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RESEARCH ARTICLE

Mathematical Modeling of Automotive Supply Chain Networks

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Abstract
The aim of this research is to set up a possible model for describing automotive supply chain networks. The work to be presented analyses the real-world environment and its changes in a broader view. The research provides the extension of the mathematical modeling domain, accordingly. The paper studies the basic relational structure of automotive supply chain, where the methodology is determined by the classical business strategic levels. This is done by combining the theories of network and theories for traffic systems. We uncover the hidden relational parallelism between the automotive supply chain and positive systems, complex networks and find that the industry network shows some similar scale-free structural properties indicating the existence of a typical dependence and dominance structure with complex dynamical behavior.

Keywords
modeling, automotive supply chain, scale-free network, graph theory

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1 Introduction

The vehicle industry is facing tremendous amount of challenges mainly caused by complexity constraints. Customization patterns are resulting high competition along the supply chain, which requires flexible and intelligent engineering solutions.

Due to the complexity only those organisations can be efficient, which are striving to reduce risks and complexity, such as setting up proper forecasting and monitoring requirements closely. (Kopeček and Pinte, 2014). In spite of these elements there are also some unforeseeable, stochastic influencing factors which are unpredictable, hence uncertainty avoidance on each strategic level of the entire supply chain plays an important role.

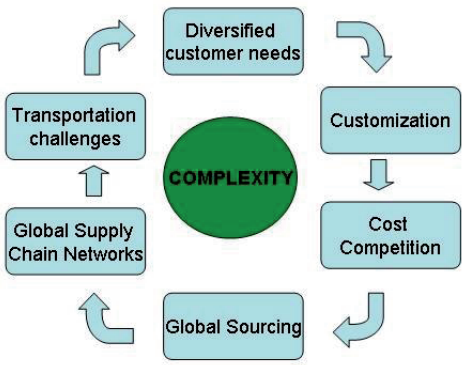

Fig. 1 Integrated complexity model for automotive supply chain networks (Own editing)

To map these uncertain factors we used network theory implications in order to define the main characteristics of such a system. Before we start the detailed analysis of each operating level, we describe first the basic model of an automotive supply chain network.

2 Defining automotive supply chain networks – mathematical approach

Every type of network is characterised by its specific structure, participants, relations, links, hierarchy etc. To describe an automotive supply chain network we return to the basics of graph theory, where the nodes are the firms and the links (edges) are representing the relationships between organisations. In practise these elements are embedded in the wider economic environment. Thus an automotive network can be described as a subsystem of vehicle industry, a part of national, international and global economy.

The node includes suppliers, distribution centre nodes, warehouse nodes and the client node. The more edges are connected to the nodes, the more new chances of cooperation with the node exist. When a new node joins into the network, it does not connect with all the original nodes, but will be preferential to connecting with the most optimistic one, through mastering the information, selecting the nodes which have more connecting edges. By constantly selecting preferential connection, it forms ultimately a complex network. (Zhang, 2014)

The basic structure of the automotive industry is characterized by few Original Equipment Manufacturers (OEMs) and numerous suppliers that can be component manufactures (often SMEs) or big multinational enterprises which assemble entire systems that are supplied to the large car producers. During the last decade more and more value creation, and with it relevant know-how, has been shifted from the OEMs to specialized suppliers also including R&D. In addition as mentioned, increased complexity is another challenging topic in the automotive industry. Electronic systems linking various units of a car need to communicate with a common language and be able to interact without interference in a perfect reliable manner. (Buchmann and Pyka, 2013)

Main problem is to determine or identify the initial conditions of the network, which are not trivial and affect the explanation of dynamics. Albert et al. explains a discrete mechanism whereby a new node connects to others based on their level of connectivity or degree. (Albert et al., 1999) When one includes the geospatial location of nodes and edges, the probability to connect to others depends not only on the degree but also on the Euclidian distance. That is, depending on the type of transportation mode, nodes prefer linking to higher degree nodes or closer nodes. (Ducruet and Lugo, 2013) Translated onto automotive supply chain networks, all the participants would like to join or work with an OEM.

