A simple dynamic model for the dispersion of motorway traffic emission

Alfréd Csikós, István Varga and Katalin M. Hangos

***Abstract*— In this work a modeling approach is introduced for the dispersion of motorway traffic emissions. The process model is developed for a distributed parameter system, and is derived based on the conservation law within the balance volumes between the road and the rural area, specified by the wind direction. Parallel to the wind, plug flow is considered, and for the absorption of pollution a simplified version of the Gaussian plume model is used. For the boundary conditions of the model, the output of the macroscopic emission model, introduced in [Csikós et al. (2012)] is substituted. A sensitivity analysis is performed on the proposed model which justifies the preconception on the future control system structure.**

Keywords: traffic emission, emission dispersion, sensitivity analysis.

I. Introduction

The dispersion of vehicular emissions is a significant environmental problem. Therefore, its dynamic modeling and use in designing pollution-aware traffic controllers is of primary importance, that is widely investigated in the literature.

Traffic emission dispersion models are mainly developed for urban networks: so-called street-canyon models (see e.g. [Buckland et al. (1999)] and [Tominaga et al. (2011)]) describe the accumulation of concentration of pollutants being stuck in the canyons of urban streets. These models are not adaptable for freeways because of the fundamentally different topology. Several dispersion models (such as Gaussian plume models (see e.g. [Johnson et al. (1973)], [Benson (1979)]) or CFD models (e.g. [Gosman (1999)]) that are applicable for the freeway topology provide high resolution description of the pollutant dispersion, but are not suitable for on-line use because of their high computational demands. In [Zegeye et al. (2011)] a grid-based dispersion model is suggested for traffic control purposes, but the use of boundary conditions (the emission) is not discussed properly.

In this paper a novel model is suggested for the dispersion of motorway traffic emissions. The motivation for the research is a model based control for the regulation of pollutant concentrations emerging at rural areas in the proximity of a motorway stretch. Thus a simple yet accurate model is needed to characterize the dispersion of exhaust gases. The dispersion process is described as a distributed parameter system (DPS) in which the law of mass conservation conservation holds assumming isothermal conditions. The elements of the mathematical model (the assumptions of the initial value problem) are specified using topological considerations, and the boundary conditions originate from the macroscopic traffic emission framework introduced in [Csikós et al. (2012)]. The created partial differential equation is converted to a set of ordinary differential equations by lumping the system. These differential equations are then reformalized by finite difference approximation resulting in a model being discrete both in space and time. Then an analytical approximation of the decay rate (modeling the dissolution of pollution) is proposed. For the suggested model a sensitivity analysis of the existing motorway control measures (i.e. ramp metering, variable speed limits) is performed to justify the choice of the control structure.

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II. Emission dispersion modeling

The dispersion model aims at modeling the evolving concentration of the pollutants with local effects (CO, HC, NO_X) in order to design a model based control for the regulation of pollutant concentrations. The contemplated control framework involves a feedback controller that keeps the instantaneous concentration of the target zone under a specified state constraint using dynamic speed limits and ramp metering. Sudden squalls are treated as additive uncertainties.

The topographic layout of the problem is illustrated in figure 1: the area between the motorway and the rural area is divided to constant cross-section channels of equal width, parallel to the wind direction. The process of emission dispersion is considered a distributed parameter process system [Hangos and Cameron (2001)] described for the flow channels separately.

A. Assumptions

- In the flow channels (also called as balance volumes) the conservation law for component masses is satisfied.
- For boundary parameters, the concentration is calculated from the spatiotemporal emission model proposed in [Csikós et al. (2012)].
- The pollution is ideally mixed over the cross section of the flow channels.
- Constant wind direction is supposed. The analysis can be carried out for different dedicated wind directions most characteristic for the area.

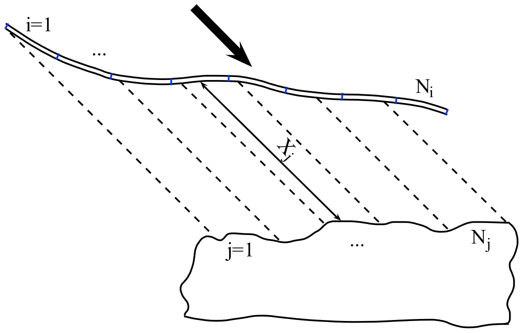

Fig. 1. System layout

- The flow channels are parallel with the wind direction and are of equal cross-section. (The whole length of the rural area is considered by separated balance volumes of equal significance.)
- Only axial dispersion is present through the channels.
- Plug flow is assumed within the balance volumes.
- The dissolution of gases is modeled by a simplified version of the Gaussian plume model, considering the vertical diffusion of high temperature gases. The proposition for the decay rate coefficient is stated in section II-F.

