Freeway shockwave control using ramp metering and variable speed limits

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Abstract— **In this work a novel controller design method is suggested for motorway shockwave management using ramp metering and variable speed limit (VSL) control. The proposed controller has a feedforward-feedback control structure that is designed for a motorway arterial model. For the feedback design the nonlinear model predictive control is used. The feedforward control is utilized to enhance the operability of the control system to high disturbances. Two different controllers are designed: while controller A uses continuous VSL signs, VSL input values of controller B is chosen from a discrete set. For the latter, a two-step optimization is used to decrease oscillations. The designed controllers are tested in a case study, in which a total traffic breakdown situation is modeled. In the uncontrolled case, the initial perturbation leads to a traffic jam with zero traffic speed, whereas the proposed control design is capable of preventing the congestion.**

Keywords: traffic breakdown, traffic shockwaves, nonlinear model predictive control

I. Introduction

In this work a motorway control system is proposed for the management of shockwaves. Shockwaves are homogeneous high density areas of freeway traffic flow appearing at bottlenecks and propagating backwards, leading to a congesting traffic. The phenomenon is best described by the kinematic wave theory, for details see [Lighthill and Whitham (1955)] and [Richards (1956)]. The problem has been addressed by different approaches. In [Hegyi et al. (2003)] a nonlinear model predictive control approach is used for variable speed limit (VSL) control. Later, in [Hegyi et al. (2007)] a static solution for the problem is proposed using the kinematic wave theory and the phenomenon of constant propagating velocity of shockwaves. In [Zhang et al. (2005)] the control method introduced in [Hegyi et al. (2003)] is evaluated in a microscopic traffic simulator called Corsim. In [Hou et al. (2007)], an iterative learning approach is designed for the use of ramp metering and VSL signs.

The applied mathematical model for control studies is the second order macroscopic model (see [Papageorgiou et al. (1990)]), which has served as a basis model for ramp metering control designs several times (e.g. see [Papamichail et al. (2010)]). It is also important to notice that a simplified model is created for the consideration of variable speed limits.

Although the above control approaches offer potentially strong results to manage the motorway traffic in near-to-congestion situation, the low sensitivity of the potentially available control inputs, ramp metering and VSL control make it difficult to reach satisfactory results by applying a feedback type controller alone. Therefore the aim of this study is to use a novel control design approach that involves a feedforward and a feedback controller, in the latter one the nonlinear model predictive control (NMPC) technique is used. Two different controllers are designed regarding the input constraints of variable speed limits: continuous valued inputs and VSL inputs of a discrete set is compared. The controllers are evaluated in a case study of a rush hour traffic breakdown situation.

The paper is structured as follows: after the introductory section, in section II-A the system model is reviewed. Then, in section III the control design is outlined. In section IV simulation results of a case study are discussed. The concluding remarks are summarized in section V.

II. Case study: a motorway strech

The simplest but still important simple case study is used to show the design and performance of the proposed controller that is described in this section.

A. System description

The case study system is a 10 km long motorway stretch divided to 10 segments of equal length. The applied mathematical model is the second order macroscopic model (see [Papageorgiou et al. (1990)]), however a simplified model is created for the consideration of variable speed limits. The control is realized using a metered ramp on segment no. 4 (r_4) and variable speed limits of segment 1 through 9 (VSL_1 to VSL_5). Segment no. 10 is uncontrolled. Upstream flow (q_0), mean speed of the upstream traffic (v_0), and also downstream density (ρ_{11}) are considered as disturbances. The system layout is illustrated in figure 1.

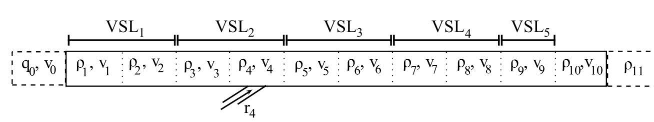

Fig. 1. System layout

Ramp placement The ramp is deliberately located at the middle of the network. It would clearly have the highest

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influence on the whole system if allocated at segment 1, but in this case the potential upstream congesting effect of the ramp is not observable: placing at segment 4 provides an opportunity to analyze the ramp control's upstream effect throughout segment 1 to 3.