Majority of the concerns are caused nowadays by complexity which leads to complex processes also in logistics, including transportation of goods, material handling, picking, line feeding, warehousing etc. Customization requirements by end-users are forcing the participants to apply intelligent pioneer solutions, which further increase the complexity, on the other hand to avoid uncertainty along the supply chain.

Therefore the analysis and modelling of supply chain networks are needed for planning purposes. Using the pragmatic model of an operating company we analysed the penetration degree and the appearance of stochastic elements on each level related to supply chain processes.

Those levels are:

- Operational level
- Tactical level
- Strategic level

2.1 Network dynamics

Automotive supply chain networks are constantly changing. Organisations are taking part and falling out based on business principles. In mathematical way these phenomena can be modelled with random graphs where nodes are stochastically selected. This statement is only valid within a certain degree (operational level) in case of the supply chain, since the collaboration between emerging and disappearing companies are determined on strategic and tactical levels (long term agreements). Considering that a car is built from more than 10.000 components, the network consists of an innumerable amount of nodes (organisations) and linkages between the nodes (collaborative agreements). Elaboration of new agreements as well as termination of existing co-operations influences the growth, fragmentation and therefore the structure of the supply chain network. From an aggregate network perspective this leads to constantly changing structures, hence in order to model supply chain networks and their development, an inherently dynamic framework is required.

Evolutionary models are based on the assumption that the object of analysis is continuously changing. Compared to traditional static or comparative-static economic models, evolutionary models capture the causes, underlying mechanisms, consequences of change processes and strong uncertainty. Due to the uncertainty and nonlinearities, guideposts for future development remain opaque and vague. Stochastic actor-based models for network dynamics enable us to analyse the process of network evolution and to disentangle different driving factors in this complex process. Contrariwise, standard regression models can hardly be applied for network data since the independence of observations is explicitly excluded from the network case. The network properties of one node are not independent of the other nodes' network attributes (Buchmann and Pyka, 2013). In the following section we describe the variables determining the basic of supply chain network in the automotive industry.

2.2 Basic model

In the initial automotive supply chain network, there are n connected nodes, which have an OEM, and there are i suppliers and customers.

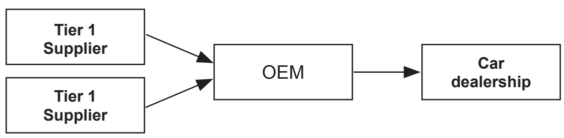

Fig. 2 Basic automotive supply chain netwotk model
(Own editing based on Zhang, 2014)

For every time interval T, with a probability p of joining a supplier or car dealership node, its degree is m ($m \leq i$) if it wants to connect with the OEM nodes, its new node coordinates are randomly assigned (x_1, y_1). As shown in Fig. 3. and Fig. 4.

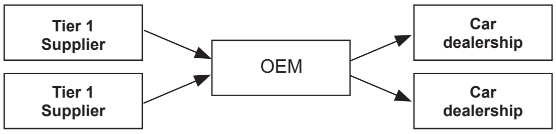

Fig. 3 Extended automotive supply chain network model with multiple actors (Own editing based on Zhang, 2014)

For every time interval T, with a probability $1-p$ of joining an OEM node, its degree is m ($2 \leq m \leq n - i$), if it wants to be one of the supplier and the original nodes to connect with dealerships, and with at least one supplier node is connected with a client, its new node coordinates are randomly assigned (x_1, y_1). P is generally greater than 0.5, because the speed of the joining node should be less than that of the OEM to join supplier nodes. (Zhang, 2014)

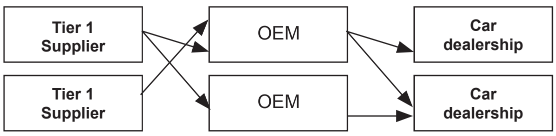

Fig. 4 Extended network model with multiple relationship between actors (Own editing based on Zhang, 2014)

When starting the evaluation of the network based on the above model, the first step is to record the relevant participants for the analysis. All the vendors who have a relationship with a car manufacturer should be recorded. Based on the records of the relationship among the nodes, all suppliers supply relationship with OEMs and record the relationship between them. All OEM As and OEM Bs have a business relationship, the relationship between them should be recorded. All OEM nodes have a business relationship with customers and the relationship between them should also be recorded.