Remark: The number and position of the flow channels depends on the wind direction. Thus, the indices of the balance volumes (j) do not necessarily check up with the indices of the road segments (i). In most cases, $N_i \neq N_j$, i.e. the number of motorway segments and flow channels differ. Grouping of the balance volumes is a delicate matter: on the one hand, the aim is to minimize the extra dimensions of the system, coming from the imission modeling. On the other hand, the practical choice of flow channel positioning follows a rule that segments of coherent control decisions (e.g VSL signs) should be grouped in the same flow channel.

B. Conservation of pollution masses

The outlined process system can be modeled through the conservation of pollutant masses within the balance volumes. Each flow channel is considered as an autonomous balance volume with individual dynamics. The mass balance for pollutant p in flow channel j is described through the variable $m_j^p(x,t)$ (measured in units $[g]$) as a bivariate function of time ($0 \leq t$) and space ($0 \leq x_j \leq X_j$), where X_j denotes the length of flow channel j.

The conservation equation of pollutant p for balance volume j is:

$$\frac{\partial m_j^p}{\partial t} = \phi_{j,in}^p - \phi_{j,out}^p - \psi_{j,dis} \tag{1}$$

where $\phi_{j,in}^p$, $\phi_{j,out}^p$ and $\psi_{j,dis}$ are the inflow, outflow and dissolution rate of the pollutant, respectively.

The inflow of pollutants at the border of the balance volume comes from the emission of the segments involved in the balance volumes:

$$\phi_{j,in}^p = \sum_{j,i=1}^{N_{j,i}} E_{j,i}^p \cdot L_{j,i} \tag{2}$$

where $N_{j,i}$ denotes the number of segments involved in balance volume j; $E_{j,i}^p = [g/m{\cdot}s]$ denotes the spatiotemporal emission of motorway segment j,i, $L_{j,i}$ denotes the length of segment j,i $\left(\sum_{j,i=1}^{N_{j,i}} L_{j,i} = L_j\right)$.

The outflow of pollutants is the direct effect of the wind:

$$\phi_{j,out}^p = u\frac{\partial m_j^p}{\partial x_j} \tag{3}$$

where u (in units $[m/s]$) denotes the wind speed.

The dissolution of pollutants is described by the following formula:

$$\psi_{j,dis}^p = \lambda_p m_j^p \tag{4}$$

where λ_p (in units $[s^{-1}]$) denotes the decay rate of the pollutant p. For the approximation of λ_p see section II-F

In the followings, the conservation of the pollutant is formalized by means of concentration in a balance volume increment ΔV_j. This volume increment represents the flow within the balance volume using the assumption of plug flow and considering a constant height and cross-section of the flow channel. The volume of the flow channel is calculated using the surface of the rural zone and the length of the channel. The size of volume increment $\Delta V_j = \Delta X_j L_j H_j$ can be obtained similarly. For the parameters of a flow channel see figure 2. The relationship between the mass of

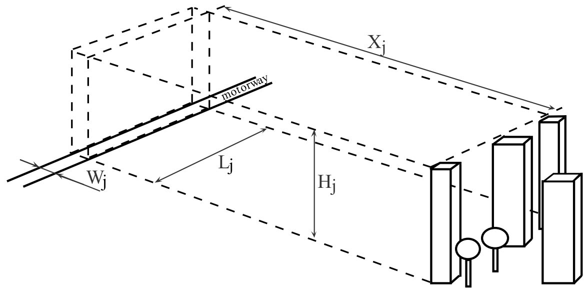

Fig. 2. Flow channel parameters

pollutant p and its concentration in an infinitesimal segment ΔV_j of balance volume j is described as:

$$\Delta m_j^p = c_j^p \cdot \Delta V_j = c_j^p \cdot H_j L_j \Delta x_j \tag{5}$$

where $c_j^p = c_j^p(x_j + \Delta x_j, t)$ denotes the concentration of pollutant p in the balance volume increment ΔV_j, measured in units $[kg/m^3]$.

