Nonlinear analysis of actuator interventions using robust controlled invariant sets

P. Gáspár and B. Németh
Systems and Control Laboratory, MTA SZTAKI Institute for Computer Science and Control, Hungarian Academy of Sciences, Kende u. 13-17, H-1111 Budapest, Hungary

ABSTRACT: The paper analyzes the maximum robust Controlled Invariant Sets of lateral vehicle dynamical actuators. In the analysis the nonlinear polynomial Sum-of-Squares (SOS) programming method is applied. It considers the nonlinear characteristics of the lateral tire force with a polynomial approximation. Since the tire force characteristics depend on several conditions (e.g. vertical loads, road frictions), their effects occur in the modeling as uncertainties. Parametric uncertainties of the tire model are considered in the robust stability of vehicle dynamics. The maximum robust Controlled Invariant Sets approximate the regions of the tire side-slip angles in which the vehicle can be robustly stabilized by constrained control inputs. As an example the maximum robust Controlled Invariant Sets of the steering and the brake control systems are estimated at various velocities and road conditions. The results of the analysis concerning the steering and the brake control will be illustrated through simulation examples.

1 INTRODUCTION AND MOTIVATION

In the paper the maximum Controlled Invariant Sets of lateral vehicle dynamical actuators are analyzed to determine their intervention capacity and provide a theoretical basis for their coordination in the integrated vehicle control. In the analysis the nonlinear characteristics of the lateral tire force are considered. Since the tire force characteristics depend on several conditions (e.g. vertical loads, road frictions), their effects occur in the modeling as uncertainties. Parametric uncertainties of the tire model are considered in the robust stability of vehicle dynamics.

In the analysis the nonlinear polynomial Sum-of-Squares (SOS) programming method is applied. The Controlled Invariant Sets approximate the regions of the tire side-slip angles in which the vehicle can be robustly stabilized by constrained control inputs. The design of lateral stability control based on set-theoretical methods was proposed by Palmieri et al. 2011. In another method the uncertain effects of the driver were also considered, see Carvalho et al. 2013. A control method in which there was a large operating region accessible by the driver and smooth interventions at the stability boundaries was proposed by Kritayakirana & Gerdes 2012, Beal & Gerdes 2013.

Our preliminary results of the set-based analysis were presented in Németh & Gáspár 2013, Németh et al. 2014. The previous analysis proposes the maximum Controlled Invariant Sets of the vehicle with differential braking and steering actuators. It is proposed that the size of the sets depends on the velocity, the tire-road adhesion coefficient and the steering speed.

In the paper the variation of the tire force characteristics is handled as an uncertainty of the system formulated in a parameter-dependent way. The proposed method results in the maximum robust Controlled Invariant Sets of the actuators. The resulting sets provide information about the states of the vehicle, in which it can be robustly stabilized using a finite control input. Stability is guaranteed against the change of the tire characteristics in a predefined range. The method does not require knowledge about the current tire characteristics to guarantee the stability of the vehicle. As an example the maximum robust Controlled Invariant Sets of the steering and the brake control systems are estimated at various velocities and road conditions.

The paper is organized as follows. In Section 2 the lateral vehicle dynamics which is used for the analysis is presented. In Section 3 the effect of the parameter variation of the polynomial tire model is analyzed. In Section 4 the computation method of robust Controlled Invariant Sets is developed. In Section 5 an illustration example is presented. Finally, Section 6 presents some concluding remarks.

2 NONLINEAR MODELING OF LATERAL VEHICLE DYNAMICS

In the section the nonlinear lateral vehicle model, on which the analysis of the actuator efficiency is based, is presented. The formulation of the lateral dynamical model, which incorporates the nonlinearities of the tire characteristics, is detailed.

