Sensor blending – a geometric view

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The topic of this research has been initiated in the discussions with Prof. Gary Balas at the Department of Aerospace Engineering and Mechanics, University of Minnesota. This paper is a tribute to his memory.

Abstract – *Based on an input/output perspective, this paper investigates the internal stability property of the control loop when the interaction between the plant and sensor is considered explicitly. As a main result of the paper, we provide a Youla type characterization of all the sensors that renders the loop stable for a fixed plant and controller. By using ideas borrowed from projective geometry, the paper places sensor blending problem in a general geometric context: an operation on stable sensors is provided under which feedback stability is preserved and under which sensors form a group, the sensor group. Under the action of this group operation changes in the relevant sensitivities can be also given in explicit terms.*

Keywords: Sensor blending, YOULA parameterization, Sensor group, Projective geometry

1. Introduction and motivation

Sensors play an important role in the control loop. The choice of the sensor configuration affects considerably the achievable performance and the robustness of this performance. This motivates the sensor selection problem, see, e.g., [9], [11].

When none of the sensors can provide the necessary performance it is time for sensor blending or a sensor reconfiguration. The simplest form of sensor fusion, i.e., a linear combination of the sensors, does not preserve stability of the control loop, in general. In order to design efficient algorithms that operate on the set of controllers or a set of sensors that fulfill a given property, e.g., stability or performance (a norm bound), it is important to have an operation that preserves that property, i.e., a suitable blending method.

There are already a lot of applications for controller blending: both in the LTI system framework, [7], [12] and in the framework using gain-scheduling, LPV techniques, see [10], [1], [4], [5]. While these approaches exploits the Youla parameterization of stabilizing controllers, they do not provide an exhaustive characterization of the topic, since the plant should be strictly proper. The approach presented in this paper does not only provide a remedy for this problem but also shows that the proposed operation leaves invariant the strongly stabilizing controllers and defines a group structure on them, too. For sensor blending there are considerably less results, for an elementary sensor blending approach see [6].

Despite the important role played by sensors in the control feedback loop, their effect often remain implicit by embedding them into the plant. Even, when they are considered explicitly as an element of the basic feedback loop, see, e.g. chapter 3 in [2], in the thorough investigation concerning internal stability of the loop their effect is completely ignored.

In sensor selection and blending problems, however, it is necessary to keep track the role of the sensors in the feedback loop, i.e., to give the sensor set whose elements keep the loop stable for a fixed pair of plant and controller, and to give a method that for two members of this set associates a new element of the set, i.e., an operation on sensors that leaves the stability of the loop invariant.

In this paper we investigate stability properties of the control loop when the interface between plant and sensor is considered explicitly. This study is made from an input/output perspective and its aim is to give a parametrization of all sensors that keep the loop stable for a fixed plant and controller configuration.

While for plant and controller pairs this is a well-known topic and the corresponding stabilizing controller set is given by the Youla parametrization, for the general configuration presented in this paper the situation is more involved. In order to obtain a Youla type characterization for the stabilizing sensor set we need to assume stability of the sensors.

Section 2 gives the basic definition of feedback stability containing the sensor. Then Section 3 provides the parametrization of the stabilizing sensor set. Then, it is assumed that the sensors are stable and Section 4 introduces an operation on controllers that preserves stability and presents the sensor group.

2. Problem setup

2.1 Feedback stability: the (P, K) loop

A well-known concept of control theory is that of the feedback and the stability of the feedback loop, as a continuity property. For practical reasons the systems, i.e., plants and controllers are causal. Thus continuity is formulated as a property of boundedness and causality of the corresponding map. In what follows we consider linear systems, i.e., the signals are elements of some normed linear spaces and an operator means a linear map that acts between signals. Thus, boundedness of the systems is regarded as boundedness in the induced operator norm.

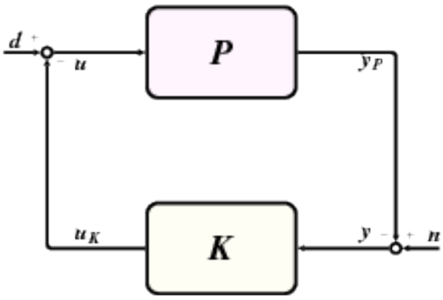

Figure 1: Feedback connection

To fix the ideas let us consider the feedback-connection depicted on Figure 1. It is convenient to consider the signals

$$w = \begin{pmatrix} d \\ n \end{pmatrix}, p = \begin{pmatrix} u \\ y_P \end{pmatrix}, k = \begin{pmatrix} u_K \\ y \end{pmatrix}, z = \begin{pmatrix} u \\ y \end{pmatrix} \in \mathcal{H},$$

where $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2$ and we suppose that the signals are elements of the Hilbert space $\mathcal{H}_1, \mathcal{H}_2$ (e.g., $\mathcal{H}_i = \mathcal{L}^{n_i}[0, \infty)$) endowed by a resolution structure defined by a nest algebra which determines the causality concept on these spaces. For more details on nest algebras and causality, see [3].