The outflow of the pollutant can be reformalized using (5):

$$\phi_{j,out}^p = u\frac{H_j L_j \Delta x_j c_j^p}{\Delta x_j} = u \cdot c_j^p H_j L_j \tag{6}$$

The dissolution increment can also be easily calculated as

$$\psi_{j,dis}^p = \lambda_p \cdot c_j^p H_j L_j \Delta x_j \tag{7}$$

The inflow of the pollutant is described differently for the origin of the balance volume and an arbitrary volume increment ΔV_j within the balance volume.

At the origin of the balance volume, the inflow of pollutant p is an external excitation, formalized as the total emission of traffic emerging above the motorway in the volume $H_j L_j W_j$.

$$\phi^p_{j,in} = \frac{\sum_{j,i=1}^{N_{j,i}} E^p_{j,i} \cdot L_{j,i}}{H_j L_j W_j} \tag{8}$$

For an arbitrary volume increment ΔV_j within the flow channel, the inflow to ΔV_j equals to the outflow of the previous increment:

$$\phi^p_{j,in}(x_j + \Delta x_j, t) = \phi^p_{j,out}(x_j, t) = u c^p_j(x_j, t) H_j L_j \tag{9}$$

Using the above formulae, the conservation for the balance volume increment ΔV_j can be formalized also for two cases: *At the origin of the balance volume*, substituting (2), (5), (6), (7), and (8) to the conservation equation (1):

$$H_j L_j \Delta x_j \frac{\partial c^p_j}{\partial t} = \sum_{j,i=1}^{N_{j,i}} E^p_{j,i} \cdot L_{j,i} - u \cdot c^p_j H_j L_j - \lambda_p \cdot c^p_j H_j L_j \Delta x_j \tag{10}$$

By further arrangement:

$$\frac{\partial c^p_j}{\partial t} = \frac{\sum_{j,i=1}^{N_{j,i}} E^p_{j,i} \cdot L_{j,i}}{H_j L_j \Delta x_j} - u \frac{\partial c^p_j}{\partial x_j} - \lambda_p c^p_j \tag{11}$$

Within the balance volume, substituting (2), (5), (6), (7), and (9) to the conservation equation (1):

$$\begin{aligned} H_j L_j \Delta x_j \frac{\partial c^p_j}{\partial t} &= u c^p_j(x_j, t) H_j L_j - u \cdot c^p_j(x_j + \Delta x_j, t) H_j L_j \\ &- \lambda_p \cdot c^p_j(x_j + \Delta x_j, t) H_j L_j \Delta x_j \end{aligned} \tag{12}$$

Further arrangement results in the following form:

$$\frac{\partial c^p_j}{\partial t} = u \frac{(c^p_j(x_j + \Delta x_j, t) - c^p_j(x_j, t))}{\Delta x_j} - \lambda_p c^p_j(x_j + \Delta x_j, t) \tag{13}$$

Formalizing (11) and (13) in one continuous partial differantial equation:

$$\frac{\partial c^p_j}{\partial t} = -u \frac{\partial c^p_j}{\partial x_j} - \lambda_p c^p_j \tag{14}$$

The boundary condition of the PDE in (14):

$$-u \frac{\partial c^p_j}{\partial x_j}\bigg|_{x_j=0} = \frac{\sum_{j,i=1}^{N_{j,i}} E^p_{j,i} \cdot L_{j,i}}{H_j L_j W_j} \tag{15}$$

C. Normed form

In the followings, a normed form of the dynamic equation (14) is developed. The reasons for this step are the following:

- The distances X_j are not equal.
- For the pollutants, the maximal allowed concentrations are different. In the non-dimensional normed form, ratios to the concentration present in the regulations are used. Thus the equal weighting of the pollutants in the control design is simplified.

The following normed terms are introduced:

$$\hat{x}_j = \frac{x_j}{X_j}, \quad \hat{x}_j \in [0, 1] \forall j$$

For the concentrations we use:

$$\hat{c}^p_j = \frac{c^p_j}{c^p_{max}}, \hat{c}^p_j \in [0, 1] \forall j$$

where c^p_{max} denotes the maximal allowed concentration of pollutant p.