The modeling of tire forces is a crucial point of vehicle dynamics. Several tire models have been published, see e.g., Pacejka 2004, Kiencke & Nielsen, 2000, de Wit et al. 1995. These models formulate the nonlinearity of longitudinal and lateral tire forces accurately. In the paper a polynomial tire modeling approach is presented, by which the nonlinearities of the tire characteristics are considered in a given operation range.

$$\mathcal{F}(\alpha) = \sum_{k=1}^{k=n} c_k \alpha^k = c_1\alpha + c_2\alpha^2 + \ldots + c_n\alpha^n \tag{1}$$

The lateral dynamics of the vehicle is formulated as follows

$$J\ddot{\psi} = \mathcal{F}_1(\alpha_1)l_1 - \mathcal{F}_2(\alpha_2)l_2 + M_{br} \tag{2a}$$

$$mv\left(\dot{\psi} + \dot{\beta}\right) = \mathcal{F}_1(\alpha_1) + \mathcal{F}_2(\alpha_2) \tag{2b}$$

where m is the mass of the vehicle, J is yaw-inertia, l_1 and l_2 are geometric parameters. β is side-slip angle of the chassis, $\dot{\psi}$ is the yaw rate. $\mathcal{F}_1(\alpha_1)$ and $\mathcal{F}_2(\alpha_2)$ represent lateral tire forces, which depend on tire side-slip angles α_1 and α_2. The relationships between the tire side-slip angles for the front and rear axles, the steering angle of the vehicle and the side-slip angle of the chassis are

$$\alpha_1 = \delta - \beta - \frac{\dot{\psi}l_1}{v}, \quad \alpha_2 = -\beta + \frac{\dot{\psi}l_2}{v}. \tag{3}$$

In the following (3) is used to transform (2) into a polynomial state-space representation $\dot{x} = f(x) + gu$, where x is the state vector, u is the control input signal, f and g are matrices.

Note that in several control applications the lateral forces are approximated with linear functions, such as $\mathcal{F}_i(\alpha_i) = c_i\alpha_i$, $i = [1, 2]$, where c_i is cornering stiffness. The advantage of this formulation is the simple description although the linear tire model can be used in a narrow tire side-slip range. In this case the states of the systems are $\dot{\psi}$ and β.

The yaw rate and the side slip of the vehicle can be expressed from (3) in the following forms:

$$\dot{\psi} = v\frac{\alpha_2 - \alpha_1 + \delta}{l_1 + l_2}, \quad \beta = -\frac{\alpha_1 l_2 + \alpha_2 l_1 - l_2\delta}{l_1 + l_2}. \tag{4}$$

Equation (2) contains the time derivatives of $\dot{\psi}$ and β and they must be differentiated to obtain $\dot{\beta}$ and $\ddot{\psi}$. $\ddot{\psi} = v\frac{\dot{\alpha}_2 - \dot{\alpha}_1 + \dot{\delta}}{l_1+l_2}$, $\dot{\beta} = -\frac{\dot{\alpha}_1 l_2 + \dot{\alpha}_2 l_1 - l_2\dot{\delta}}{l_1+l_2}$. Now the vehicle model (2) is reformulated using (4):

$$\dot{\alpha}_2 - \dot{\alpha}_1 = \left[\frac{l_1 + l_2}{Jv}\left(\mathcal{F}_1(\alpha_1)l_1 - \mathcal{F}_2(\alpha_2)l_2\right)\right] - \dot{\delta} + \frac{l_1 + l_2}{Jv}M_{br} \tag{5a}$$

$$\dot{\alpha}_1 l_2 + \dot{\alpha}_2 l_1 = v(\alpha_2 - \alpha_1) - \frac{l_1 + l_2}{mv}\left[\mathcal{F}_1(\alpha_1) + \mathcal{F}_2(\alpha_2)\right] + v\delta + l_2\dot{\delta} \tag{5b}$$

The rearrangement of the vehicle model shows that the new states of the model are the tire slip angles α_1 and α_2. In this way the nonlinearity of the lateral tire forces $\mathcal{F}_1$, $\mathcal{F}_2$ can be considered.