Variable speed limit signs To describe the effect of variable speed limit signs, a simplified and conservative model is considered. In order to obtain smooth input affine dynamics, the following assumption is used based on the equilibrium speed function (1) of METANET (for the original function, see [Papageorgiou et al. (1990)]): by using variable speed limit VSL_i on segment i, the free flow speed v_{free} is modified to VSL_i (thus, in case of VSL control a fraction of the uncontrolled equilibrium speed-density function is considered n in Eq. (1)). This assumption, however, neglects an additional positive effect of variable speed limits: the slight increase in critical density, thus the extension of the stable domain of the system.

$$V(\rho_i(k)) = VSL_i(k) \cdot exp\left(\frac{-1}{a} \cdot \left(\frac{\rho_i(k)}{\rho_{cr}}\right)^a\right) \qquad (1)$$

B. State space model

The state equations are given in the form of the following discrete time nonlinear difference equations, that are in input-affine form:

$$\begin{pmatrix} \rho_1(k+1) \\ v_1(k+1) \\ \dots \\ \rho_4(k+1) \\ v_4(k+1) \\ \dots \\ \rho_{10}(k+1) \\ v_{10}(k+1) \end{pmatrix} = \begin{pmatrix} \rho_1(k) + \frac{T}{L}[-\rho_1(k)v_1(k)] \\ v_1(k) - \frac{T}{\tau}v_1(k) - \frac{T}{L}v_1(k)v_1(k) - \frac{\eta T}{\tau L}\frac{\rho_2(k)-\rho_1(k)}{\rho_1(k)+\kappa} \\ \dots \\ \rho_4(k) + \frac{T}{L}[\rho_3(k)v_3(k) - \rho_4(k)v_4(k)] \\ v_4(k) - \frac{T}{\tau}v_4(k) + \frac{T}{L}v_4(k)\left(v_3(k) - v_4(k)\right) - \frac{\eta T}{\tau L}\frac{\rho_5(k)-\rho_4(k)}{\rho_4(k)+\kappa} \\ \dots \\ \rho_{10}(k) + \frac{T}{L}[\rho_9(k)v_9(k) - \rho_{10}(k)v_{10}(k)] \\ \left\{ \begin{matrix} v_{10}(k) + \frac{T}{\tau}\left(v_{free} \cdot exp\left(\frac{-1}{a} \cdot \left(\frac{\rho_{10}(k)}{\rho_{cr}}\right)^a\right) - v_{10}(k)\right) \\ +\frac{T}{L}v_{10}(k)\left(v_9(k) - v_{10}(k)\right) + \frac{\eta T}{\tau L}\frac{\rho_{10}(k)}{\rho_{10}(k)+\kappa} \end{matrix} \right\} \end{pmatrix}$$

$$+ \begin{pmatrix} 0 \\ \frac{T}{\tau} \cdot VSL_1(k) exp\left(\frac{-1}{a} \cdot \left(\frac{\rho_1(k)}{\rho_{cr}}\right)^a\right) \\ \dots \\ \frac{T}{L}r_4(k) \\ \frac{T}{\tau} \cdot VSL_2(k) \cdot exp\left(\frac{-1}{a} \cdot \left(\frac{\rho_4(k)}{\rho_{cr}}\right)^a\right) - \frac{\delta T}{\tau L}\frac{r_4(k)v_4(k)}{\rho_4(k)+\kappa} \\ \dots \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{T}{L}q_0(k) \\ \frac{T}{L}v_1(k)v_0(k) \\ 0 \\ \dots \\ \dots \\ \dots \\ 0 \\ -\frac{\eta T}{\tau L}\frac{\rho_{11}(k)}{\rho_{10}(k)+\kappa} \end{pmatrix} \qquad (2)$$

where τ, v_{free}, a, ρ_{cr}, η, δ, κ are constant model parameters, L denotes the segment length, T denotes the sample time. In our case, the sample time $T = 10s$. This choice, considering the spatial discretization step $L = 1km$ satisfies the numerical stability conditions (see [Caligaris et al. (2010)]).

