origin in the z-plane. The output of such a filter for white noise input is an autoregressive (AR) process

$$\text{AR(p):} \qquad y_t + a_1 y_{t-1} + ... + a_p y_{t-p} = e_t$$

where e_t and y_t denote respectively the white noise input with variance λ and the output of the pth order system at discrete time step t. The AR methods tend to adequately describe spectra of data that is "peaky," that is, data whose PSD is large at certain frequencies. With a given model order, AR model parameters are estimated in this paper by using a least-squares based method called *modified covariance method*. The modified covariance method is based on minimizing the forward and backward prediction errors [2, Chapter 9.].

An alternative model structure is the auto regressive moving average (ARMA) process

$$\text{ARMA(p,q):}$$
$$y_t + a_1 y_{t-1} + ... + a_p y_{t-p} = e_t + c_1 e_{t-1} + ... + c_q e_{t-q}.$$

again with e_t being a white noise process of variance λ. With given n and p, ARMA model parameters are estimated in this paper by iteratively minimizing a quadratic prediction error criterion [5]. With the choice $q = p$, the model will be denoted a single argument by ARMA(p).

A. Parametric spectral estimation

Suppose we have N regularly sampled measurements of the scalar output, $y = \{y_1, y_2, ..., y_N\}$, with sampling frequency f_s and suppose that an AR or ARMA model is identified from the data. Once the model structure (AR(p) or ARMA(p)) and the noise variance λ are given, the estimated PSD of the output can be expressed by

$$\Phi_{yy}(f) = |W(f)|^2 \lambda \qquad (1)$$

as a function of frequency f, $f \in \{-\frac{f_s}{2}, -\frac{f_s}{2} + \frac{f_s}{N}, -\frac{f_s}{2} + \frac{2f_s}{N}, ..., \frac{f_s}{2}\}$ where

$$W(f) = \frac{1}{1 + \sum_{k=1}^{p} a_k e^{-j2\pi fk}} \qquad \text{AR(p)} \qquad (2)$$

$$W(f) = \frac{1 + \sum_{k=1}^{p} c_k e^{-j2\pi fk}}{1 + \sum_{k=1}^{n} a_k e^{-j2\pi fk}} \qquad \text{ARMA(p)} \qquad (3)$$

is the transfer function of the modell. The output of the model

B. The problem of parameter estimation

Whenever the model order p is selected, the goal of the parameter estimation is to find the model parameters

$$\theta = [a_1, a_2, ..., a_p]^T \qquad \text{AR(p)} \qquad (4)$$
$$\theta = [a_1, a_2, ..., a_p, c_1, c_2, ..., c_p]^T \quad \text{ARMA(p)} \quad (5)$$

which minimize the variance λ of the prediction error $\varepsilon_t(\theta) = y_t - \hat{y}_t(\theta)$. This problem is equivalent to minimizing the variance of the PSD estimate.

For AR model parameter identification, the modified covariance method is applied which minimizes the sum of the squares of the forward and backward prediction errors, [4, Chapter 6.5].

For ARMA model parameter identification an iterative search algorithm is used that minimizes a robustified quadratic prediction error criterion [8], [5, Chapter 10.2].

C. Model order selection

Model order selection is an important problem, because if the order that is used is too small, the resulting spectral estimate will be smoothed and will have poor resolution. If the model order is too high, the spectral estimate may contain spurious peaks and lead to spectral line splitting.

In selecting order, one approach would be to increase the order until the modeling error is minimized. The difficulty with this is that the error is a monotonically nonincreasing function of the model order p. This problem may be mitigated by incorporating a penalty function that increases with the model order. Several criteria proposed include a penalty term that increases linearly with n. One of them is Akaike Information Criterion (AIC)

$$\text{AIC}(p) = \log V + \frac{2d}{N} \qquad (6)$$

where d is the number of parameters, it increases linearly with p, V denotes the loss function

$$V = \frac{1}{N} \sum_{t=1}^{N} \varepsilon_t(\theta)^2 \qquad (7)$$

where $\varepsilon_t(\theta)$ is the prediction error at time t with parameter vector θ. It has been observed that the AIC gives an estimate for the order p that is too small when applied to non-AR processes and it tends to overestimate the order as N increases.

Further help in selecting the model order is looking at the pole zero map of the transfer function $W(f)$. Poles with negative real part; and of very high frequencies (out of the bandwidth of the system and/or the measurements); poles with large confidence ellipsoids; pole-zero cancellation, all may refer to overestimation of the model order. On the other hand, since the search for the optimal parameters may stuck in local optima of the criterion function, overparameterization can also be advantageous as it may result in the better estimation of the physically meaningful poles of the model.