The next step is to make the use of the relationship between each two nodes to make a relation matrix. For example, suppliers A and OEM A have a relationship in business, or OEM A and OEM B have a relationship in business (intercompany), or OEM B and car dealership A have a shipping relationship, which in the matrix are labelled as 1, while no relationship is represented as 0.

When computing the node degree and degree of distribution and assuming that the network is a scale-free network, the statistical parameters must be according to the scale-free network node degree distribution function.

The probability that describes nodes with the same edges is $P(k)$.

$$P(k) = c \cdot k^{-r} \tag{1}$$

where c is a constant, k is the node degree, r is an index. When a network node degree obeys an index between (2, 3) the power-law distribution means that the network is a scale-free network. (Barabási, 2003; Bollobás, 2001)

This kind of modelling is important on strategic and tactical level in order to evaluate the strength, flexibility and robustness of the whole system.

2.3 Parallelism with positive systems

Reaching the operational level of an automotive supply chain network the logistics elements became more relevant factors in the workflow. Part of the logistics processes are the transportation related tasks, which carry on many stochastic supply processes during the execution, since the traffic has big impact on efficiency. There we assume that parallelism can be considered between the analysis of transportation process and positive systems.

The first definition of positive systems was given by Luenberger (Luenberger, 1979): A positive system is a system in which the state variables are non-negative. In the majority of the transportation systems the original physical meaning of the states meet this requirement (Péter, 2012).

To describe classical transportation processes, the literature sets up in most cases general linear system of equations, without using the positive qualities of the process. We might think that the properties recognized in the general linear systems are also valid for positive systems without any restrictions; however, the controllability and observability conditions of positive systems cannot be clearly derived with methods known from general systems. This problem is particularly true if a non-negative co-domain is required not only for the states, but for the actuator signal, as well. Therefore, the description of the traffic processes as purely positive systems is not trivial.

3 The creation of new model on operational level

3.1 Macroscopic modelling of traffic

In the publications of Boothby, W.M. and Sachkov, Y.L. the following theorem regarding a real matrix applied in control theory can be declared: the system is positive if and only if the matrix A is a Metzler matrix, i.e. the elements outside the main diagonal are non-negative (the elements in the main diagonal may be arbitrary) (Boohtby, 1982; Sachkov, 1997).

The most complex part of the topic is the proper modelling. The deeper understanding of the transport processes is essential for performing advanced traffic and routing design. The application of the traditional modelling approach raises a lot of unanswered questions and always struggles with problems of dimensions.

The systems describing the processes of automotive supply networks on operational level are large stochastic dynamic systems. Obviously, the road traffic network model as part of the whole supply scheme is also a very complex dynamic system:

- A number of infrastructure related characteristics impose conditions.
- A number of specific control functions operate in the system. (like AETR agreement, maximal load weight, vehicle restriction etc.).
- The oncoming vehicles also interact with each other. This interaction prevails naturally in the case of uncertain drivers, but manifests itself mainly in the disruptions due to overtaking, and in the interference due to the lights of oncoming vehicles in the night.
- The defined parking lots and parking lanes beside roads are "foreign elements" of the classic network operation, at the same time the parked vehicles are also interacting with the network sections, arcs, which they are directly linked to. This relationship with time-varying intensity has such an ability to create, for example, peak load by itself on the analysed network without any traffic received from a defined external network.
- Internal automations concerning the transfer of vehicles operate between the linked network elements. For example, despite the green light the transfer does not take place when the vehicle density is too high on the receiving section, or zero on the transferring section.
- A large number of participants play an important role.
- The human factor has a significant influence.
- There are many external factors such as seasonal effects, weather, road quality, road width, topography, etc.

However, the basic requirement of the applicable models is efficiency:

- The model must take into account all the elements of the system, which cause real impact during the operation and the neglect those which would distort the results.
- It must be mathematically correct and valid.
- The model must be numerically fast during simulation.
- Real-time control must be achieved.