The feedback connection is called well-posed if for every $w \in \mathcal{H}$ there is a unique p and k such that $w = p + k$ (causal invertibility) and the pair (P, K) is called stable if the map $w \to z$ is a bounded causal map, i.e., the operator matrix

$$\begin{pmatrix} I & K \\ P & I \end{pmatrix}^{-1} = \begin{pmatrix} S_u & S_c \\ S_p & S_y \end{pmatrix} = \begin{pmatrix} (I - KP)^{-1} & -K(I - PK)^{-1} \\ -P(I - KP)^{-1} & (I - PK)^{-1} \end{pmatrix}$$

should be stable, i.e., all the block elements are stable.

The equation

$$y = -P(I - KP)^{-1}d + (I - PK)^{-1}n = S_p d + S_y n$$

reflects the disturbance rejection property of the feedback loop, thus S_p and S_y bear a specific meaning, they are the plant sensitivity and (output) sensitivity functions, while $L = -PK$ is termed as loop transfer function. Analogously the input sensitivity and controller sensitivity functions is S_u and S_c, i.e., $u = S_u d + S_c n$.

In order to obtain another classical interpretation of this loop we should consider the noise as $n + r$, where r stands for a reference signal. By considering $e = r - Pu$ as a tracking error, i.e., a performance signal, we have

$$e = P(I - KS_p)d + PKS_y n + (I + PKS_y)r.$$

This motivates the introduction of the output complementary sensitivity as

$$T_y = -PKS_y, \quad \text{with} \quad S_y + T_y = I_y.$$

Basically two questions are related to feedback connections: stabilisability, i.e., whether there is any controller that makes the feedback loop stable and, if it the case, to provide a characterisation of the stabilising controllers. Here we are interested in the second question.

We are not very restrictive if it is assumed that among the stable factorizations there exists a special one, called double coprime factorization, i.e., $P = NM^{-1} = \widetilde{M}^{-1}\widetilde{N}$ and there are causal bounded systems $U, V, \widetilde{U}$ and $\widetilde{V}$ such that

$$\begin{pmatrix} \widetilde{V} & -\widetilde{U} \\ -\widetilde{N} & \widetilde{M} \end{pmatrix} \begin{pmatrix} M & U \\ N & V \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \tag{1}$$

an assumption which is often made when setting the stabilization problem, [14], [3]. Recall that $\begin{pmatrix} M \\ N \end{pmatrix}$ and $\begin{pmatrix} U \\ V \end{pmatrix}$ are determined only up to stably invertible factors (invertible with bounded causal inverse) T and T'.

The existence of a double coprime factorization implies feedback stabilizability. In most of the usual model classes actually there is an equivalence. Given a double coprime factorization the set of the stabilizing controllers is provided through the well-known Youla parametrization: $\mathcal{K}_{stab} =$

$$= \{K \mid K(U + MQ)(V + NQ)^{-1}, Q\ stable, (V + NQ)^{-1} exists\}.$$

2.2 Feedback stability: the (P, K, S) loop

In what follows we set up a feedback loop containing a sensor and we are to study stability properties of the resulting feedback loop. Let us consider the configuration depicted on Figure 2, where P is the plant, S is the sensor while K is the controller.

At the connection interface we have the following linear relations:

$$d = u + Ky \tag{2}$$
$$n = y + Sy_s, \tag{3}$$
$$n_s = y_s - Pu, \tag{4}$$

where d, n and n_s are the external perturbations, u contains the control inputs and y represents the measured outputs while y_s are the signals measured by the sensors.