D. Model structure selection

We know about the experimental system that it suffers from many time-varying uncertain effects about which we have no information (road excitation, roll, pitch, lateral and longitudinal motion of the vehicle during the experiments). We, therefore, cannot expect from the identification procedure to result in an accurate model that stands the test of common model validation tools with different data. We only expect that that *some* significant properties of the physical system can be consistently captured.

A simplified model of a suspension system, the so called quarter car model, consists of two rigid masses, one for the *sprung* mass and one for the *unsprung* mass, which are connected by a spring and a damper (possibly of nonlinear characteristics) and the unsprung mass is connected to the road with a spring and a small negligible damper (tire). The linearized system with realistic parametrization has two lightly damped modes, each with a complex pair of poles. It is expected from the model identification procedure that at least the two frequencies of these two modes can be obtained. Unfortunately, the frequency corresponding to the idle speed of the engine is about 10Hz, it is close to the one eigenfrequency of the suspension system.

In this situation the following can be said. If the system that generates the data can be characterized by the chosen model structure, then there exists a minimal model order such that the further increase of the model order will leave some significant properties of the model untouched. To be more specific, we expect from the good model structure that the low frequency behavior of the model (frequencies corresponding to stable complex pole pairs with positive real parts) will not change significantly if the model order is increased.

III. EXPERIMENTAL CONDITIONS AND PREPROCESSING

In order to test the behavior of a commercial city bus (test vehicle, see Figure 1), the bus was driven in real traffic conditions. On the test bus a 10-channel accelerometer and a specific computerized data acquisition system has been installed. Near the four spindle on each of the Z-axis direction a PCB ICP-type accelerometer was mounted to measure the unsprung mass vertical acceleration. On the floor plate, over
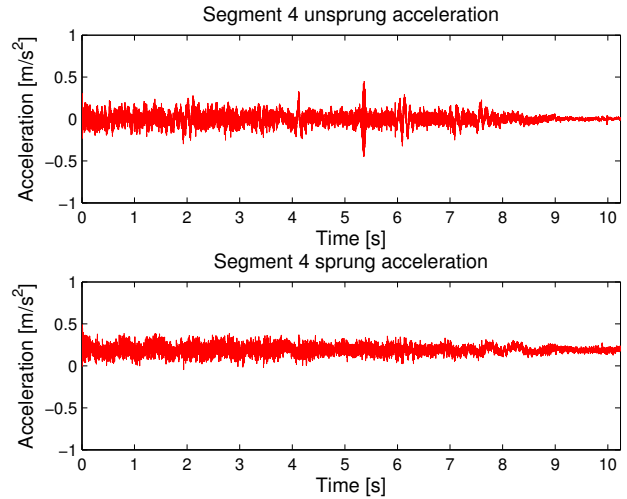

Fig. 1. The test vehicle

Fig. 2. Measured time-domain acceleration data at the front right. Unsprung mass (top) and sprung mass (down).

these sensors, differential DC MEMS accelerometers were mounted along the Z-axis direction to measure the vertical acceleration of the sprung mass.

The data collected from the route-way experiments was segmented into smaller records. Eight pieces, each of length $10.24\ sec$, were cut. The measurements are affected by both the road surface and the vehicle dynamics. In order to minimize the disturbing effects caused by the lateral and longitudinal acceleration of the bus, the criteria in selecting the segments was the possible constant speed and the straight track. A sample data record can be observed in Figure 2.

The row data was sampled by $20\ kHz$ resulting in a large data set. In order to reduce the amount of data, a new sampling frequency was chosen to be $f_s = 1/T_s = 200Hz$ which is sufficiently high to capture the frequency content of the suspension system. However, resampling must be taken with care, due to aliasing effects [2]. An ideal filter (rectangular window) was used as antialiasing filter. First, the DFT components of the $20\ kHz$ signal which correspond to frequencies higher than 25Hz were zeroed out. Then the filtered $20\ kHz$ signal was reconstructed by using inverse DFT. The resulted data set consisted $N = 2048$ samples. More details about the

experimental conditions and preprocessing can be found in [10].

IV. PARAMETRIC SPECTRAL ANALYSIS OF THE SUSPENSION SYSTEM

The overall goal of the modelling of the suspension system can be the following.

- To characterize the vibration properties of the system in order to evaluate the vehicle in terms of ride comfort. In a long-rage period it can serve diagnostic/maintenance purposes.
- To evaluate the technical condition of the road, particularly in terms of ride comfort.