System variables As *state variables*, the traffic density ρ_i and traffic mean speed v_i are modeled of each segment:

$$x(k) = \begin{pmatrix} \rho_1(k) \\ v_1(k) \\ \dots \\ \dots \\ \rho_{10}(k) \\ v_{10}(k) \end{pmatrix} \in \mathbb{R}^{20}$$

The boundary conditions of (2) are considered as *disturbances*. They are collected in the following vector

$$d(k) = \begin{pmatrix} q_0(k) \\ v_0(k) \\ \rho_{11}(k) \end{pmatrix} \in \mathbb{R}^3$$

where q_0 and v_0 denote the traffic flow and speed of the upstream segment of the motorway stretch, ρ_{11} denotes the traffic density of the downstream segment of the network. These boundary variables of the network are considered as disturbances.

The *input* vector is in the form:

$$u(k) = \begin{pmatrix} r_4(k) \\ VSL_1(k) \\ VSL_2(k) \\ VSL_3(k) \\ VSL_4(k) \\ VSL_5(k) \end{pmatrix} \in \mathbb{R}^6$$

where r_4 denotes the on-ramp of the 4th segment, VSL_i denotes the dynamic speed limit of segment $(2i-1)$ (see the system layout in Fig. 1.

Outputs are in the output vector $y(k)$

$$y(k) = x(k)$$

That means, that a full information control is assumed during the control design: all disturbances and system variables are measured (thus the boundary conditions of the problem are stated in the disturbances).

III. Control design

The control task is to eliminate the effect of a sudden growth of downstream density. Normally, this perturbation leads to a backwards propagating shockwave of a high density area. The controller aims at preventing the shockwave formation and stabilizing it by keeping traffic state near critical conditions. The regulator control for this setpoint also maximizes the traffic capacity of the motorway arterial.

The proposed controller uses both feedback and feedforward measures. The feedback controller by itself is not capable of preventing the shockwave propagation and the traffic breakdown, thus it is basically used for the regulation to critical traffic conditions. The prevention of shockwaves is solved by utilizing a feedforward controller as well which modifies density values by downstream density information for the feedback design - thus the anticipation of the high density downstream traffic is involved in the control design.

Two controllers are designed and compared: while controller A uses continuous values for VSL control, controller B applies VSL values of the following set: $VSL_{set} = \{60, 70, 80, 90, 100, 110, 120, 130\}$. The difference of the two controllers are realized in the feedback optimization algorithm.

a) Feedback controller: For feedback, the nonlinear model predictive control algorithm (see [Grüne and Pannek (2011)]) is used with the optimization method of active set algorithm (via $fmincon$ in MatLab). For prediction horizon length, $K = 5$ is chosen, any further expansion of the prediction horizon provides no significant improvement of control performance, however leads to high computational requirements and thus long simulation periods.

The following cost function $J(k)$ is used for the controller design:

$$\begin{aligned} J(k) \quad = \quad & \sum_{k=1}^{K} \Big(\sum_{i=1}^{10} \|\bar{\rho_i}(k) - \rho_{crit}\|_2^2 \\ & + w_1 \|d_4(k) - r_4(k)\|_2^2 \\ & + w_2 \sum_{i=1}^{4} \|VSL_i(k) - VSL_i(k-1)\|_2^2 \\ & + w_3 \| \sum_{i=1}^{4} VSL_i(k) - VSL_{i+1}(k)\|_2^2 \Big) \end{aligned} \tag{3}$$

where $\bar{\rho_i}(k)$ denotes the modified traffic density via feedforward at time step k respectively, N_n denotes the number of freeway segments, in this case $N_n = 10$ and K denotes the control horizon. For the feedforward design and the analytic form of $\bar{\rho_i}(k)$ see section III-.0.b.

The cost function (3) formalizes the objective of the maximization of traffic capacity at each segment - this is done by minimizing the quadratic difference from the critical traffic conditions (in traffic congestion situations this is also the traffic stabilizing objective). The cost function also contains the penalizing on the differences of the ramp demand ($d_4(k)$) and the ramp input ($r_4(k)$) and the penalizing of the alternation of inputs - both spatially (i.e. of neighbouring segments) and temporally. For the choice of weighting parameters w_1, w_2, w_3 see section III-.0.d.

The following constraints were set for the system variables.

Disturbances: Throughout the control horizon, constant disturbance values were considered. Thus,

$$\begin{aligned} q_0(k+j) &= q_0(k) \\ v_0(k+j) &= v_0(k) \\ \rho_{11}(k+j) &= \rho_{11}(k) \end{aligned}$$

for each k sample step and each $j = 1, ..., K$ control horizon step.