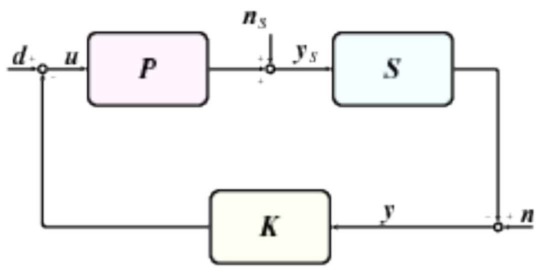

Figure 2: Feedeback stability

It is convenient to consider the signals

$$w = \begin{pmatrix} n \\ n_s \\ d \end{pmatrix}, z = \begin{pmatrix} y \\ y_s \\ u \end{pmatrix} \in \mathcal{H},$$

i.e., we have

$$\begin{pmatrix} n \\ n_s \\ d \end{pmatrix} = \begin{pmatrix} I_y & S & 0 \\ 0 & I_y & -P \\ K & 0 & I_u \end{pmatrix} \begin{pmatrix} y \\ y_s \\ u \end{pmatrix}, \quad (5)$$

where $\mathcal{H} = \mathcal{H}_1 \oplus \mathcal{H}_2 \oplus \mathcal{H}_3$ and we suppose that these signals are elements of the Hilbert spaces $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3$ (e.g., $\mathcal{H}_i = \mathcal{L}^{n_i}[0, \infty)$) endowed by a resolution structure defined by a nest algebra which determines the causality concept on these spaces. For more details on nest algebras and causality, see [3].

The loop is stable, if the map $w \mapsto z$ is a causal bounded map, i.e., the matrix

$$\begin{pmatrix} I_y & S & 0 \\ 0 & I_y & -P \\ K & 0 & I_u \end{pmatrix}^{-1} =$$

$$\begin{pmatrix} \begin{pmatrix} I_y & S \\ PK & I_y \end{pmatrix}^{-1} & \begin{pmatrix} -SP \\ P \end{pmatrix} (I_u - KSP)^{-1} \\ (I_u - KSP)^{-1}(-K \quad KS) & (I_u - KSP)^{-1} \end{pmatrix}$$

is stable.

It is known that for an internally stable standard control feedback loop, i.e., a stable pair (P, K) we have

$$\begin{pmatrix} I & K \\ P & I \end{pmatrix}^{-1} = \begin{pmatrix} \Sigma_u & \Sigma_c \\ \Sigma_p & \Sigma_y \end{pmatrix} =$$
$$= \begin{pmatrix} (I - KP)^{-1} & -K(I - PK)^{-1} \\ -P(I - KP)^{-1} & (I - PK)^{-1} \end{pmatrix}, \quad (6)$$

and the elements of the matrix bear a specific meaning:

Σ_p and Σ_y are the plant sensitivity and (output) sensitivity functions, while Σ_u and Σ_c are the input sensitivity and controller sensitivity functions. They reflect the disturbance rejection property of the feedback loop.

An important role plays the different complementary sensitivity functions, e.g., the output complementary sensitivity $\Theta_y = -PK\Sigma_y$, for which we have $\Sigma_y + \Theta_y = I_y$. They are related to the tracking properties of the loop. The map $\Lambda = -PK$ is termed as loop transfer function.

As for the standard loop, in our case one can also define the different control relevant sensitivities:

$$\begin{pmatrix} I_y & S & 0 \\ 0 & I_y & -P \\ K & 0 & I_u \end{pmatrix}^{-1} = \begin{pmatrix} \Phi_{yn} & \Phi_{yn_s} & \Phi_{yd} \\ \Phi_{y_s n} & \Phi_{y_s n_s} & \Phi_{y_s d} \\ \Phi_{un} & \Phi_{un_s} & \Phi_{ud} \end{pmatrix}, \quad (7)$$

i.e., we have for example, that

$$u = -(I_u - KSP)^{-1}Kn + (I_u - KSP)^{-1}KSn_s +$$
$$+(I_u - KSP)^{-1}d = \Phi_{un}n + \Phi_{un_s}n_s + \Phi_{ud}d.$$

It turns out that feedback stability of the triple (P, K, S) is equivalent to feedback stability of (PK, S), (SP, K) and (P, KS), i.e., stability of the matrices

$$\begin{pmatrix} I_y & S \\ PK & I_y \end{pmatrix}^{-1}, \begin{pmatrix} I_y & K \\ SP & I_y \end{pmatrix}^{-1} \text{ and } \begin{pmatrix} I_y & KS \\ P & I_y \end{pmatrix}^{-1}. \quad (8)$$