To achieve these goals we need a mathematical model based on which we can distinguish the effects of vertical road disturbances, vehicle motion (roll and pitch effects due to lateral and longitudinal acceleration), and other vibrations due to engine, gear and the flexible chassis.

Parametric spectral analysis helps to estimate the order of the model required to characterize the measurements of the suspension system. It provides an initial guess of the modes of the system and gives a more accurate estimate about the eigenfrequencies and PSD amplitudes of it. The latter is the base of the ride comfort analysis, see [10].

The suspension system at one wheel can be modelled by at least a forth order model which has two lightly damped second order modes. The physical interpretation of the input of the model is the vertical displacement of the road. The outputs of the model are the vertical accelerations of the wheel axis and of the chassis. Further disturbing inputs are present which represent the vibration of the engine, gear system and the flexible chassis excited by the roll and pitch motion of the vehicle. Furthermore, nonlinear effects in the spring and damper are present. These disturbing effects are generated by a complex, nonlinear and unknown dynamics.

It is assumed in this paper that the measured outputs are generated by a linear time-invariant dynamics driven by a fictitious white noise process. The goal of the analysis is to find the frequencies of the modes corresponding to the suspension system and determine the order of the additional dynamics of disturbing effects.

In the remaining of the section the results of AR(p) and ARMA(p) model identification are presented for one representative road segment and one sensor mounted at the sprung mass at the front right.

A. Model order estimation

Consider first the evaluation of AIC criterion as the model order is increased. Note that the physical (quarter car) model has the same structure as ARMA(4). The set of model orders ranges from 4 to 500 in case of AR, and from 4 to 50 in case of ARMA. As it will be seen, ARMA models of significantly lower degree show appropriate fit to the data in the low frequency domain, while only much higher order AR models provide similar fit. This difference between the model structures can be seen in Figures 3 and 4. Figure 3 shows
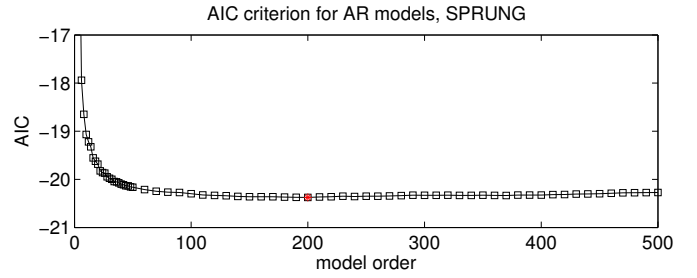

Fig. 3. Akaike information criterion of AR models as function of model order. The minimal value is marked with a red ⋆.
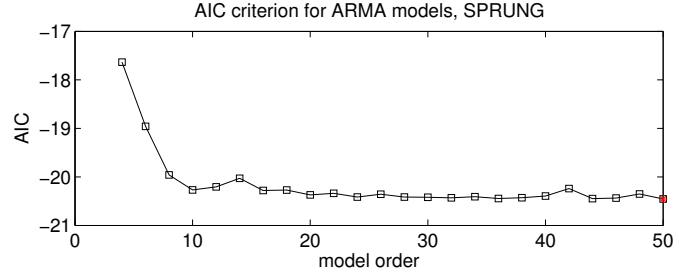

Fig. 4. Akaike information criterion of ARMA models as function of model order.

that the minimum of the AIC is achieved by AR(200), but the criterion does not change significantly from $p > 50$. The AIC plot for ARMA models show that a $p = 10$th order model may suffice to represent the data. The contribution of higher order models is not significant in the AIC criterion.

B. Spectral analysis

The PSD estimate based on the identified AR and ARMA models are plotted together with the periodogram of the data and a smoothed PSD estimate produced by Welch's method. An analysis of the data based on the periodogram and Welch's method is discussed in [10].

We are interested in the resonant modes of the physical system, therefore the stable complex poles with positive real part are selected, and their associated frequency is computed by

$$f_i = \frac{|\ln(z_i)|}{2\pi T_s} \tag{8}$$

where z_i denotes a complex pole of the discrete time model and $T_s = 1/f_s$ is the sampling time. Actually, f_i is the frequency of the step response equivalent (zero order hold equivalent) continuous-time resonant mode.

The PSD estimates and the pole frequencies of AR(10), AR(80), ARMA(10) and ARMA(20) models are presented respectively in Figures 5, 7, 9 and 11. The figures show that AR(80) and ARMA(10) models have comparable fit to the data.