Inputs:

For the *ramp control*: as the system model considers a single lane of the motorway, ramp input is constrained between 0 and the minimum of $0.5 \cdot Q_{cap} = 1100$ veh/h or the ramp demand, where Q_{cap} denotes the traffic flow capacity of a single lane.

$$0 < r_4 < min(d_4, 1100)$$

where d_4 denotes the ramp demand on the ramp of motorway section 4.

For *dynamic speed limits*:

$$60 < VSL_i < 130$$

for $i = 1, ..., 5$.

Controller A: the VSL_i values are continuous values in the constrained interval.

Controller B: VSL_i may take values from the following discrete set: $VSL_{set} = \{60, 70, 80, 90, 100, 110, 120, 130\}$. The design of the optimal VSL input is still carried out in a continuous manner, the applied control is chosen though by rounding the designed input to the possible discrete values. Nevertheless, for the elimination of VSL input oscillation a two-step optimization is carried out. In the first step, optimal input is calculated, considering continuous set for VSL input. Then, input variables for VSL control are rounded to the elements of the discrete set. After setting the VSL values, another optimization is run for the ramp control considering the fixed VSL values as input constraints.

b) Feedforward controller: The feedforward controller uses the phenomenon that high density areas propagate backwards with constant shockwave speeds. This means that the density difference between an upstream and a downstream segment sperarated by a certain distance ($\Delta\rho_i = \rho_{i+j} - \rho_i$) can be considered in the control input design before it actually appears on the upstream segment.

For the feedback controller, the modified state information is calculated as follows:

$$\bar{\rho_i}(k+1) = \rho_i(k) + c \cdot (\rho_{i+j}(k) - \rho_i(k)) \tag{4}$$

where $\rho_i(k)$ denotes the measured traffic density of section i and c denotes the feedforward gain.

The value of c can be derived analytically, and j can be determined by manual tuning (detailed in section III-.0.d). The value of c is determined based on the dynamical model, in particular on the capacity diagram. The feedforward controller is designed so that the occurrence of a downstream congestion of ρ_{jam} density at segment $i + j$ should trigger an immediate shift of the VSL downwards. Thus, the effect of a total traffic breakdown is eased with a single shift of upstream VSL control, which provides a flow capacity drop of Δq. The shift between 130 km/h and 120 km/h VSL control is illustrated in figure 2.

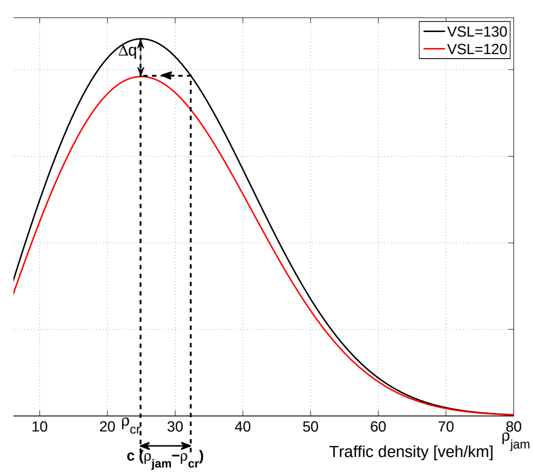

Fig. 2. Flow drop as a result of VSL shift

In view of the evoked effect, the density modification is searched proportional to the upstream and downstream density difference which leads to a capacity drop Δq that can only be managed by an immediate shift of VSL signaling. For this end, jam density (ρ_{jam} =80 veh/km) is assumed as downstream density at segment $i+j$ and the critical density (ρ_{cr} =25 veh/km) at segment i. The density increase that causes a VSL downshift is equal to $c(\rho_{jam}-\rho_{cr})$ from which c can be obtained. The value of c is different for each actual VSL input, Table I summarizes the calculated values for the feedforward gain c. As the computed feedforward gains are

VSL shift	130/120	120/110	110/100	100/90	90/80	80/70	70/60
c	0.155	0.160	0.166	0.170	0.174	0.179	0.182

TABLE I

FEEDFORWARD GAINS FOR DIFFERENT VSL SITUATIONS

nearly the same, the highest $c = 0.182$ is used for the control - at this value, in any case, the VSL shift is evoked in case of a downstream jam.

c) Control system architecture: The control system architecture is the same both for controllers A and B. The controlled system uses the feedforward and feedback measures in the following way: the feedforward controller modifies the current state elements ρ_i as shown in (4) - for each element using the upstream state information (and thus also the disturbance of the downstream density ρ_{11}), and the modified density $\bar{\rho}_i$ is fed to the feedback controller. The MPC controller carries out the optimization using the modified state information, the historic input and the disturbance data in a rolling horizon manner: the optimization is realized on a fix horizon length N, but only the first element of the optimal input (u) is applied on the system. For the control system structure see figure 3.