3.2 Dynamic operational model

A very important new structural result from the modelling point of view is, that the dynamic model of the road network is made up of the multitude of the same elements and the codomain of each x_i state parameter value is located in the interval of [0,1]. However automotive companies are located in most of the cases in suburban industrial parks, due to its subnetwork, vehicles might have to pass through municipal areas as well. Therefore, the parking lots may also be treated as generalized sections of the model and are dynamic components of the network as much as the lanes.

Another important new structural result is, that regardless of the map-graph, a unified hyper-matrix structure can be specified for the mathematical modelling of large-scale road network processes, which for a network located not necessarily in a singly connected domain describes the entire system of relations between the network elements (internal-internal, external-internal, internal-external and external-external relations, (Péter et al., 2013).

The new description of the dynamic model of the system is the base for the calculation and control of the system processes. In this context, the general network model describing the internal and external network operations and the positive non-linear differential system of equations describing the internal and external network processes for any domain bounded by a closed curve is given (Péter and Fazekas, 2014).

The general network model describing the operation of the internal and external networks simultaneously is as follows:

$$\begin{bmatrix} \dot{x} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \langle L \rangle^{-1} & \\ & \langle P \rangle^{-1} \end{bmatrix} \begin{bmatrix} K_{11}(x,s) & K_{12}(x,s) \\ K_{21}(x,s) & K_{22}(x,s) \end{bmatrix} \begin{bmatrix} x \\ s \end{bmatrix} \tag{2}$$

Where <*L*> and <*P*> are diagonal matrices containing the length of internal and external network sections:

$$\langle L \rangle = \langle l_1, l_2, ..., l_n \rangle \quad \langle P \rangle = \langle p_1, p_2, ..., p_m \rangle \tag{3}$$

K_{11}; K_{12}, K_{21} and K_{22} denote the internal-internal, the external-internal, the internal-external and the external-external connection matrices, respectively. The physical meaning of the matrix elements is the connection (transmission) speed. The elements in the main diagonal of K_{11} and K_{22} are 0 or non-negative. All other elements of the matrices are 0 or positive.

x is the state parameter vector of the internal sectors,
s is the state parameter vector of the external sectors,
$\dot{x}$ is the time derivative of x,
$\dot{s}$ is the time derivative of s.

$$x = \begin{bmatrix} x_1(t) \\ x_1(t) \\ . \\ . \\ . \\ x_n(t) \end{bmatrix}, s = \begin{bmatrix} s_1(t) \\ s_1(t) \\ . \\ . \\ . \\ s_m(t) \end{bmatrix}, \dot{x} = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_1(t) \\ . \\ . \\ . \\ \dot{x}_n(t) \end{bmatrix}, \dot{s} = \begin{bmatrix} \dot{s}_1(t) \\ \dot{s}_1(t) \\ . \\ . \\ . \\ \dot{s}_m(t) \end{bmatrix} \tag{4}$$

The positive non-linear differential equation system describing the operation of global network processes is given by the generalization of the model in publication of Péter et al. (2015) and Szauter et al. (2015).

3.3 Transportation network model

- In our model $0 \le xi(t) \le 1$; ($i = 1, ..., n$) normalized vehicle density is used as state parameter. The total length of the vehicles in one stage or section is divided by the length of the stage or section. This calculation can also be used in the case of parking lots, so the parking lots are generalized sections of the model, as well.