For each model, the pole-zero map is presented in Figures 6, 8, 10 and 12. Over-parametrization of the model can be
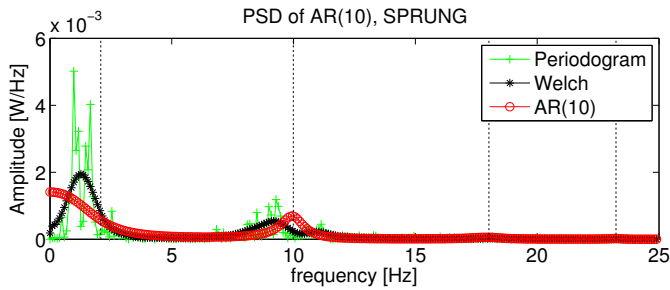
Fig. 5. Power spectrum of AR(10) model. Data: segment 1, sprung mass acceleration. Vertical black dotted lines show the frequencies of the lightly damped modes.
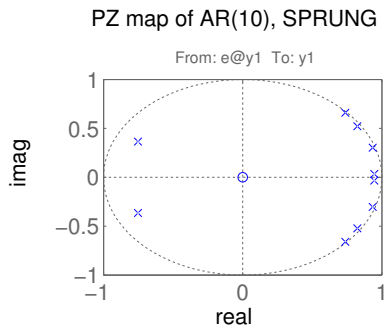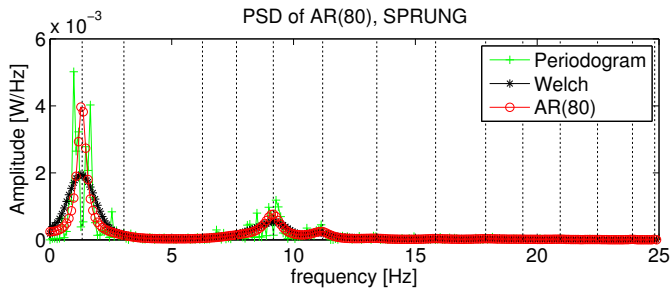

Fig. 6. Pole map of AR(10) model.

Fig. 7. Power spectrum of AR(80) model. Data: segment 1, sprung mass acceleration. Vertical black dotted lines show the frequencies of the lightly damped modes.
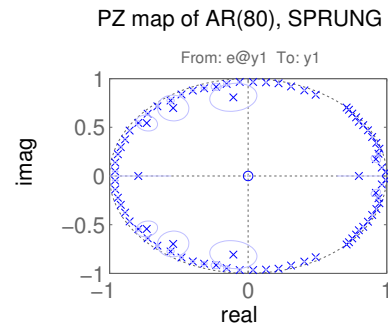

Fig. 8. Pole map of AR(80) model.

Fig. 9. Power spectrum of ARMA(10) model. Data: segment 1, sprung mass acceleration. Vertical black dotted lines show the frequencies of the lightly damped modes.
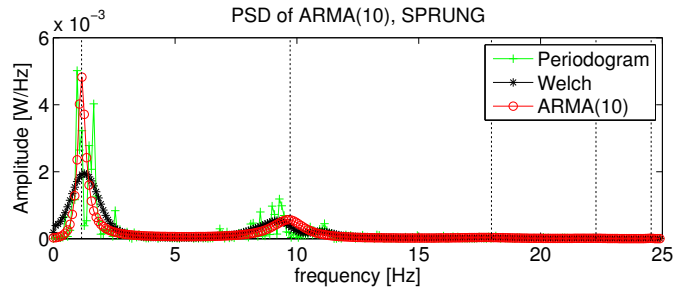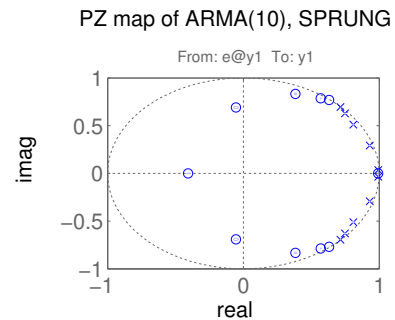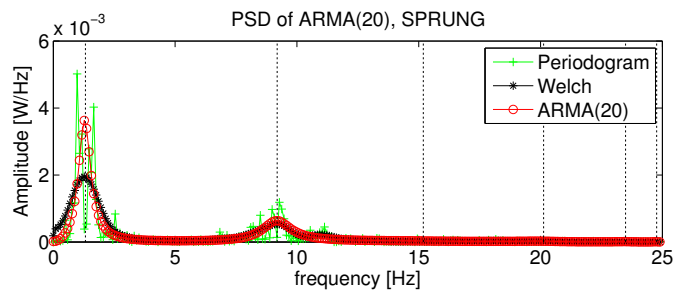

Fig. 10. Pole-zero map of ARMA(10) model.