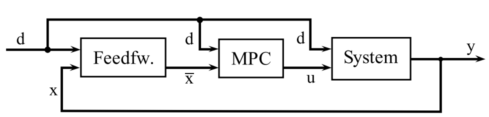

Fig. 3. Control system interconnection

d) Tuning of the control parameters: Some of the control parameters are based on theoretical considerations (i.e. the choice of the feedforward gain c). On the other hand, certain parameters are in an indirect relationship with the dynamic model, and thus need to be tuned manually. For the weight parameters w_1, w_2, w_3 of the cost function (3) the initial values are calibrated offline for different traffic situations. The choice of j (the downstream gap of the feedforward controller) is chosen by simulations so that the information of a downstream segment is used from such a distance, on which the shockwave effect can be eliminated at high traffic densities. The parameters are summarized in Table II.

IV. SIMULATION RESULTS

The efficiency of the shockwave managament is illustrated in a case study. The simulation models a rush hour situation:

w_1	w_2	w_3	j
0.02	0.005	0.01	2

TABLE II

THE TUNED CONTROL PARAMETERS

traffic of critical density is run on a motorway arterial, thus the network is performing at its capacity maximum (near critical conditions) when a sudden downstream density increase is experienced for a short period of time (at the 1000 sec, lasting for 200 sec). For boundary parameters, the upstream flow is chosen as q_0=2026 veh/h, the upstream speed is v_0 =90 km/h. Downstream and ramp disturbances are chosen as in figure 4.

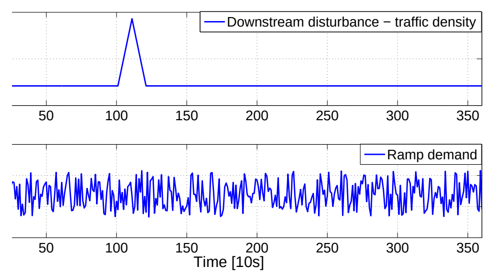

Fig. 4. Disturbances

Normally, this perturbation leads to a backwards propagating shockwave of a high density area and eventually to a congestion. The spatiotemporal presence of traffic density in the uncontrolled case is plotted in figure 5.

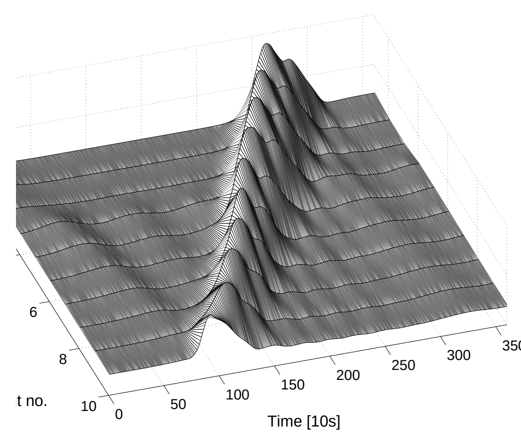

Fig. 5. Spatiotemporal traffic density in the uncontrolled case

In figure 6 the spatiotemporal profile of traffic speed is plotted. It is clear that a backwards propagating shockwave appears and leads to a bottleneck situation. At the bottleneck, the traffic slows down to zero velocity. The control task is to prevent the congestion triggered by the downstream perturbation and to increase traffic speed.