By substituting the above non-dimensional groups, the PDE (14) can be reformulated as:

$$\frac{\partial \hat{c}^p_j}{\partial t} = -u \frac{\partial \hat{c}^p_j}{X_j \partial \hat{x}} - \lambda_p \hat{c}^p_j \tag{16}$$

Eq. (16) describes the dynamics of the relative concentration balance of pollutant p in balance volume j.

D. Spatiotemporally discrete system

1) Spatial discretization: In the followings, the continuous PDE (16) is converted to a set of ordinary difference equations. First, by lumping the system, a set of ordinary differential equations are obtained. The lumped model of the DPS is a finite approximation in the spatial variable with the temporal variable remaining the only independent variable in the lumped system.

In our case, a single lump is applied for each balance volume. This choice can be justified by the effort for minimizing the number of state dimensions and the assumption of plug flow. As system dynamics are enhanced by the number of modeled lumps, by choosing only one lump per balance volume, a minimal extension of system dynamics can be achieved.

By using the first order approximation of the spatial differentiation applying backward difference we obtain:

$$f'(x^l_j) = \frac{\partial f(x^l_j)}{\partial x_j} \simeq \frac{f(x^l_j) - f(x^{l-1}_j)}{x^l_j - x^{l-1}_j},$$

where the spatial points x^l_j and x^{l-1}_j are the boundary points of the lth lump. Then the system dynamics of the lumped system in lump l of balance volume j are as follows:

$$\begin{aligned} \frac{d\hat{c}^{p,l}_j(t)}{dt} &= -u \frac{\hat{c}^{p,l}_j(t) - \hat{c}^{p,l-1}_j(t)}{X_j(\hat{x}^l_j - \hat{x}^{l-1}_j)} - \lambda_p \hat{c}^{p,l}_j(t) \\ &= u \frac{\hat{c}^{p,l-1}_j(t) - \hat{c}^{p,l}_j(t)}{X_j} - \lambda_p \hat{c}^{p,l}_j(t) \end{aligned} \tag{17}$$

In our case, the number of lumps $l = 1$, thus $x^l_j - x^{l-1}_j = X_j$, and $\hat{c}^{p,l-1}_j(t) = \hat{c}^{p,0}_j(t)$ is the relative concentration at the origin of balance volume j - the boundary condition of the PDE. Finally, we obtain the following model equation for the single lump:

$$\frac{d\hat{c}^p_j(t)}{dt} = u \frac{\hat{c}^{p,0}_j(t) - \hat{c}^p_j(t)}{X_j} - \lambda_p \hat{c}^p_j(t) \tag{18}$$

2) Temporal discretization: The ordinary differential equation (18) describes the dynamics of the relative concentration at the boundary of the rural area. However, the model structure of the motorway system is spatiotemporally discrete. To embed the imission dynamics into the existing system, the differential equations of the balance volumes need to be turned to difference equations. This is carried out by using finite difference approximation in the time domain. Differences are calculated based on the temporal increment of the original traffic system: sample time T_s. The difference equation of the conservation in balance volume j at discrete time step k:

$$\frac{\hat{c}_j^p(k+1)-\hat{c}_j^p(k)}{T_s} = u\frac{\hat{c}_j^{p,0}(k)-\hat{c}_j^p(k)}{X_j} - \lambda_p \hat{c}_j^p(k) \quad (19)$$

From the difference, the discrete dynamics of relative concentration in balance volume j for time step k is in the form:

$$\hat{c}_j^p(k+1) = \hat{c}_j^p(k) + T_s\left(u\frac{\hat{c}_j^{p,0}(k)-\hat{c}_j^p(k)}{X_j} - \lambda_p \hat{c}_j^p(k)\right) \quad (20)$$

where boundary condition $\hat{c}_j^{p,0}(k)$ is obtained from the following formula:

$$\hat{c}_j^{p,0}(k) = \frac{T_s \dfrac{\sum_{j,i=1}^{N_{j,i}} E_{j,i}^p(k) L_{j,i}}{H_j L_j W_j}}{c_{max}^p}, \quad (21)$$

i.e. the concentration appearing at sample step k is the temporal integral of the macroscopic emission rate within the volume $H_j \times L_j \times W_j$ during time step k.