Fig. 11. Power spectrum of ARMA(20) model. Data: segment 1, sprung mass acceleration. Vertical black dotted lines show the frequencies of the lightly damped modes.

concluded from the position of the poles in the complex domain: they are placed at high frequencies, outside the Nyquist frequency (close to the unit circle with $\arg z_i$ far from zero). The confidence ellipsoids of these poles reflect larger uncertainties. In case of ARMA models, Figures 10 and 12, pole-zero cancellation can also be observed. On the other hand, over-parameterization (i.e. increasing the model order) yields more accurate fit at the low frequency range (compare Fig. 9 with Fig. 11).

C. Frequencies of resonant modes

Let's consider the frequencies of the complex poles in the low frequency range, 0Hz - 15Hz. For every model for which the AIC was computed, the stable complex poles with
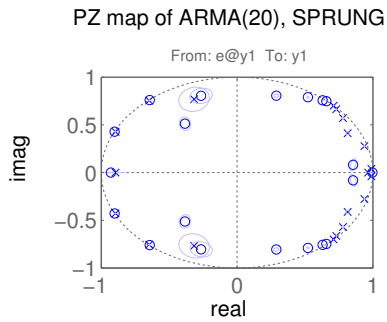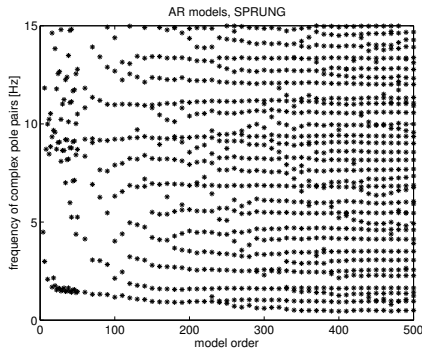
Fig. 12. Pole-zero map of ARMA(20) model.

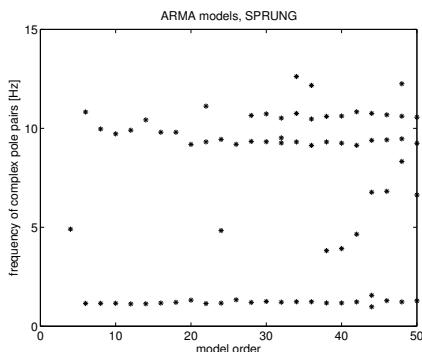Fig. 13. Frequencies of the identified resonant modes in the different AR models.

Fig. 14. Frequencies of the identified resonant modes in the different ARMA models.

positive real part are selected and the associated frequencies are computed. The results are summarized in Figure 13 for the AR models and 14 for the ARMA models. For each model order, frequencies of all resonant modes are plotted. It can be observed that with AR models, the number of low frequency modes increase heavily with the model order. Therefore, AR model structure is not sufficient to draw conclusions about the resonant modes of the system. This is in contrast to the case of ARMA models, where the number of identified modes is maximum 5. When considering all ARMA models simultaneously, it can be seen that 3 modes are surely present in the system, one about 1.5Hz and two around 10Hz.

V. Conclusion and future work

The suspension system of a city bus has been analyzed based on the AR and ARMA parametric spectral analysis of real acceleration measurements. Our experiences and conclusions are summarized as follows.

1) *Model structure.* The pole-zero model structure (ARMA) of lower model order showed better fit to the data as compared to the all-pole model structure (AR).

2) *Model order estimation.* The applied criterion for model order estimation was Akaike Information Criterion. Good fit to data was achieved with ARMA models of significantly higher order as compared to the model order suggested by the physical model of the suspension system. The explanation for this are the effects of neglected dynamics appearing in the measurements.

3) *Physical model structure.* The ARMA(4) model structure, which corresponds to a quarter car suspension model, and the spectral analysis based on that model do not reflect the true frequency domain behavior of the system. A consequence of this on the overall goals of the project (road surface classification and suspension systems diagnosis) is that identification of physical models must be carried out jointly with uncertainty modeling and classification.

The applied analysis method, as a nonparametric tool, was a preliminary step in system modeling and identification. Future work includes the identification of both physical and black box models of the system, together with uncertainty modeling for road surface classification and suspension systems diagnosis. Further goal is to construct mathematical models, as in reference [1], based on experimental data for an adaptive intelligent active suspension control system.

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