However, (5) includes the time derivative of the front-wheel steering angle. Since δ is a control input, $\dot{\delta}$ is modeled as $\dot{\delta} \cong \max\left(\frac{|\dot{\delta}|}{|\delta|}\right) \cdot \delta = \nu \cdot \delta$, where parameter ν represents the relationship between the maximum steering value and the variation speed of δ. Since $\max \delta$ is a given fixed limit at the actuator analysis, a high ν value represents a fast changing steering signal, while a slow changing steering signal is modeled with low ν.

The polynomial state-space representation of the system is formulated using (5) and the approximation of $\dot{\delta}$ is as below:

$$\dot{x} = f(x) + Gu \tag{6}$$

where

$$x = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \quad u = \begin{bmatrix} M_{br} \\ \delta \end{bmatrix}^T, \quad f(x) = \begin{bmatrix} \mathrm{f}_1(\alpha_1, \alpha_2) \\ \mathrm{f}_2(\alpha_1, \alpha_2) \end{bmatrix}^T, \quad G = \begin{bmatrix} g_1 & h_1 \\ g_2 & h_2 \end{bmatrix}$$

and

$$\begin{aligned}
\mathrm{f}_1 =& \frac{l_1}{Jv}\left[\mathcal{F}_2(\alpha_2) l_2 - \mathcal{F}_1(\alpha_1) l_1\right] + \frac{v}{l_1 + l_2}(\alpha_2 - \alpha_1) - \frac{1}{mv}\left[\mathcal{F}_1(\alpha_1) + \mathcal{F}_2(\alpha_2)\right], \\
\mathrm{f}_2 =& \frac{l_2}{Jv}\left[\mathcal{F}_1(\alpha_1) l_1 - \mathcal{F}_2(\alpha_2) l_2\right] + \frac{v}{l_1 + l_2}(\alpha_2 - \alpha_1) - \frac{1}{mv}\left[\mathcal{F}_1(\alpha_1) + \mathcal{F}_2(\alpha_2)\right], \\
h_1 =& \frac{v}{l_1 + l_2} + \nu, \quad h_2 = \frac{v}{l_1 + l_2}, \quad g_1 = -\frac{l_1}{Jv}, \quad g_2 = \frac{l_2}{Jv}.
\end{aligned}$$

The two actuators of the system are the differential braking moment and the front wheel steering angle. In the forthcoming study the system is analyzed using the actuators separately. In the steering analysis $M_{br} \equiv 0$. In the examination of braking $\delta \equiv 0$, which has an effect on the definition of the front tire side-slip α_1, see (3).

3 UNCERTAINTY OF THE TIRE MODEL

The shape and values of the lateral tire force are determined by several vehicle dynamic parameters. One of the most important one is the vertical load F_z on the tire, which is a significant component of lateral dynamics, see Pacejka 2004. In this paper the variation of the tire characteristics based on the F_z variation is analyzed in the maximum Controlled Invariant Sets computation. The change in the vertical load results in a parameter variation of the polynomial tire model, formulated in (1). In this section the impact of the vertical load on the tire force characteristics is proposed.

In the following the previously formulated polynomial tire model is reformulated depending on the vertical load of the wheel. The modified model is represented as:

$$\mathcal{F}(\alpha, \rho) = \sum_{k=1}^{k=n} c_k(\rho)\alpha^k = c_1(\rho)\alpha + c_2(\rho)\alpha^2 + \ldots + c_n(\rho)\alpha^n, \tag{7}$$

where the coefficient functions $c_k(\rho)$ are $c_k(\rho) = \sum_{j=1}^{j=m} d_j \rho^j = d_1\rho + d_2\rho^2 + \ldots + d_m\rho^m$. In the formulation $\rho = F_z$ is defined as a scheduling variable of the tire model. The proposed model is able to represent the tire force characteristics in a bounded scheduling variable region $\rho \in [\rho_{min}, \rho_{max}]$.