Note, that for $S = I$, i.e., the "ideal" case, stability of (P, K, I) is equivalent to the stability of the pair (P, K). In this particular case we have

$$\begin{pmatrix} \Phi_{yn}^i & \Phi_{yn_s}^i & \Phi_{yd}^i \\ \Phi_{y_s n}^i & \Phi_{y_s n_s}^i & \Phi_{y_s d}^i \\ \Phi_{un}^i & \Phi_{un_s}^i & \Phi_{ud}^i \end{pmatrix} = \begin{pmatrix} \Sigma_y & -\Sigma_y & \Sigma_p \\ \Theta_y & \Sigma_y & -\Sigma_p \\ \Sigma_c & -\Sigma_c & \Sigma_u \end{pmatrix}.$$

3 Parametrization of the stabilizing sensors

For a fixed pair (P, K) we are interested in the set $\mathcal{S}_{stab}$ of sensors that make the loop (P, K, S) stable. In order to obtain this set it is convenient to consider the corresponding performance problem instead, i.e., to consider the loop

$$z = \mathfrak{F}_l(G, S)w, \quad \text{where } G = \begin{bmatrix} I & 0 & 0 & -I \\ -PK & I & P & PK \\ -K & 0 & I & K \\ -PK & I & P & PK \end{bmatrix}.$$

This reveals the fact that if the loop is stabilizable, i.e., there is a stabilizing S_0, then all the stabilizing sensors S are those that make the pair (PK, S) stable.

Due to its importance we sketch a short proof: recall that the Redheffer star product can be expressed using LFTs as

$$A * B = \begin{pmatrix} \mathfrak{F}_l(A, B_{11}) & A_{12}(I - B_{11}A_{22})^{-1}B_{12} \\ B_{21}(I - A_{22}B_{11})^{-1}A_{21} & \mathfrak{F}_u(B, A_{22}) \end{pmatrix}.$$

One can easily check that

$$\begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{pmatrix} = \begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & 0 \end{pmatrix} * \begin{pmatrix} 0 & I \\ I & G_{yu} \end{pmatrix}, \qquad (9)$$

i.e.,

$$\mathfrak{F}_l(G, S) = \mathfrak{F}_l(\begin{pmatrix} G_{zw} & G_{zu} \\ G_{yw} & 0 \end{pmatrix}, \mathfrak{F}_l(\begin{pmatrix} 0 & I \\ I & G_{yu} \end{pmatrix}, S)) = \mathfrak{F}_l(\tilde{G}, \tilde{S}),$$

where $\tilde{S} = S(I - G_{yu}S)^{-1}$ has an inverse $S = (I + \tilde{S}G_{yu})^{-1}\tilde{S}$ if and only if $\mathfrak{F}_l(G, S)$ is well defined.

The key observation that leads to the desired result is that the LFT loop is stable for an S if and only if the pair $(G, \mathrm{diag}\{0, S\})$ is stable. However, fixing a particular stabilizing S_0 we have a double coprime factorization induced by the stable pair (G_{yu}, S) (inner loop):

$$S_0 = uv^{-1} = \tilde{v}^{-1} \quad \text{and} \quad G_{yu} = nm^{-1} = \tilde{m}^{-1}\tilde{n}.$$

Moreover, this induces a double coprime factorization of $\mathrm{diag}\{0, S\}$. It turns out that, by inverting the usual roles, we have a dual Youla parameterization of G. It turns out that G should have the following form

$$G = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & 0 \end{pmatrix} * \begin{pmatrix} -m^{-1}u & m^{-1} \\ \tilde{m}^{-1} & 0 \end{pmatrix} * \begin{pmatrix} 0 & I \\ I & G_{yu} \end{pmatrix},$$

where q_{11}, q_{12}, q_{21} are stable systems. Then the stabilizing controller set of an LFT loop coincides with the set of all stabilizing controllers of G_{yu}, and the closed–loop for a stabilizing controller is given by

$$\mathfrak{F}_l(G, S) = q_{11} + q_{12}qq_{21}, \qquad (10)$$

where q is the Youla parameter of S relative to the given double coprime factorization of G_{yu}.

Theorem 3.1 *Let us suppose that there exist an S_0 that makes the triple (P, K, S_0) stable. Then, for fixed P and K the triple (P, K, S) is stable if and only if the pair (PK, S) is stable.*

This results relates the set of sensors with the loop transfer Λ and for SISO systems the loop-shape. In practice we can assume that sensors are stable, therefore in what follows we investigate stability properties of the loop under this assumption.

4 Stable sensors

If a plant is stabilizable in general it is not obvious whether there exists a stable controller as a stabilizing one. If such a controller exists, then it is called a strongly stabilizing controller.