E. Initial and boundary conditions

In order to get a well-posed initial value problem, initial values need to be specified as well for the PDE (1) in addition to the boundary conditions ((21)):

$$c_j^p(x_j, 0) = 0, \quad \forall x_j \in [0, X_j] \quad (22)$$

Thus, the discrete dynamics of relative concentrations at the boundary of the rural area are formalized.

F. Determination of the decay rate λ_p

According to the modeling assumptions, the dissolution of the pollution within balance volumes is modeled as a linear function of the concentration with the coefficient 'decay rate' λ_p, see (4). It is of key importance to provide an accurate estimation for λ_p for the exact modeling of pollution dispersion. In this section an analytic approximation is proposed.

For the approximation, the Gaussian plume dispersion model [Cooper et al. (2002)] is used. The model assumes a point source of emission from which the pollution is dispersed by diffusion in plume shape and there are no chemical or removal processes taking place. The change in concentration distribution is the result of the extension of high temperature gases. The Gaussian plume model gives a distribution function of the concentration within the plume. The dispersion is modeled by the following equation:

$$C = \frac{Q}{2\pi u \sigma_y \sigma_z} exp\left[-\frac{y^2}{2\sigma_y^2}\right]\left\{exp\left[-\frac{(H_{plume}-z)^2}{2\sigma_z^2}\right] + exp\left[-\frac{(H_{plume}+z)^2}{2\sigma_z^2}\right]\right\} \quad (23)$$

where the concentration $C(x,y,z,t)$ in units $[g/m^3]$ is obtained as a function of the pollutant emission rate Q (in $[g/s]$) and the wind speed u (in $[m/s]$). Parameters σ_y and σ_z denote the crosswind- and vertical direction standard deviations of the concentration distribution at downwind distance x respectively and H_{plume} denotes the height of the plume centerline.

The Gaussian plume model is adopted under the following assumptions:

- For all pollutants, $\lambda_p = \lambda$ is assumed equal.
- A simplified model is considered: the pollution disperses only in vertical (z) direction (see figure 3). (This condition eliminates the cross-effects among the flow channels and preserves the independence of concentration dynamics within the flow channels.)
- Mass flux decrease is described via the pollution getting outside balance volume bounds.
- The origin of the plume is at half the height of the flow channel. This assumption checks up with the condition of ideal mixing over the cross section of the flow channels.
- The pollution outside the balance volume in surface direction is reflexed from the ground and remains in the balance volume.
- Wind speed is only the function of time, and is constant along the balance volume, i.e. $u(x_j,t) = u(t)$.
- Decay rate is calculated for different stability classes (i.e. approximately constant wind speeds). Thus, reaction rate is a piecewise function of wind speed.

It is important to notice, that within the plume the conservation law is satisfied as the improper integral of the Gaussian bell function equals to 1 regardless the value of σ_z. The loss of pollution is caused by the decrease of mass within the intersection of the plume and the balance volume. The decay rate λ is derived based on the downwind change of the mass flux of pollution. Mass flux is calculated using the concentrations of the intersection of the balance volume and the plume (highlighted as red in figure 3). The mass flux

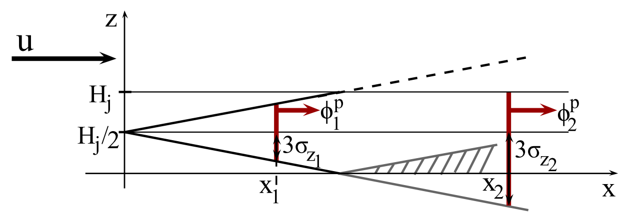

Fig. 3. Flux decrease in the simplified plume model

at point x_ℓ of the balance volume can be calculated as

$$\phi(x_\ell, t) = u(t)C(x_\ell, t) \quad (24)$$

The relative change in mass flux between point x_1 and x_2 along the balance volume, based on the Gaussian plume approach:

$$\frac{\phi(x_1)-\phi(x_2)}{\phi(x_1)} = \frac{1}{\sigma_z(x_2)2\pi}\int_{-\infty}^{-H_j/2} exp\frac{-z^2}{2\sigma_z(x_2)^2}dz \tag{25}$$