Since the front and the rear axles have their own dependence on their vertical load, in the vehicle model two scheduling variables ρ_1, ρ_2 are introduced. Thus, the polynomial vehicle model (8) is extended as

$$\dot{x} = f(x, \rho_1, \rho_2) + Gu \tag{8}$$

where

$$f(x, \rho_1, \rho_2) = \begin{bmatrix} \mathrm{f}_1(\alpha_1, \alpha_2, \rho_1, \rho_2) \\ \mathrm{f}_2(\alpha_1, \alpha_2, \rho_1, \rho_2) \end{bmatrix}$$

4 COMPUTATION METHOD OF MAXIMUM ROBUST CONTROLLED INVARIANT SETS

A vehicle model with a polynomial tire model is formulated in Section 2. In the following the maximum robust Controlled Invariant Sets of the system is computed.

The goal of the nonlinear actuator analysis is the determination of their intervention limits in addition to a constrained peak-bounded actuation. With an appropriate intervention of the actuators some of the unstable regions can be stabilized. In the next section the largest state-space region where the stability of the system can be guaranteed by a given peak-bounded control input is determined. This question leads to the computation of the Controlled Invariant Sets, see Korda et al. 2013.

The state-space representation of the system is given in the following form, see (8):

$$\dot{x} = f(\rho_1, \rho_2, x) + gu \tag{9}$$

where the state vector of the system is $x^T = [\alpha_1, \alpha_2]$. The expression $f(\rho_1, \rho_2, x)$ is a matrix, which incorporates smooth polynomial functions and $f(\rho_1, \rho_2, 0) = 0$. In the next analysis one control input is considered, thus $u = M_{br}$ or $u = \delta$. ρ_1 and ρ_2 are unknown bounded scheduling variables of the system. Since the accurate measurement of the scheduling variables may be difficult, it is necessary to find an algorithm, which does not require this knowledge. In the following a robust analysis is presented, where the Controlled Invariant Sets use only the information of the bounds. Thus, the actual values of ρ_1, ρ_2 are not required.

The global asymptotical stability of the system at the origin is guaranteed by the existence of the Control Lyapunov Function of the system defined as follows, see Sontag 1989:

Definition 1.1 *A smooth, proper and positive-definite function $V : \mathbb{R}^n \to \mathbb{R}$ is a Control Lyapunov Function for the system if*

$$\inf_{u \in \mathbb{R}} \left\{ \frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) + \frac{\partial V}{\partial x} g \cdot u \right\} < 0, \qquad x \neq 0 \tag{10}$$

According to Definition 1.1 two main cases are distinguished:

1/ If $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) < 0$ then the system is stable and $u \equiv 0$. This stability scenario is contained by the next two stability criteria.

2/ If $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) > 0$ then the system is unstable. However, the system can be stabilized

- 2/a: If $\frac{\partial V}{\partial x} g < 0$ and $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) + \frac{\partial V}{\partial x} g \cdot u_{max} < 0$. In this case the upper peak-bound of control input u stabilizes the system.
- 2/b: If $\frac{\partial V}{\partial x} g > 0$ and $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) - \frac{\partial V}{\partial x} g \cdot u_{min} < 0$. In this case the lower peak-bound of control input u stabilizes the system. Note that $u_{min} = -u_{max}$.

The Controlled Invariant Sets of the system (9) are defined as the level-set of the Control Lyapunov Function at $V(x) = 1$. Thus, the fulfilment of the previous stability criterion must be guaranteed at $V(x) \leq 1$. The defined set-emptiness conditions are transformed into greater than or equal ($\geq$) conditions. Thus, the condition $\frac{\partial V}{\partial x} g < 0$ in 2/a is rewritten to $\frac{\partial V}{\partial x} g \leq -\epsilon$, where $\epsilon \in \mathbb{R}^+$ is as small as possible. Similarly in 2/b $\frac{\partial V}{\partial x} g \geq \epsilon$ is used. Additionally, the conditions $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) \pm \frac{\partial V}{\partial x} g \cdot u_{max} < 0$ in 2/a and 2/b are also reformulated into two conditions: $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) \pm \frac{\partial V}{\partial x} g \cdot u_{max} \leq 0$ and $\frac{\partial V}{\partial x} f(\rho_1, \rho_2, x) \pm \frac{\partial V}{\partial x} g \cdot u_{max} \neq 0$.