The effect of the proposed controllers on traffic density and speed is plotted in figures 7 to 10: the shockwave is

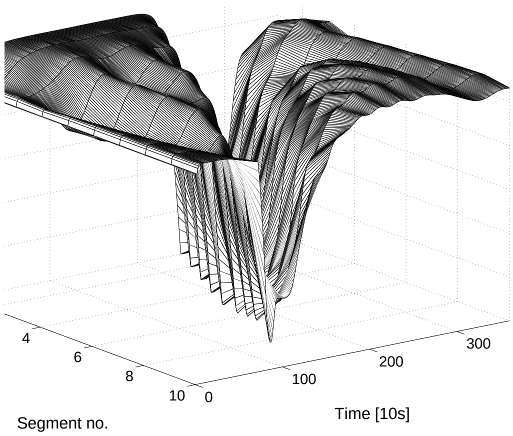

Fig. 6. Spatiotemporal traffic speed profile in the uncontrolled case

prevented by both controllers A and B.

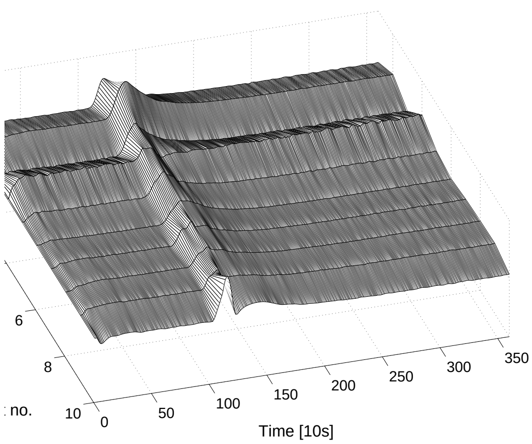

Fig. 7. Traffic density in case of controller A (continuous VSL values)

The ramp controller is already active already before the disturbance appears as the initial conditions are somewhat different from steady-state dynamics. This results in slightly different density profiles at the beginning of the simulation (figures 5, 7 and 8). The speed limit controller comes into action only at 1000s, but without a delay on the whole length of the motorway stretch which leads to an immediate density increase in all segments. The lowest traffic speed at the ultimate segment is 30 km/h by both controllers, in the controlled case the traffic speed never goes below 40 km/h, thus the traffic breakdown is successfully eliminated. Apart from the jam situation, the controllers tend to maintain the conditions of critical density without insufficiently slowing down traffic for flow stabilization. Low VSL values at steady states are only present at segment 1-2. (see figures 12 and 13) where the joint flow of downstream ramp flow and the main lane flow create metastable conditions. Nevertheless, slight oscillation of controller b. is present upstream the merging ramp which remains a problem to solve. Apparently, the

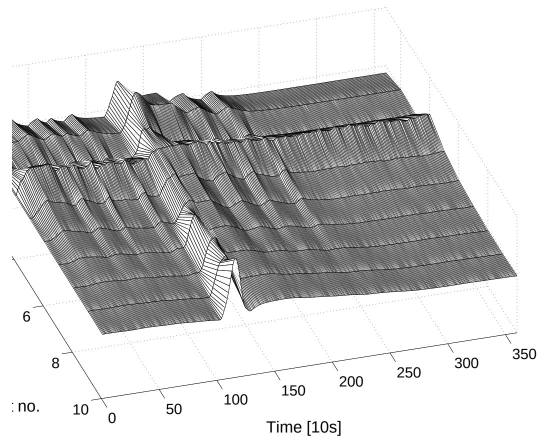

Fig. 8. Traffic density case of controller B (discrete VSL values)