For a general characterization of strong stability of a pair (P, K) one can observe that strongly stabilizing controller exists if and only if there is a stable Q such that $V + NQ$ is unimodular, or, equivalently, if there is a stable K such that the matrix $-\tilde{N}K + \tilde{M} = \tilde{M}(I - PK)$ is unimodular, where $P = NM^{-1} = \tilde{M}^{-1}\tilde{N}$ is a double coprime factorization of P. It follows that for any two stable controllers we have $(I - PK_1)^{-1}(I - PK_2)$ is stable.

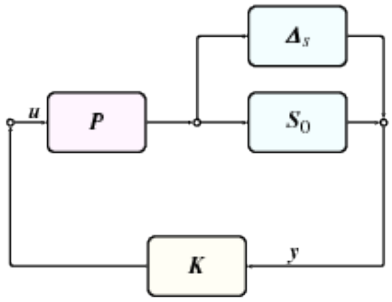

Figure 3: Sensor loop with additive perturbation

We have already seen that stable sensors should render the pair (PK, S) is stable. Let us consider now that for fixed P and K there exists at least one sensor S_0 that makes the loop stable. Then a general sensor can be written as $S = S_0 + \Delta_s$, where Δ_s is a stable perturbation, see Figure 3.

It is immediate that $-\Phi^0_{ysn}$ and Φ^0_y define a double coprime factorization of $PK = (-\Phi^0_{ysn})(\Phi^0_y)^{-1}$, i.e.,

$$\begin{pmatrix} I & -S_0 \\ -R_0 & I + R_0S_0 \end{pmatrix}\begin{pmatrix} I + S_0R_0 & S_0 \\ R_0 & I \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix},$$

with $R_0 = -\Phi^0_{ysn}$. Thus stable sensors S can be written as

$$S = S_0 + Q(I + R_0Q)^{-1} = S_0 + (I + QR_0)^{-1}Q,$$

i.e., $\Delta_s = (I + QR_0)^{-1}Q, \quad Q = \Delta_s(I - R_0\Delta_s)^{-1}.$

Therefore we can conclude this section with the following theorem:

Theorem 4.1 *For fixed P and K with stable sensor S the triple (P, K, S) is stable if and only if the pair (R_0, Δ_s) is stable, where $\Delta_s = S - S_0$ and*

$$R_0 = -\Phi^0_{ysn} = PK(I - S_0PK)^{-1},$$

i.e., Δ_s is a strongly stabilizing controller for R_0.

5 The sensor group

As it can be verified by a simple calculation, we have the factorization

$$\begin{pmatrix} I_y & S & 0 \\ 0 & I_y & -P \\ K & 0 & I_u \end{pmatrix} = \begin{pmatrix} I_y & S_0 & 0 \\ 0 & I_y & -P \\ K & 0 & I_u \end{pmatrix}$$
$$\begin{pmatrix} I_y & (I-S_0PK)^{-1}\Delta_s & 0 \\ 0 & I_y - PK(I-S_0PK)^{-1}\Delta_s & 0 \\ 0 & -K(I-S_0PK)^{-1}\Delta_s & I_u \end{pmatrix} =$$

$$\begin{pmatrix} I_y & S_0 & 0 \\ 0 & I_y & -P \\ K & 0 & I_u \end{pmatrix} \begin{pmatrix} I_y & \Phi^0_{yn}\Delta_s & 0 \\ 0 & I_y + \Phi^0_{y_sn}\Delta_s & 0 \\ 0 & \Phi^0_{un}\Delta_s & I_u \end{pmatrix}. \qquad (11)$$

Since

$$I_y + \Phi^0_{y_sn}\Delta_s = I - PK(I-S_0PK)^{-1}\Delta_s =$$

$$= I_y - (I - PKS_0)^{-1}PK(S - S_0) =$$

$$= (I - PKS_0)^{-1}(I - PKS),$$

it follows that for stable sensors the last factor of the factorization is unimodular.

It follows that the movement from a stable sensor S_0 to a stable sensor S is provided by the action of the matrix

$$T_{\Delta_s} = \begin{pmatrix} I & \Phi^0_{yn}\Delta_s & 0 \\ 0 & I - R_0\Delta_s & 0 \\ 0 & \Phi^0_{un}\Delta_s & I \end{pmatrix}.$$

We will show that under the operation

$$\Delta_{1,2} = \Delta_1 \boxplus_{S_0} \Delta_2 = \Delta_1 + \Delta_2 - \Delta_2 R_0 \Delta_1$$

we have a group and an induced group action. Indeed, it is immediate, that $I - R_0\Delta_{1,2} = (I - R_0\Delta_2)(I - R_0\Delta_1)$, and a short calculation gives $T_{\Delta_1}T_{\Delta_2} = T_{\Delta_1 \boxplus_{S_0} \Delta_2}$.