Above the stability criterion of the polynomial system has been formed. Based on these constraints it is necessary to find a Control Lyapunov Function V which meets the following set emptiness conditions:

$$\Big\{-\frac{\partial V}{\partial x}g-\epsilon\geq 0,\ 1-V(x)\geq 1,\ l_1(x)\neq 0, \frac{\partial V}{\partial x}f(\rho_1,\rho_2,x)+\frac{\partial V}{\partial x}g\cdot u_{max}\geq 0,$$
$$\frac{\partial V}{\partial x}f(\rho_1,\rho_2,x)+\frac{\partial V}{\partial x}g\cdot u_{max}\neq 0\Big\}=\emptyset \tag{11a}$$
$$\Big\{\frac{\partial V}{\partial x}g-\epsilon\geq 0,\ 1-V(x)\geq 1,\ l_2(x)\neq 0, \frac{\partial V}{\partial x}f(\rho_1,\rho_2,x)-\frac{\partial V}{\partial x}g\cdot u_{max}\geq 0,$$
$$\frac{\partial V}{\partial x}f(\rho_1,\rho_2,x)-\frac{\partial V}{\partial x}g\cdot u_{max}\neq 0\Big\}=\emptyset \tag{11b}$$

Note that the relations in the third inequality are inverted to guarantee the emptiness of the sets. The role of $l_{1,2}(x)\neq 0$ is to guarantee the condition $x\neq 0$ in (1.1). $l_{1,2}(x)$ is chosen as a positive definite polynomial, see Jarvis-Wloszek et al. 2003. Further constraints on the stabilization problem are the validity ranges of the scheduling variables ρ_1,ρ_2. The stability condition must be fulfilled in a limited validity range $\rho_{1,min}\leq\rho_1\leq\rho_{1,max}$, $\rho_{2,min}\leq\rho_2\leq\rho_{2,max}$, where $\rho_{1,min}$, $\rho_{2,min}$ and $\rho_{1,max}$, $\rho_{2,max}$ are the bounds of the scheduling variables.

Since it is necessary to find the maximum Controlled Invariant Sets, another set emptiness condition is also defined to improve the efficiency of the method, see Jarvis-Wloszek et al. 2003:

$$\{p(x)\leq\beta, V(x)\geq 1, V(x)\neq 1\}=\emptyset \tag{12}$$

where $p\in\Sigma_n$ is a fixed and positive definite function. β defines a $P_\beta:=\{x\in\mathbb{R}^n|\,p(x)\leq\beta\}$ level set, which is incorporated in the actual Controlled Invariant Set. Thus, the maximization of β enlarges P_β together with the Controlled Invariant Set.

In the followings the set-emptiness conditions are reformulated to SOS conditions based on the generalized S-procedure, see Tan & Packard 2008. Thus, the next optimization problem is formed to find the maximum Controlled Invariant Set:

$$\max\beta \tag{13}$$

over $\quad s_i\in\Sigma_n, i=[1\dots 13];\ V,p_1,p_2\in\mathcal{R}_n;\ V(0)=0$
such that