- The object of the model is a positive non-linear system. In the network the medium flows with variable speed and according to the defined time-dependent distribution factors is denoted by $\alpha_{ij}(t)$. The medium is embodied in road vehicles. The speed depends on the vehicle density, the maximum value of which is limited in all sections. In addition, the speed function is influenced by weather and visibility conditions, road geometry, quality and width.
- $\beta_{ij}(t)$ denotes the obstruction $(0 \leq \beta_{ij}(t) < 1)$ or facilitation $(1 < \beta_{ij}(t))$ occurring at the transition between certain sections.
- $0 \leq u_{ij}(t) \leq 1$ switching function takes into account the effect of traffic lights operating at the section transitions.
- Vehicles are exchanged between the parallel running sections (lanes), as well as between parking lots and sections in the network. This transfer is taken into account by the proportionality function $0 \leq \gamma_{ij}(t)$ or $0 \leq \gamma_{ij}(x_i(t), x_j(t), t)$.
- Internal prohibiting automatisms operate on the network, as well: vehicles cannot be transferred from j to i, if i is full, $x_i(t) = 1 \rightarrow S(x_i(t)) = 0$. Vehicles also cannot be transferred from j to i, if j is empty $x_j(t) = 0 \rightarrow E(x_j(t)) = 0$. These conditions are easy to follow applying the normalized state parameters, which provide that in the model vehicles cannot be taken from empty sections (the density does not enter into the negative range), and also cannot be transferred to sections where the density has already reached 1.
- The network is analysed on a not necessarily singly connected domain bounded by a closed curve "G". In the external sections, which are in direct transfer relation with a network section, the normalized traffic density $0 \leq S_i(t) \leq 1; (i = 1,. . . ,m)$ is measured.
- The traffic model is the so-called macroscopic model.
- The mathematical model is a non-linear, non-autonomous differential equation system.

The model predictive application of the mathematical network model (2) provides the pre-calculation of the traffic in the given domain, which takes into consideration the expected complex changes as well. The determination of the optimal trajectory between arbitrary pairs of points in the given domain is performed based on the above.

The task is defined as follows: we depart from point „***A***" to target point „***B***" on the possible trajectories of the network at time t_0. The distance x travelled to time t along a trajectory results in the route function $x(t)$ to which the travel time T belongs upon arrival at point „***B***". This mapping is a real functional J: $x(t) \rightarrow T$, where in case of time of arrival t_1, $T = t_1 - t_0$, the minimization of which is the primary goal regarding transportations.

Using the model-predictive method and taking into account the real traffic conditions (the expected congestions, traffic light settings, etc. typical of the seasonality and the time of the day occurring along the route), the respective bivariate $V(t, x)$ velocity function can be calculated along all $x(t)$ trajectories. Applying the above in the solution of the integral equation (5) the function $x(t)$ can be determined:

$$x(t) = \int_{t_0}^{t} V(\tau, x(\tau))d\tau \tag{5}$$

Finally, the trajectory belonging to the minimal travel time T is chosen from the considered trajectories.

Along the optimal trajectory the pre-calculated tracking (definition of distance-time and speed-realization) increases transportation safety as well. Other important criteria can also be taken into account from the transportation planning point of view. It will be possible, for example, to calculate and consider the optimal power demand and minimal emission of vehicles depending on the expected traffic.

Also depending on the traffic it may take more or less time to reach the destination. The transportation time is represented by a probability variable with exponential distribution. At the destination the goods are unloaded. The arrived vehicles as well as the resources necessary to unload the vehicle are waiting in the corresponding queues. The loading and unloading process is represented by a probability variable with exponential distribution. After the unloading is finished the demand is considered to be completed (Vadvári and Várlaki, 2015).

4 Conclusion

In this paper we studied the operation of the transportation chain and the road network environment affecting it. The studies take into account the transportation chain graph and the traffic environment during the transport process, which affects the dynamic performance of the supply chain. The complex network dynamical models created in this manner serve for the more exact analysis of the real-world processes. This provides additional new opportunities for the optimal control of the processes.

Despite the increasing complexity caused by customization, operational efficiency, global sourcing etc. on nodes, the dynamic modelling helps to identify and understand the relationship between network configuration, market-economic behaviour and changes in the customer's needs.

The science of complexity and positive systems provides the relevant framework to merge models of supply chain management set ups and transportation modes. In particular the graph theory could also support correlation to analyse network dynamics. Therefore, transport networks and supply chain management studies can include mutually and explicitly their characteristics in the research.

One of the most important limitations of this paper lies in the fact that it was analysed mostly with mathematical tools, however it may require scientists to combine interdisciplinary approach to extend the field of applicability. Future research should be conducted to investigate how practical and theoretical methods can be combined to manage and control the entire system taken into consideration the environmental safety perspectives, changes in travelling pattern and growth of digitalization.

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