Using the assumption of small variations in wind speed, the propagation time from point x_1 to point x_2 can be calculated based on the distance of the points $(x_2 - x_1)$:

$$T_{wind12} = \frac{x_2 - x_1}{u} \tag{26}$$

The decay rate λ can be stated as the relative change in mass flux during the propagation time (using (25) and (26)):

$$\lambda = \frac{(\phi(x_1)-\phi(x_2))/\phi(x_1)}{T_{wind12}} = \frac{\frac{1}{\sigma_z(x_2)2\pi}\int_{-\infty}^{-H_j/2} exp\frac{-z^2}{2\sigma_z(x_2)^2}dz}{x_2 - x_1}u \tag{27}$$

For a particular flow channel j, $x_1 = 0$ and $x_2 = X_j$ is substituted to (27). Then the decay rate of flow channel j is a function of its length and the wind speed:

$$\lambda = \frac{u}{X_j\sigma_z(X_j)2\pi}\int_{-\infty}^{-H_j/2} exp\frac{-z^2}{2\sigma_z(X_j)^2}dz \tag{28}$$

Finally, the parameter σ_z is considered as a function of the centerline distance from the source (x) in the form [EPA ISC3 guide]:

$$\sigma_z = ax^b \tag{29}$$

Based on the wind speed, different dispersion stability classes are specified and for them, a and b parameter values are stated (see table I).

Stability class	Wind speed [m/s]	a	b
A	0-2	158.08	1.0542
B	2-4	109.3	1.0371
C	4-6	90.673	0.93198
D	6-8	61.141	0.91465

TABLE I

PARAMETER VALUES OF EQUATION (29). SOURCE: [EPA ISC3 GUIDE]

Note that the parameter σ_z and thus λ is a bivariate function of wind speed u and downwind distance X_j, piecewise in u and continuous in X_j.

By using the formula of σ_z (29), k can be written as follows:

$$\lambda(X_j, u) = \frac{u}{X_j a(u) X_j^{b(u)} 2\pi}\int_{-\infty}^{-H_j/2} exp\frac{-z^2}{2a(u)X_j^{2b(u)}}dz \tag{30}$$

For the bivariate function plot of λ see figure 4.

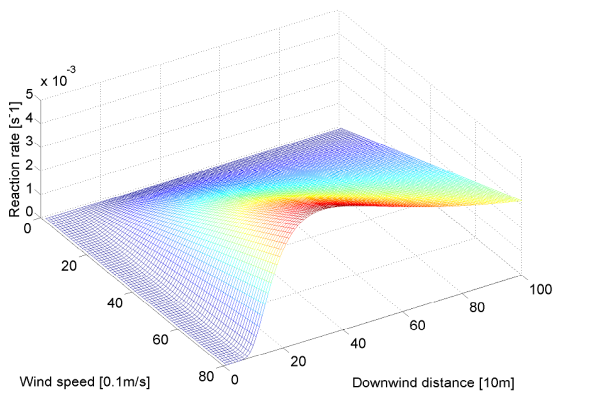

Fig. 4. Reaction rate as a function of wind speed and downwind distance

Remark: Using the formula stated in (30) and substituting it to (20) a nonlinear process model is obtained, in which the relative concentration of pollutant p in flow channel j ($\hat{c}_j^p$) is the state variable, wind speed u is the disturbance variable, and boundary condition $\hat{c}_j^{p,0}$ is computed from the macroscopic traffic-emission model framework. Using these assumptions, the dynamics of imission can be described applying the existing traffic-emission system model detailed in [Csikós et al. (2012)].

G. On the consistency of the solution

As the proposed process model describes the DPS with finite difference approximations both in space and time, the Courant-Friedrichs-Lewy condition [Courant et al. (1928)] needs to be analyzed for the consistency. Considering, that a maximum of $u_{max} = 8m/s = 29km/h$ wind speed is assumed (Pasquill-Gifford stability class D - during daytime, with moderate solar radiation), the CFL condition reads:

$$u_{max} \leq \frac{X_j}{T_s}, \forall j$$

Thus, the shortest distance for X_j can be calculated as follows:

$$u_{max} \cdot T_s \leq X_j$$

In our case, $X_j \geq 80.56m$ - this is the minimal distance of rural areas from the motorway for which the lumping and sampling assumptions respect the CFL condition.