$$-\left(\frac{\partial V}{\partial x}f(\rho_1,\rho_2,x)+\frac{\partial V}{\partial x}g\cdot u_{max}\right)-s_1\left(-\frac{\partial V}{\partial x}g-\epsilon\right)-s_2\left(1-V\right)-s_3\left(\rho_1-\rho_{1,min}\right)$$
$$-s_4\left(\rho_{1,max}-\rho_1\right)-s_5\left(\rho_2-\rho_{2,min}\right)-s_6\left(\rho_{2,max}-\rho_2\right)-p_1l_1\in\Sigma_n \tag{14a}$$
$$-\left(\frac{\partial V}{\partial x}f(\rho_1,\rho_2,x)-\frac{\partial V}{\partial x}g\cdot u_{max}\right)-s_7\left(\frac{\partial V}{\partial x}g-\epsilon\right)-s_8\left(1-V\right)-s_9\left(\rho_1-\rho_{1,min}\right)$$
$$-s_{10}\left(\rho_{1,max}-\rho_1\right)-s_{11}\left(\rho_2-\rho_{2,min}\right)-s_{12}\left(\rho_{2,max}-\rho_2\right)-p_2l_2\in\Sigma_n \tag{14b}$$
$$-\left(s_{13}(\beta-p)+(V-1)\right)\in\Sigma_n \tag{14c}$$

where Σ_n represents SOS, which is defined as

$$\Sigma_n:=\left\{p\in\mathcal{R}_n\,\middle|\,p=\sum_{i=1}^{t}f_i^2,\ f_i\in\mathcal{R}_n,\ i=1,\dots,t\right\}. \tag{15}$$

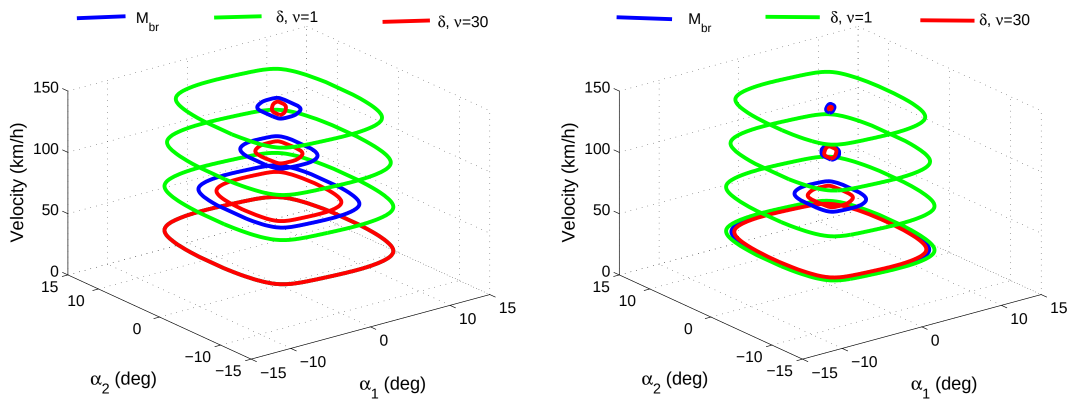
During the optimization it is necessary to find V,p_1,p_2 functions and $s_i, i=[1\dots 13]$ Sum-of-Squares, which guarantee the conditions (14).

5 ILLUSTRATION OF ROBUST INVARIANT SETS

In this section the results of the analysis are illustrated during an example. The maximum robust Controlled Invariant Sets of the steering and braking actuators are analyzed at different velocities and tire-road conditions. Moreover, the robust results are compared to the nominal analysis, published in Németh et al. 2014. The vertical load change as an uncertainty of the tire model is considered. The vehicle of the proposed example is a medium-size passenger car with the mass $m = 1823kg$. The variation of the vertical load on each wheel is derived from $\pm 75kg$ dynamic load transfer.

Figure 1(a) shows the maximum robust Controlled Invariant Set at $\mu = 1$ (dry asphalt) road conditions. Three scenarios are illustrated in the figure: differential braking (M_{br}), slow steering $(\delta, \nu = 1)$ and fast steering actuation $(\delta, \nu = 30)$. Figure 1(a) shows that the sets of the steering actuation depend significantly on the speed of the actuation ν. Although the $\nu = 1$ scenario has increased sets, it is necessary to actuate with a fast steering intervention in emergencies. The differential braking M_{br} represents a compromise between the two steering scenarios. It can be stated that the velocity has a significant effect on the sizes of the sets. The vehicle can be stabilized with a constrained control input in a tighter region at high velocity.