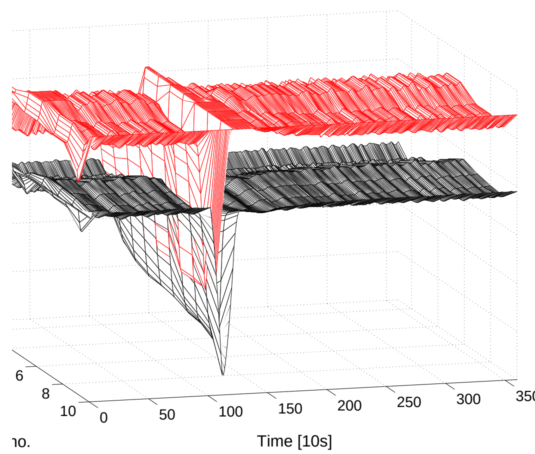

Fig. 9. Traffic speed (black) and VSL values (red) - controller A

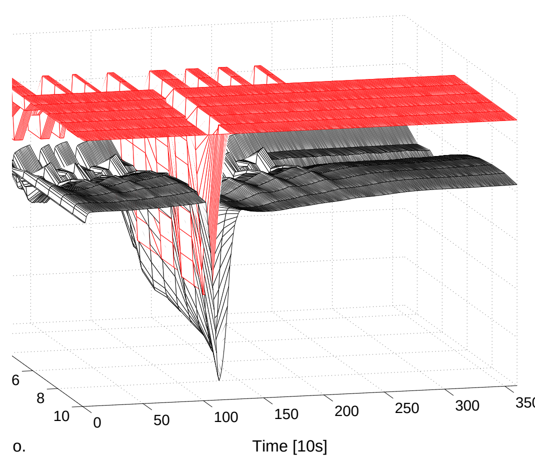

Fig. 10. Traffic speed (black) and VSL values (red) - controller B

oscillation of the ramp demand leads to an oscillating ramp

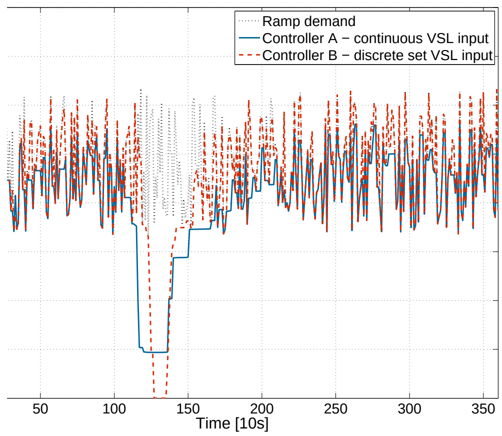

Fig. 11. Ramp control comparison

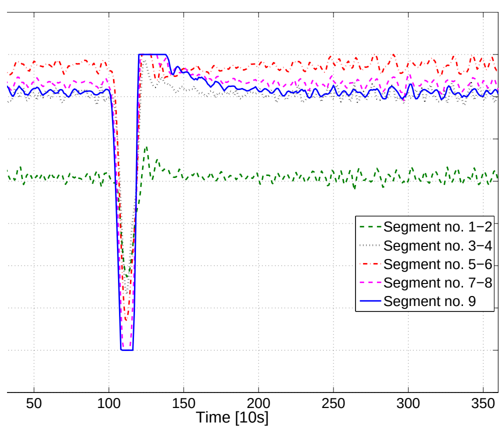

Fig. 12. VSL control - controller A, continuous input set

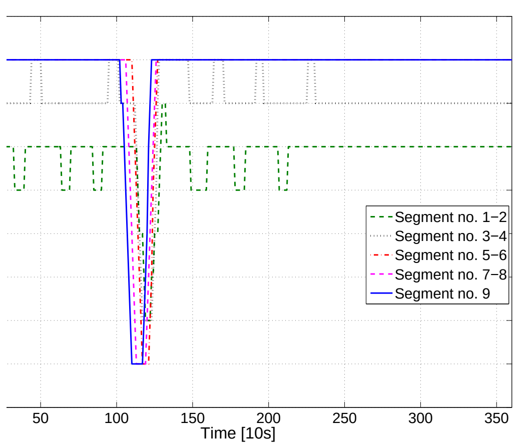

Fig. 13. VSL control - controller B, discrete input set

input and also an oscillating VSL control. This alternation however could be reduced by smaller differences in the input set values. The VSL control is in both cases obeyed during the whole simulation (see figures 9-10). The ramp metering signal of the controllers (see figure 11) tend to follow the current ramp demand, and is only reduced significantly during the presence of the perurbation.

V. Conclusion

In this work a control design method was proposed for motorway shockwave management using ramp metering and variable speed limit (VSL) control. A feedforward-feedback control structure was created for a 10-km-long motorway arterial model. For feedback design the nonlinear model predictive control was used. The feedforward control was utilized to enhance the operability of the control system to high disturbances. Two different controllers were designed: while controller A used continuous VSL signs, VSL input values of controller B came from a discrete set. For the latter, a two-step optimization was used to decrease oscillations. The designed controllers were tested in a case study, in which a total traffic breakdown situation was modeled. In the uncontrolled case, the initial perturbation leads to a traffic jam with zero traffic speed, whereas the proposed control design was capable of preventing the congestion. Future research involves the analysis and comparison of different control designs for the problem (e.g. control design for the system formalized in LPV).

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