Thus the set of Δ_s that corresponds to stable sensors form a group under the operation $\boxplus_{S_0}$ with the (identity) null element 0 and inverse $\Delta_s^- = -\Delta_s(I - R_0\Delta_s)^{-1}$.

Note that for the Youla parameter Q_s that corresponds to S we have $Q_s = -\Delta_s^-$, where $\Delta_s = S - S_0$.

In case of $\Delta_{1,\bar{2}} = \Delta_1 \boxplus_{S_0} \Delta_2^-$ we obtain

$$I - R_0\Delta_{1,\bar{2}} = (I - R_0\Delta_2)^{-1}(I - R_0\Delta_1).$$

Remark 5.1 *It is obvious, that the operation defined for Δ_s is inherited by Q_s:*

$$Q_{1,2} = Q_1 \boxplus_{S_0} Q_2 = Q_1 + Q_2 - Q_2R_0Q_1,$$

defining a subgroup of the additive group formed by the corresponding stable Youla parameters. One has the following identity:

$$Q_1 \boxplus_{S_0} Q_2 = -(I - \Delta_2R_0)^{-1}(\Delta_1 \boxplus_{S_0} \Delta_2)(I - R_0\Delta_1)^{-1}.$$

In contrast to the mare addition, which preserves stability of the loop, this new blending preserves also the stability property of the sensor.

The operation defined above can be also expressed directly in terms of the sensors. It follows that the group operation for the sensors, with a slight abuse of the notation, can be equally written as

$$S = S_1 \boxplus_{S_0} S_2 = S_1 + S_2 - S_0 - (S_2 - S_0)R_0(S_1 - S_0).$$

Thus we can conclude this section with the assertion:

Theorem 5.1 *The set $S_{stable,s}$ of stable sensors S that make the triple (P, K, S) stable form a group, called the sensor group, for the operation*

$$S = S_1 \boxplus_{S_0} S_2 = S_1 + S_2 - S_0 - (S_2 - S_0)R_0(S_1 - S_0). \qquad (12)$$

with $R_0 = PK(I - S_0PK)^{-1}$ and arbitrary $S_0 \in S_{stable,s}$. The unit of this group is S_0 and the inverse element is

$$S^{\boxplus_{S_0}^{-1}} = S_0 - (S - S_0)[I - R_0(S - S_0)]^{-1}. \qquad (13)$$

Note that $S^{\boxplus_{S_0}^{-1}} - S_0 = -Q_s$, where Q_s is the parameter of S from Theorem 4.1.

5.1 Transformation of sensitivities

In (7) we have already introduced the sensitivity matrix Φ_S that corresponds to the sensor S. Let us denote by Φ_i the matrix that corresponds to the sensor S_i. Thus, we have

$$\Phi_S = R_{\Delta_s}\Phi_0, \quad R_{\Delta_s} = T^{-1}_{\Delta_s},$$

with

$$R_{\Delta_s} = \begin{pmatrix} I & -\Phi^0_{yn}\Delta_s\Phi_{y_sn_s}(\Phi^0_{y_sn_s})^{-1} & 0 \\ 0 & \Phi_{y_sn_s}(\Phi^0_{y_sn_s})^{-1} & 0 \\ 0 & \Phi^0_{un_s}\Delta_s\Phi_{y_sn_s}(\Phi^0_{y_sn_s})^{-1} & I \end{pmatrix}.$$

Thus

$$\Phi_{S_1 \boxplus_{S_0} S_2} = R_{\Delta_2}\Phi_1 = R_{\Delta_2}R_{\Delta_1}\Phi_0 = R_{\Delta_1 \boxplus_{S_0} \Delta_2}\Phi_0.$$

Note that $R_{\Delta_1 \boxplus_{S_0} \Delta_2} = R_{\Delta_2}R_{\Delta_1}$.

7. Conclusions

In this paper we have investigated stability properties of the control loop when the interface between plant and sensor is considered explicitly. This study was made from an input/output perspective and its aim was to give a parametrization of all sensors that keep the loop stable for a fixed plant and controller configuration. In this setting we provide the set of stabilizing sensors.

Considering stable sensors, a global operation is given under which feedback stability is preserved and under which sensors form a group, the sensor group. The transformation law of the sensitivities corresponding to the control loop is also provided under the action of this group operation.

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