Figure 1(b) illustrates the robust sets at $\mu = 0.4$ (wet asphalt) road conditions. It shows that the robust sets at low μ are significantly smaller than at good road conditions. At high velocity the reduction is considerable: it is possible to robustly stabilize the vehicle only in a very small region. Thus, the change of μ has a significant role in guaranteeing lateral stability.



(a) Actuation limits $\mu = 1$

(b) Actuation limits $\mu = 0.4$

Figure 1. Maximum robust Controlled Invariant Sets

6 CONCLUSION

In the paper the maximum robust Controlled Invariant Sets have been proposed to estimate the intervention capacity of the different actuators. In the analysis the nonlinear polynomial SOS programming method has been applied. The uncertainties of the nonlinear characteristics of the lateral tire force are considered. The results of the analysis concerning the steering and the brake control have been illustrated through simulation examples. It can be stated that the consideration of the tire uncertainty reduces the sizes of the sets, consequently, the vehicle can be robustly stabilized with a constrained actuation in a tighter region compared to the nominal case. However, this consideration is necessary, due to the significant vertical load transfer at most of the vehicle maneuvers. In this way the advantage of the robust analysis is the improvement of safety in the vehicle control.

ACKNOWLEDGEMENT

This paper was supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

REFERENCES

Beal, C. E. & Gerdes, J. C. 2013. Model predictive control for vehicle stabilization at the limits of handling. *IEEE Transactions on Control Systems Technology*. 21(4):1258–1269.

Carvalho, A. & Palmieri, G. & Tseng, E. H. & Glielmo, L. & Borrelli, F. 2013. Robust vehicle stability control with an uncertain driver model. *IEEE European Control Conference*. Zürich, Switzerland.

de Wit, C. C. & Olsson, H. & Astrom, K. J. & Lischinsky, P. 1995. A new model for control of systems with friction. *IEEE Transactions on Automatic Control*, 40(3):419–425.

Jarvis-Wloszek, Z. & Feeley, R. & Tan, W. & Sun, K. & Packard, A. 2003. Some controls applications of sum of squares programming. *Proceedings of 42nd IEEE Conference on Decision and Control, Maui, USA*, 5:4676–4681.

Kiencke, U. & Nielsen, L. 2000. *Automotive control systems for engine, driveline and vehicle*. Springer.

Korda, M. & Henrion, D. & Jones, C. N. 2013. Convex computation of the maximum controlled invariant set for polynomial control systems. *Proceedings of 52nd IEEE Conference on Decision and Control, Maui, USA* 7107-7112.

Kritayakirana, K. & Gerdes, J. C. 2012. Autonomous vehicle control at the limits of handling. *International Journal of Vehicle Autonomous Systems*, 10(4):271–296.

Németh, B. & Gáspár, P. 2013. Analysis of vehicle actuators based on reachable sets. *European Control Conference, Zurich, Switzerland*.

Németh, B. & Gáspár, P. & Péni, T. 2014. Nonlinear automotive actuator analysis based on sum of squares programming. *European Control Conference, Strasbourg, France*.

Pacejka, H. B. 2004. *Tyre and vehicle dynamics*. Elsevier Butterworth-Heinemann, Oxford.

Palmieri, G. & Baríc, M. & Glielmo, L. & Tseng, E. H. & Borrelli, F. 2011. Robust vehicle lateral stabilization via set-based methods for uncertain piecewise affine systems: Experimental results. *50th IEEE Conference on Decision and Control, Orlando, Florida*, pages 3252–3257.

Sontag, E. D. 1989. A ”universal” construction of Artstein's theorem on nonlinear stabilization. *Systems & Control Letters*, 13:117–123.

Tan, W. & Packard, A. 2008. Stability region analysis using polynomial and composite polynomial Lyapunov functions and sum-of-squares programming. *IEEE Transactions on Automatic Control*, 53(2):565–571.