III. SENSITIVITY ANALYSIS

In this section, the effect of the motorway traffic control inputs (i.e. dynamic speed limits and ramp metering) are analyzed on the pollutant concentrations using a simplified version of dynamic sensitity analysis. The boundary parameters of the analysis are provided by the second-order traffic model and the motorway traffic emission framework detailed in [Csikós et al. (2012)]. The case study is utilized to obtain preliminary information on controler structure design, i.e. to test the efficiency of the control inputs. Low concentration values are expected to be present in case of low main lane densities, which can be provided by low ramp inputs or high dynamic speed limits in case of no congestion. The sensitivity analysis also serves to quantify the effect of control measures on the states.

The analysis was carried out for a single, 1 km long four-lane motorway stretch with all four lanes heading to the same direction, and the effect on a rural area 1 km far from the road for a 1.5 hour long period was investigated. The parameters of the flow channel were $X_j = 1000m$; $H_j = 30m$; $L_j = 1000m$.

A rush hour situation was modeled without any congestion. Traffic variables without control were chosen to correspond to an average flow on the main lane and the ramp: q=1800 veh/h/lane and r=400 veh/h/lane. An average traffic density with ρ=25 veh/km/lane was used with traffic mean speed v=90 km/h. Throughout the simulation, constant wind direction and speed (4 m/s) was present, perpendicular to the road.

In case of ramp metering (see figure 5), the control input was changed in 100 veh/h steps. The simulation clearly shows, that the lower ramp input is set, the lower concentration can be maintained. Lower boundary conditions of low main lane densities explain this phenomenon. It is clear, that for the minimization of concentrations, the minimal ramp input is required. By the total withdrawal of the ramp traffic (400 veh/h, which is 1/4 of the total traffic), almost 25% decrease of the concentration can be achieved. Thus, the reduction of ramp input can result in proportionally lower concentrations values, however, with transients. The length of the transients (appearing at input switches around 1000 s, 2000 s, etc) depends on the wind speed and needs further analysis. The results of variable speed limit analysis (shown

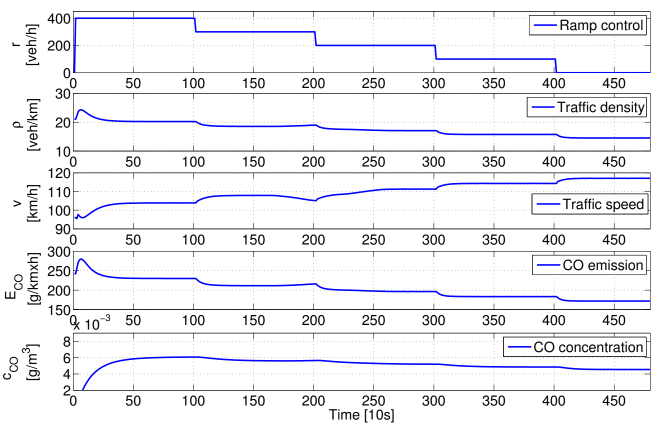

Fig. 5. Simulation of different ramp inputs

in figure 6) is similar to those of the ramp metering: best concentration levels can be achieved by keeping the main lane density as low as possible. However, this needs an opposite regulation to that of the ramp metering by keeping speed limits high: as the decreasing of speed limits increases traffic density, the no speed limit case provides best concentration values. This means that while the traffic stabilizing intervention of ramp metering reduces concentrations as well, the traffic stabilizing effect of variable speed limits entails extreme concentration increases (almost 300% based on the simulation results).

The sensitivity analysis thus agrees with the engineering expectations. In addition, the case study shows that it is hard to handle imission as a control objective in a control system structure as traffic stabilizing interventions do not necessarily improve concentration levels. This observation suggests not to incorporate the minimization of the pollutant concentrations into the control goal, but keeping it instead under constraints, specified by the international legislation limits. Before finalizing the control structure, further sensitivity analyses need to be performed in different traffic situations.

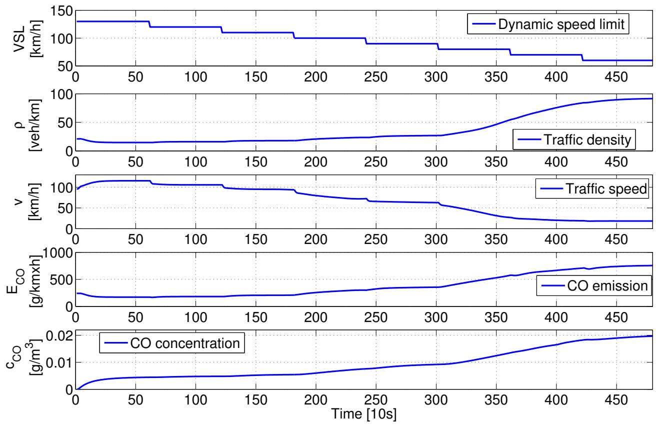

Fig. 6. Simulation of different VSL inputs

IV. Conclusion

In this work a novel dispersion model is introduced for motorway traffic emissions. The process model is developed for a distributed parameter system, and is derived based on the mass conservation laws within balance volumes, specified by the wind direction. In the balance volumes, plug flow is considered, and for the absorption of pollution a simplified version of the Gaussian plume model is used. For the boundary conditions of the model, the output of the macroscopic emission model, introduced in [Csikós et al. (2012)] is used.

On the proposed model a sensitivity analysis is performed which justified the idea of considering the states of concentration dynamics to be kept under constraints in the future control framework. Future work involves the control of the joint system of traffic and traffic emission dispersion based on the concepts of parameter-dependent system formulation and model predictive control design.

References

[Benson (1979)] P. E. Benson. CALINE3 - A Versatile Dispersion Model for Predicting Air Pollutant Levels Near Highways and Arterial Streets. Office of Transportation Laboratory California Department of Transportation, Research Report No. FHWA/CA/TL-79/23 Nov. 1979.

[Buckland et al. (1999)] A. T. Buckland and D. R. Middleton. Nomograms for calculating pollution within street canyons. Atmospheric Environment, 33(7):1017 - 1036, March 1999.

[Cooper et al. (2002)] C. D. Cooper and F. C. Alley, Air Pollution Control: A Design Approach, Waveland Press Inc., 2002.

[Courant et al. (1928)] R. Courant, K. Friedrichs and H. Lewy. Uber die partiellen Differenzengleichungen der mathematischen Physik. Mathematische Annalen, 100(1): 32-74, 1928.

[Csikós et al. (2012)] A. Csikós, I. Varga: Real-time Modeling and Control Objective Analysis of Motorway Emissions. Procedia - Social and Behavioral Sciences 54: pp. 1027-1036. (2012).

[EPA ISC3 guide] D. B. Schwede: User's guide for the industrial source complex (ISC3) dispersion models for use in the multimedia, multipathway and multireceptor risk assessment (3MRA) for HWIR99. Volume 2: Description of model algorithms. U.S. Environmental Protection Agency, Office of Solid Waste, June 30, 1999.

[Gosman (1999)] A. D. Gosman. Developments in CFD for industrial and environmental applications in wind engineering. Journal of Wind Engineering and Industrial Aerodynamics, 81 (1-3):21-39, May 1999.

[Hangos and Cameron (2001)] K. M. Hangos, I. T. Cameron: Process Modelling and Model Analysis. Academic Press, 2001. ISBN-10: 0121569314.

[Johnson et al. (1973)] W. B. Johnson, F. L. Ludwig, W. F. Dabberdt, and R. J. Allen. An urban diffusion simulation model for carbon monoxide. Air Pollution Control Association, 23(6): 490-498, June 1973.

[Tominaga et al. (2011)] Y. Tominaga and T. Stathopoulos. CFD modeling of pollution dispersion in a street canyon: Comparison between LES and RANS. Journal of Wind Engineering and Industrial Aerodynamics, Volume 99, Issue 4, April 2011, Pages 340-348.

[Vardoulakis et al. (2003)] S. Vardoulakis, B. E. A. Fisher, K. Pericleous, and N. G. Flesca. Modelling air quality in street canyons: a review. Atmospheric Environment, 37(2): 155-182, 2003.

[Zegeye et al. (2011)] S. K. Zegeye, B. De Schutter, J. Hellendoorn, and E. A. Breunesse. Nonlinear MPC for the improvement of dispersion of freeway traffic emissions. Proceedings of the 18th IFAC World Congress, Milan, Italy, August 28 - September 2, 2011.